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TABLES OF MINOR PLANETS DISCOVERED BY JAMES C. WATSON

PART II

ON v. ZEIPEL'S THEORY OF THE PERTURBATIONS OF THE MINOR PLANETS OF THE HECUBA GROUP

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THIRD MEMOIR.

TABLES OF MINOR PLANETS DISCOVERED BY JAMES C. WATSON.

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ON v. ZEIPEL'S THEORY OF THE PERTURBATIONS OF THE MINOR PLANETS OF THE HECUBA GROUP.

BY

ARMIN O. LEUSCHNER, ANNA ESTELLE GLANCY, AND SOPHIA H. LEVY.

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CONTENTS.

	Page.
Preface	7
Introduction	9
I. Formulæ and tables for the *Hecuba* group, according to the theory of Bohlin–v. Zeipel, and an example of their use	10
Determination of constant elements and of perturbations of the mean anomaly	10
Perturbations of the radius vector	20
Perturbations of the third coordinate	21
Check computation	22
Computation of the perturbations for the time t	22
Comparison of the revised with v. Zeipel's original tables	27
Table A	28
Table B	30
Table C	31
Table D	34
Table E_1	35
Table E_2	35
Table F	36
Table G	38
II. Tables for the determination of the perturbations of the *Hecuba* group of minor planets	41
Development of the differential equations for W and for the third coordinate	41
Integration of the differential equation for W	78
Comparison of tables	120
Perturbations of the mean anomaly	121
Comparison of tables	134
Perturbations of the radius vector	137
Perturbations of the third coordinate	140
Comparison of tables	146
Constants of integration in $n\delta z$ and ν	146
Comparison of tables	155
Errata in "Angenäherte Jupiter-Störungen für die *Hecuba*-Gruppe," H. v. Zeipel	155
Errata in "Sur le Développement des Perturbations Planétaires," § 1–7 and Tables I–XX, Karl Bohlin	157

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PREFACE.

Part I of "Tables of Minor Planets Discovered by James C. Watson," containing tables for 12 of the 22 Watson planets, was published in 1910 in the Memoirs of the National Academy of Sciences, Volume X, Seventh Memoir, with a preface by Simon Newcomb, in which he gives an account of the early history of the investigations of the perturbations of the Watson planets under the auspices of the Board of Trustees of the Watson Fund.

In the introduction to Part I [1] reference is made to the Watson planets of the *Hecuba* group, for which it was found necessary to construct special tables on the plan of Bohlin's tables for the group 1/3. A comparison of these tables with similar tables by v. Zeipel remained to be made before applying either of them to the development of perturbations of planets of the *Hecuba* group. This comparison was completed in 1913 with the assistance of Miss A. Estelle Glancy and Miss Sophia H. Levy, with the results set forth in the following pages.

Publication of these results was delayed, partly because it seemed desirable to verify the tables by application to a number of planets and partly on account of interruptions caused in recent years by war conditions. Miss Glancy, in particular, had undertaken to test the accuracy of our tables, which we had applied to v. Zeipel's example, (10) *Hygiea*, by further investigations on this example after joining the Observatorio Nacional at Córdoba in 1913. This test has now been completed with highly satisfactory results. The tables have also been successfully applied to the Watson planets of the *Hecuba* group, including (175) *Andromache*, which, on account of unusually large perturbations and other unfavorable conditions, forms so far the most striking example of the applicability of the Bohlin-v. Zeipel method and of our revised tables for the *Hecuba* group.

The plan of work included conferences, in which Miss Glancy and Miss Levy took a leading part, for the discussion of the Bohlin-v. Zeipel method, involving verification of all mathematical developments and formulation of plans for the construction of tables, and, after the appearance of v. Zeipel's tables, for the comparison of v. Zeipel's original, and our revised tables. The numerical work was carried out by Miss Glancy and Miss Levy, who have also contributed very largely to the theoretical part of the work, and have prepared the principal details of the manuscript.

To avoid confusion v. Zeipel's notation and method of procedure have been followed throughout in completing our tables for the *Hecuba* group, which were well under way when v. Zeipel's memoir appeared.

To aid computers in the use of the formulæ and of the revised tables, Miss Glancy has prepared detailed directions illustrated by an application to (10) *Hygiea*, the example first chosen by v. Zeipel. These are contained in the first section of the present memoir.

Miss Glancy's contributions to this investigation and her work on (*10*) *Hygiea* were accepted by the University of California in partial fulfillment of the requirements for the degree of doctor of philosophy.

Miss Levy's contributions and her work on (*175*) *Andromache* were similarly accepted for the same degree.

It seems highly desirable to make the tables for the development of the perturbations of minor planets of the *Hecuba* group at once available to astronomers. They are therefore published herewith, in advance of the perturbations and tables of the remaining Watson planets, as Part II of "Tables of Minor Planets Discovered by James C. Watson." One or two parts, which are to follow, will contain all the numerical results for the perturbations and tables of Watson planets not published in Part I (1910).

This memoir is presented in two sections. The first section, entitled "Formulæ and Tables for the *Hecuba* Group, according to the Theory of Bohlin-v. Zeipel, and an Example of their

[1] Pp. 200–201.

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Use," contains a collection of the formulæ to be used for any planet of the *Hecuba* group, the general tables of the perturbations which must be employed, and a more complete application of the formulæ and the revised tables to the planet (10) *Hygiea*, than v. Zeipel gives. The second and more extensive section, entitled "Tables for the Determination of the Perturbations of the *Hecuba* Group of Minor Planets," concerns the construction of the tables and their discussion with reference to the corresponding tables by v. Zeipel. It forms the preliminary part of the investigation but is presented last as supplementary to the final results given in the first section.

In the second section the tabular values which differ from the corresponding numbers in v. Zeipel's tables are placed in brackets. The general Tables XXXV, XXXVIII, XLIII, LIV, LVI, LVII, LVI, LVII, of the second section, which, in order, are required to compute the perturbations of any planet of the *Hecuba* group, are repeated without brackets at the end of the first section as Tables A, B, C, D, E_1, E_2, F, G, so that the first section is complete in itself for use in developing the perturbations of any planet of this group without the necessity of reference to the second section.

A general account of the investigations of the perturbations of the Watson planets was presented to the Academy on April 16, 1916, and is published in the "Proceedings of the National Academy of Sciences," Volume 4, No. 12, March, 1919.

ARMIN O LEUSCHNER.

WASHINGTON, D. C., *1918, December.*

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TABLES OF MINOR PLANETS DISCOVERED BY JAMES C. WATSON.

By Armin O. Leuschner, Anna Estelle Glancy, and Sophia H. Levy.

INTRODUCTION.

Those planets whose mean daily motions are approximately 600″ are classed with the planet *Hecuba*, or, in the group for which

$$\mu = \frac{n'}{n} = \tfrac{1}{2}(1-w)$$

where n' and n are the mean daily motions of Jupiter and the planet, respectively, and w is a small quantity.

Among the minor planets discovered by James C. Watson there are several of this type. In the course of the general program of determining the perturbations of the Watson asteroids, there arose the necessity of computing special tables for the *Hecuba* group in preparation for the application of Bohlin's method to individual planets.

General tables for the group ½ were in the process of construction, under the direction of Professor Leuschner,[1] according to the method of Bohlin,[2] when tables for this group were published by H. v. Zeipel.[3] The computers, Dr. Sidney D. Townley and Miss Adelaide M. Hobe, made a comparison of their tables with those of v. Zeipel and found certain discrepancies Because of this fact the completion of the tables for the *Hecuba* group was deferred. These discrepancies have been explained, as a result of a careful investigation, and the tables have been completed by Miss A. Estelle Glancy and Miss Sophia H. Levy, under the direction of Professor Leuschner.

In the completion of the tables, v. Zeipel's method and order of procedure have generally been followed. There are numerous discrepancies between our tables and v. Zeipel's. As far as possible, with the aid of the original manuscript, kindly forwarded by the author, we have traced the source of these disagreements. In some of the more complicated functions it was not possible to do so, and these discrepancies remain unexplained. Our own results, however, are substantiated by the employment of independent developments. Further, where we found terms omitted which were of the same order as those which were included, we frequently extended the tables. In this connection, it is pertinent to remark that it is very difficult to set up a consistent criterion for the omission of terms. With the exception of a few scattered negligible terms, our tables are published in full. They contain terms which may ordinarily be omitted, yet their numerical magnitudes depend upon the elements of the particular planet under consideration, and their use is left to the computer's judgment. Many of them are incomplete, i. e., the tabulated coefficients do not necessarily include all the terms of a given degree in the eccentricities or mutual inclination or of the small quantity w, which depends upon the difference between the planet's and twice *Jupiter's* mean motion. In other words, the coefficients may not contain all the terms of a given degree having the factors

$$w^s,\ \eta^p,\ \eta'^q,\ j^{2t}$$

which are defined on page 12. But, assuming certain numerical limits for the fundamental auxiliary functions, the coefficients are of this magnitude. The value of the additional terms will be shown best in an application of our tables to the same planet for which v. Zeipel computed the perturbations.

Unless stated otherwise, the references to Bohlin refer to the French edition and are designated by *B*. References to v. Zeipel are designated by *Z*.

[1] Memoirs of the National Academy of Sciences, Vol. X, Seventh Memoir, p. 200.

[2] Formeln und Tafeln zur gruppenweisen Berechnung der allgemeinen Störungen benachbarter Planeten (Upsala, 1896).
Sur le Développement des Perturbations Planétaires (Stockholm, 1902).

[3] Angenäherte Jupiterstörungen für die Hecuba-Gruppe (St. Pétersbourg, 1902).

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I. FORMULAE AND TABLES FOR THE HECUBA GROUP, ACCORDING TO THE THEORY OF BOHLIN-v. ZEIPEL, AND AN EXAMPLE OF THEIR USE.

DETERMINATION OF CONSTANT ELEMENTS AND OF PERTURBATIONS OF THE MEAN ANOMALY.

The planet (10) *Hygiea* was selected by v. Zeipel as an example of the use of his tables for the group $\frac{1}{2}$. We have used it as a preliminary example for the application of our own tables, so as to provide further comparison of our tables with those of v. Zeipel.

This example is presented with the direct purpose of meeting the needs of the computer. For this reason, no attempt is made to explain the significance of the functions involved, yet their use will be less mechanical, if, in a general way, some of the essential principles underlying their development are understood. The theory of v. Zeipel is taken up in the second section of this memoir.

The method proposed by v. Zeipel is a practical adaptation of Bohlin's method of computing the perturbations by *Jupiter* upon planets whose mean motions bear nearly commensurable ratios to that of *Jupiter*. In particular, the formulae are derived for the planets of the *Hecuba* group. Tracing the history of this method one step further back, Bohlin's method is a modification of the theory of Hansen for the indeterminate case of nearly commensurable mean motions. Or, concisely, in v. Zeipel's own words, "Die benutzte Methode kann einfach dadurch charakterisirt werden, dass die Differentialgleichungen von Hansen mittels des Integrationsverfahrens des Herrn K. Bohlin gelöst worden sind."[1]

Certain principles of Hansen are fundamental to an understanding of some of the important equations. Briefly, the perturbations are reckoned in the plane of the orbit and perpendicular to it. In the plane of the orbit $n\delta z$ signifies the displacement in the planet's mean anomaly (δz is the perturbation in the time); ν gives the disturbed radius vector through the relation

$$r = \bar{r}(1+\nu)$$

and the displacement in the third coordinate is denoted by $\frac{u}{\cos i}$. With Hansen's choice of ideal coordinates, the fundamental analytical relations are:

$$\begin{aligned} &\epsilon - e\sin\epsilon = nt + c + n\delta z \\ &\bar{r}\cos f = a\,(\cos\epsilon - e) \\ &\bar{r}\sin f = a\sqrt{1-e^2}\sin\epsilon \\ &r = \bar{r}(1+\nu) \end{aligned} \tag{1}$$

$$\begin{aligned} &d\beta = \frac{u}{\cos i}\, a \sin 1'' \\ &\Delta x = d\beta\cos a \\ &\Delta y = d\beta\cos b \\ &\Delta z = d\beta\cos c \end{aligned} \tag{2}$$

$$\begin{aligned} x &= r\sin a\sin(A'+f) + \Delta x \\ y &= r\sin b\sin(B'+f) + \Delta y \\ z &= r\sin c\sin(C'+f) + \Delta z \end{aligned} \tag{3}$$

where $\epsilon, f, \bar{r}$ are fictitiously disturbed coordinates, which, in connection with constant elements and the perturbations $n\delta z$, ν, and $\frac{u}{\cos i}$ give the true position of the body. A', B', C', $\sin a$, $\sin b$, $\sin c$ are the constants for the equator. The notation for the eccentric anomaly and the true anomaly is v. Zeipel's; in Hansen's notation they would be written $\bar{\epsilon}, \bar{f}$.

[1] Angenäherte Jupiterstörungen für die Hecuba-Gruppe, p. I.

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When *Jupiter's* mean motion and that of the planet are nearly commensurable, the integration of Hansen's differential equations becomes impracticable through the presence of large integrating factors. The integrals are of the form:

$$\frac{1}{n^2\left(i-\frac{i'n'}{n}\right)^2}\begin{matrix}\sin\\ \cos\end{matrix}\left\{(in-i'n')t\right\}\begin{matrix}-\infty<i<+\infty\\ 0<i'<+\infty\end{matrix}$$

For the *Hecuba* group the mean motion is approximately twice the mean motion of *Jupiter*. Hence, for exact commensurability,

$$\mu=\frac{n'}{n}=\frac{1}{2};\quad \frac{1}{n^2\left(i-\frac{i'n'}{n}\right)^2}=\infty\,;\, i'=2,\, i=1.$$

By introducing the exponential in place of the sine and cosine, the indeterminateness can be removed, for if $in-i'n'=0$, then $e^{\sqrt{-1}(in-i'n')t}=1$. This is one of Bohlin's modifications.

For any given planet the ratio is not exactly commensurable, and the developments are originally made for the case of exact commensurability. They are then expressed, for a given case, by Taylor's series in ascending powers of a small quantity w, which depends upon the difference between the real ratio and exact commensurability. In addition to positive powers of w there will occur negative powers. They are due to the following causes. An argument θ is introduced (see p. 13), from which the mean anomaly of *Jupiter* is eliminated through the introduction of w. It is a necessary consequence of the form of the partial differential equations in which $\frac{d\theta}{dt}$ appears, that the integration of first-order terms shall contain w^{-1} and that higher order terms shall contain other negative powers. Hence the integrals are series in both positive and negative powers of w.

In distinction to the method of Hansen the elements appear explicitly in the arguments or as factors in the terms of the series.

An important feature of v. Zeipel's theory is his treatment of the constants of integration. Since the method is essentially Hansen's, the constants of integration must be determined consistently with that method. Given osculating elements, the constants of integration are determined by the condition that, at the date of osculation, $(t=0)$, the perturbations and their velocities shall be equal to zero.

v. Zeipel adopts osculating elements as his initial elements. With these elements and the perturbations and their velocities at the date of osculation, he computes elements, designated by the subscript unity, in which the constants of integration are absorbed. They are analogous to Hansen's constant elements, i. e., the fundamental equations of Hansen are valid.

Our transformations of the elements differ from v. Zeipel's in two respects. First, the constants in $\frac{u}{\cos i}$, and in its velocity have not been introduced into the elements i, Ω, but are treated in the usual Hansen manner. Second, v. Zeipel introduces certain terms in the perturbations which have the same period as the planet (argument ϵ), into the elements to form mean elements. This has not been done.

The general tables, XXXV, XXXVIII, XLIII, LIV, LVI, LVII, LVI, LVII, which are required in computing the perturbations, are given at the conclusion of the formulae. The formulae for any planet of the group $\frac{1}{2}$ are given completely, and they are supplemented by numerical values for the planet (*10*) *Hygiea*.

The references to v. Zeipel's paper are indicated briefly by *Z*, followed by the number of the page.

The osculating elements of the planet are taken from *Z* 139; the elements for *Jupiter* are taken from Astronomical Papers of the United States Nautical Almanac Office, Vol. VII, p. 23.

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(10) *Hygiea.*			*Jupiter.*		
Epoch, 1851, Sept. 17.0, Ber. M. T.			*Epoch, 1851, Sept. 17.0, Ber. M. T.*		
OSCULATING ELEMENTS.			MEAN ELEMENTS.		
$n_0 = 634''.850$	$=$	$0°.176347$	$n' = 299''.1284$	$=$	$0°.0830912$
$\varphi_0 = 5°46'.28$	$=$	$5°.7713$	$\varphi' = 2°45'.95$	$=$	$2°.7658$
$\pi_0 = 227\ 46.61$	$=$	227.7768	$\pi' = 11\ 54.45$	$=$	11.9075
$\Omega_0 = 287\ 37.19$	$=$	287.6198	$\Omega' = 98\ 55.97$	$=$	98.9328
$\omega_0 = 300\ 9.42$	$=$	300.1570	$\omega' = 272\ 58.48$	$=$	272.9747
$i_0 = 3\ 47.14$	$=$	3.7857	$i' = 1\ 18.70$	$=$	1.3117
$c_0 = 126\ 59.81$	$=$	126.9968	$c' = 199\ 57.70$	$=$	199.9617

Mean equinox and ecliptic, 1850.0.
Epoch, 1851, Sept. 16.96279 Gr. M. T.

The following notes in regard to these elements are of importance:

Jupiter's elements were first taken from *Z* 139. They were used only in the equations numbered (1). In these equations either set of elements may be used with sufficient accuracy. In fact, it is not necessary to know *Jupiter's* elements as accurately as those of the planet, for they appear only in the arguments of the perturbations. We have adopted Hill's values of the elements and Newcomb's value of the mass of *Jupiter*. The tables of the perturbations are based, however, on Bessel's value for m'. To correct the perturbations for the improved value, it is only necessary to multiply them by 1.0005, and this is done in the formulæ which follow.

The original epoch of Jupiter's elements was 1850.0 Gr. M. T. It was changed by the formula

$$c' = 148°\ 1'.97 + n't \tag{4}$$

The elements of *Hygiea* are very good osculating elements, computed by Zech. They include perturbations by *Jupiter*, *Saturn*, and *Mars* and are based on five oppositions. The reference for these elements is doubtful, for in Astronomische Nachrichten **39**, 347, the elements given by Zech are not identically the same, although the differences are very small. The values given by v. Zeipel were probably taken from Zech's manuscript, to which he had access. They may, therefore, contain some later corrections.

The auxiliary quantities Ψ, Φ, J are first computed by the formulæ:

$$\begin{aligned}
\sin\tfrac{1}{2}J\sin\tfrac{1}{2}(\Psi+\Phi) &= \sin\tfrac{1}{2}(\Omega_0-\Omega')\sin\tfrac{1}{2}(i_0+i') \\
\sin\tfrac{1}{2}J\cos\tfrac{1}{2}(\Psi+\Phi) &= \cos\tfrac{1}{2}(\Omega_0-\Omega')\sin\tfrac{1}{2}(i_0-i') \\
\cos\tfrac{1}{2}J\sin\tfrac{1}{2}(\Psi-\Phi) &= \sin\tfrac{1}{2}(\Omega_0-\Omega')\cos\tfrac{1}{2}(i_0+i') \\
\cos\tfrac{1}{2}J\cos\tfrac{1}{2}(\Psi-\Phi) &= \cos\tfrac{1}{2}(\Omega_0-\Omega')\cos\tfrac{1}{2}(i_0-i')
\end{aligned} \tag{5}$$

$$\text{Check: } \frac{\sin\Psi}{\sin i_0} = \frac{\sin\Phi}{\sin i'} = \frac{\sin(\Omega_0-\Omega')}{\sin J}$$

Then follow

$$\Pi_0 = \pi_0 - \Omega_0 - \Phi;\ \eta_0 = \frac{e_0}{2}; \qquad \Pi' = \pi' - \Omega' - \Psi;\ \eta' = \frac{e'}{2}.$$

$$j^2 = \sin^2\frac{J}{2}\cos^2\frac{1}{2}\varphi_0\cos^2\frac{1}{2}\varphi';\ \iota = \sin J\cos^2\tfrac{1}{2},\varphi \tag{6}$$

$$\Delta_0 = \Pi_0 - \Pi';\ \Sigma_0 = \Pi_0 + \Pi'$$

$$w_0 = \frac{n_0 - 2n'}{n_0}$$

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and the arguments for the date of osculation:

$$\theta_0 = \frac{1}{2}c_0 - g' \text{ where } g' = c' + [n'\delta z']; \ [n'\delta z'] = (9.5215) \sin 115°.326, \tag{7}$$

where the coefficient in parentheses is logarithmic in degrees.

$$\epsilon_0 - e_0 \sin \epsilon_0 = c_0; \ \Gamma = \frac{1}{2}\epsilon_0 + \theta_0 + \Delta_0 \tag{8}$$

---

(10) *Hygiea.*

$\Psi = 186°.4792$	$\Pi_0 = 302°.3984$	$\Delta_0 = 215°.8679$	
$\Phi = 357.7586$	$\Pi' = 86.5305$	$\Sigma_0 = 28.9289$	
$J = 5.0856$			
$\log \eta_0 = 8.70139$	$\log j^2 = 7.29275$	$\theta_0 = 223.2334$	(a)
$\log \eta' = 8.38238$	$\log \iota = 8.94739$	$\frac{1}{2}c_0 - (c' + [n'\delta z']) = 223.2445$	(b)
$\log w_0 = 8.76072$		$\frac{1}{2}c_0 - c' = 223.5448$	(c)

See footnote.[1]

$$\epsilon_0 = 131°.3236; \ \Gamma = 145°.0746$$

With these initial quantities all the arguments and factors in Table LVI or F are computed. The required function, $w - w_0$, is computed by successive approximations, the first approximation being

$$w = w_0$$

In the first trial the smallest terms and the last digit may be omitted; the second trial should be accurate; a third trial, if necessary, will require only corrections to the largest terms.

The mean motion n is then given by

$$n = \frac{2n'}{1 - w} \tag{9}$$

---

(10) *Hygiea.*

The three successive trials for w give

$w - w_0$	
+0.00388	$w = +0.061208$
+0.003541	$\log w = 8.78681$
+0.003568	$n = 637".2633$

Designating by C and S series to be computed next from Table LVII or G, it is evident by inspection of Table LVII that

$$C \cos \phi + S \sin \phi = \Sigma c \cos (\phi + X) = \Sigma c \cos X \cos \phi - \Sigma c \sin X \sin \phi$$

from which

$$C = \Sigma c \cos X; \ S = -\Sigma c \sin X \tag{10}$$

---

[1] Three numerical values for the argument θ_0 are given. According to the theory (see footnote, Part 2, p. 147), (a) is rigid; (b) is rigid within the accuracy of the developments by v. Zeipel; (c) is an approximation which v. Zeipel used and which is used here. The value (b) is preferable.

In equation (b), $[n'\delta z'] = +0".3114$ and is the complete perturbation of Jupiter by Saturn taken from Hill; in all other parts of the computation $n'\delta z']$ is only the long period term used by v. Zeipel.

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To make the order of computation evident, the successive steps for a group of terms for *Hygiea* are given.

X	c	X	$-S$	$+C$
	″	°	″	″
$-5\Gamma+6\theta_0+6\Delta_0$	+ 0.4	111.10	+ 0.4	− 0.1
$-4\Gamma+6\theta_0+6\Delta_0$	+ 1.9	256.18	− 1.8	− 0.5
$-3\Gamma+6\theta_0+6\Delta_0$	+ 4.6	41.25	+ 3.0	+ 3.5
$-2\Gamma+6\theta_0+6\Delta_0$	+ 6.8	186.33	− 0.7	− 6.8
$-\Gamma+6\theta_0+6\Delta_0$	+21.5	331.40	−10.3	+18.8
$6\theta_0+6\Delta_0$	−63.0	116.48	−56.4	+28.1
$\Gamma+6\theta_0+6\Delta_0$	− 4.0	261.55	+ 4.0	+ 0.6
$2\Gamma+6\theta_0+6\Delta_0$	− 3.1	46.63	− 2.2	− 2.1
$3\Gamma+6\theta_0+6\Delta_0$	− 1.9	191.70	+ 0.4	+ 1.9

The second column contains the sum of the numerical coefficients multiplied by their respective factors $w^s\eta^p\eta'^q j^{2t}$. The columns $-S$ and $+C$ contain the required terms from this group in the table. They can be computed at the same time if a traverse table is used.[1]

From S and C the elements π and φ can be computed by the formulæ:

$$\begin{aligned} e\sin(\pi-\pi_0) &= S\cos\varphi_0 \\ e\cos(\pi-\pi_0) &= e_0 + C\cos^2\varphi_0 \\ e &= \sin\varphi \end{aligned} \qquad (11)$$

In place of η_0, Δ_0, Σ_0 the following are used hereafter:

$$\eta=\frac{e}{2} \qquad \Delta=\Delta_0+(\pi-\pi_0) \qquad \Sigma=\Sigma_0+(\pi-\pi_0) \qquad (12)$$

(*10*) *Hygiea.*

$S=+1215''.0$	$\pi-\pi_0=+3°.0203$	$\log\eta=8.74517$
$C=+2191.1$	$\tau=230.7971$	$\log\sin\varphi=9.04620$
$\Delta=218°.8882$	$\Sigma=31°.9492$	$\varphi=6°.3858$

There remains one more element to determine, namely, c, but the computation must be deferred until we know the perturbation $n\delta z$ at $t=0$. (See equation (1), page 10 or page 16.)

The fictitiously disturbed eccentric anomaly at the time $t=0$ denoted by ε_1, is determined through the relations:

$$\varepsilon_0-e_0\sin\varepsilon_0=c_0 \qquad (13)$$

where ε_0 is calculated with the aid of Astrand's table[2];

$$tg\tfrac{1}{2}(v_0-\pi_0)=\sqrt{\frac{1+e_0}{1-e_0}}\,tg\tfrac{1}{2}\varepsilon_0;\quad tg\tfrac{1}{2}\varepsilon_1=\sqrt{\frac{1-e}{1+e}}\,tg\tfrac{1}{2}(v_0-\pi) \qquad (14)$$

(*10*) *Hygiea.*

$\varepsilon_0=131°.3236$ $\quad v_0=3°.2968$ $\quad \varepsilon_1=127°.6064$ $\quad \varepsilon_1-e\sin\varepsilon_1=122°.5578$

The perturbation $n\delta z$ is computed as follows:

The function $1+\Phi(\vartheta)$ is computed from Table XXXVIII or B. The coefficients are multiplied by their respective factors, the trigonometric functions of the arguments are expanded, and the coefficients of $\begin{matrix}\cos\\ \sin\end{matrix} j\vartheta$ are collected, (j is the numerical coefficient of ϑ).

[1] Memoirs of the National Academy of Sciences, Vol. X, Seventh Memoir, p. 218.

[2] Hülfstafeln zur leichten und genauen Auflösung des Kepler'schen Problems (Leipzig, 1890).

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(10) Hygiea.

$$1+\phi(\vartheta)=(1-0.008064)\{1-0.055937\sin 2\vartheta+0.017170\cos 2\vartheta$$
$$+0.016057\sin 4\vartheta+0.012244\cos 4\vartheta$$
$$+0.000905\sin 6\vartheta-0.005081\cos 6\vartheta+\ldots\ldots$$
$$+(\vartheta-\vartheta_0)(+0.000007-0.000490\sin 2\vartheta-0.001266\cos 2\vartheta$$
$$-0.000361\sin 4\vartheta+0.000409\cos 4\vartheta+\ldots\ldots)\}$$

where the coefficients are in radians, and ϑ_0 is the value of ϑ at $t=0$.

Let $1+\sigma$ be the nontrigonometrical term in $1+\phi(\vartheta)$, take it out as a common factor, and denote the numerical coefficients by $A_2, B_2, A_4, B_4, A_6, B_6, b_0, a_2, b_2, a_4, b_4$, respectively.

With these coefficients the following are computed:

$$K=\frac{1}{1-w}\,\frac{1}{\sin 1''}$$

$$S_2=K\left\{A_2-\frac{5}{4}(A_2A_4+B_2B_4)+\frac{1}{4}A_2(A_2^2+B_2^2)+\frac{1}{2}a_2\right\} \qquad S_2'=K\frac{w}{2}b_2$$
$$C_2=K\left\{-B_2+\frac{5}{4}(A_2B_4-B_2A_4)-\frac{1}{4}B_2(A_2^2+B_2^2)+\frac{1}{2}b_2\right\} \qquad C_2'=-K\frac{w}{2}a_2$$
$$S_4=K\left\{\frac{1}{2}A_4+\frac{1}{4}(A_2^2-B_2^2)\right\} \qquad S_4'=K\frac{w}{4}(b_4+A_2b_2-B_2a_2) \qquad (15)$$
$$C_4=K\left\{-\frac{1}{2}B_4-\frac{1}{2}A_2B_2\right\} \qquad C_4'=-K\frac{w}{4}(a_4+A_2a_2+B_2b_2)$$
$$S_6=K\left\{\frac{1}{3}A_6+\frac{5}{12}(A_2A_4-B_2B_4)-\frac{1}{12}A_2(3B_2^2-A_2^2)\right\} \qquad C_0''=K\frac{w^2}{4}(b_0-A_2b_2-B_2a_2)$$
$$C_6=K\left\{-\frac{1}{3}B_6-\frac{5}{12}(A_2B_4+B_2A_4)-\frac{1}{12}B_2(3A_2^2-B_2^2)\right\}$$

There are check formulae for these quantities in *Z* 134, equation (153), (161′). In equation (153) there is a misprint; in equation (161′) there are two misprints. The errors and their corrections are noted in the list of errata which accompanies the second section of this paper.

A part of the long period terms in $n\delta z$, denoted by $[n\delta z]_1$, is expressed by

$$[n\delta z]_1=S_2\sin 2\zeta+C_2\cos 2\zeta+S_4\sin 4\zeta+C_4\cos 4\zeta+S_6\sin 6\zeta+C_6\cos 6\zeta+\ldots$$
$$+\frac{2}{w}(\zeta-\zeta_0)(S_2'\sin 2\zeta+C_2'\cos 2\zeta+S_4'\sin 4\zeta+C_4'\cos 4\zeta+\ldots)+\left(\frac{2}{w}\right)^2(\zeta-\zeta_0)^2C_0''+\ldots \qquad (16)$$

---

(10) Hygiea.

$$1+\phi(\vartheta)=(1-0.008064)\{1-0.056384\sin 2\vartheta+0.017308\cos 2\vartheta+0.016186\sin 4\vartheta$$
$$+0.012342\cos 4\vartheta+0.000912\sin 6\vartheta-0.005122\cos 6\vartheta+\ldots$$
$$+(\vartheta-\vartheta_0)(+0.000007-0.000494\sin 2\vartheta-0.001276\cos 2\vartheta-0.000364\sin 4\vartheta$$
$$+0.000412\cos 4\vartheta+\ldots\ldots)+\ldots\ldots\}$$

$$b_0=+0.000007$$
$$A_2=+0.017308 \qquad a_2=-0.000494$$
$$B_2=-0.056384 \qquad b_2=-0.001276$$
$$A_4=+0.012342 \qquad a_4=-0.000364$$
$$B_4=+0.016186 \qquad b_4=+0.000412$$
$$A_6=-0.005122$$
$$B_6=+0.000912$$

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Unit of A_n, etc., is one radian

$$\begin{aligned}[n\delta z]_1 = {} & (3.59592)\sin 2\zeta \\ & + (4.09785)\cos 2\zeta \\ & + (3.0783)\ \sin 4\zeta \\ & + (3.2230_n)\cos 4\zeta \\ & + (2.4390_n)\sin 6\zeta \\ & + (1.494_n)\ \cos 6\zeta \\ & + \ldots\ldots\ldots \end{aligned} \qquad \begin{aligned} & + \frac{2}{w}(\zeta-\zeta_0)\ [(0.933_n)\sin 2\zeta \\ & \qquad + (0.521)\cos 2\zeta \\ & \qquad + (0.085)\sin 4\zeta \\ & \qquad + (0.005)\cos 4\zeta \\ & \qquad + \ldots\ldots\ ] \\ & + \ldots\ldots\ldots \end{aligned}$$

in which the coefficients are logarithmic in seconds of arc. For this planet it is not necessary to include C_0''.

In equation (16) let

$$\begin{aligned} S_n &= \ \ k\cos K & C_n &= k\sin K \\ S'_n &= -k'\sin K' & C'_n &= k'\cos K' \end{aligned} \tag{17}$$

Then

$$[n\delta z]_1 = \Sigma k\sin(n\zeta+K) + \frac{2}{w}(\zeta-\zeta_0)\Sigma k'\cos(n\zeta+K') + \ldots\ldots \tag{18}$$

The argument ζ is given by the relation:

$$\zeta = \frac{1+\sigma}{1+\frac{1}{2}(A_2^2+B_2^2)}\left(\frac{w}{2}\varepsilon - [n'\delta z']\right) + \frac{1-w}{2}\,c - c' \tag{19}$$

and ζ_0 is the value of ζ at $t=0$, in which, $[n'\delta z']$, the long period term between *Jupiter* and *Saturn* is:

$$[n'\delta z'] = (9.5215)\sin\{(9.58539)\,T + 115°.326\} \tag{20}$$

where the numerical coefficients are logarithmic in degrees, and T is measured from the date of osculation in Julian years.

The complete expression for the long period term in $n\delta z$ is:

$$[n\delta z] = [n\delta z]_1 + \frac{2}{1-w}\,\frac{\sigma-\frac{1}{2}(A_2^2+B_2^2)}{1+\frac{1}{2}(A_2^2+B_2^2)}\left(\frac{w}{2}\varepsilon - [n'\delta z']\right) \tag{21}$$

It is important to remark that, in equations (19), (21), the eccentric anomaly is computed by the usual formula,

$$\varepsilon - e\sin\varepsilon = c + nt + n\delta z \tag{1}$$

in which the multiples of 2π must be retained, for ε is used here as if it were the time. Since $n\delta z$ is unknown, the computation is by successive approximations.

---

(10) Hygiea.

$$\begin{aligned}[n\delta z]_1 = {} & (4.11837)\sin\ (2\zeta + 72°.5246) \\ & + (3.3130)\sin\ (4\zeta + 305.627) \\ & + (2.442)\sin\ (6\zeta + 186.48) \\ & + \ldots\ldots\ldots\ldots\ldots \\ & + \frac{2}{w}(\zeta-\zeta_0)\{(0.963)\cos\ (2\zeta + 68.83) \\ & \qquad + (0.199)\cos\ (4\zeta + 309.75) + \ldots\ldots\} + \ldots\ldots \end{aligned}$$

in which the coefficients are logarithmic in seconds of arc.

$$\log\frac{1+\sigma}{1+\frac{1}{2}(A_2^2+B_2^2)} = 9.99572;\quad \log\left\{\frac{2}{1-w}\,\frac{\sigma-\frac{1}{2}(A_2^2+B_2^2)}{1+\frac{1}{2}(A_2^2+B_2^2)}\right\} = 8.31918_n$$

The argument ϑ in $(n\delta z - [n\delta z])$, the short period part of $n\delta z$, is given by

$$\vartheta = \frac{1-w}{2}\,[n\delta z]_1 + \zeta \tag{22}$$

and the function itself is computed from Table XXXV or A.

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The numerical coefficients in Table XXXV or A are multiplied by their respective factors

$$w^{x}\eta^{p}\eta'^{q}j^{2t}$$

and the terms are then collected in the form

$$n\delta z-[n\delta z]=\Sigma C\begin{matrix}\sin\\ \cos\end{matrix}(i\tfrac{1}{2}\varepsilon+j\vartheta+k\Delta-l\Sigma) \tag{23}$$

By expanding the trigonometric functions, the known part of the argument, namely,

$$k\Delta-l\Sigma$$

is incorporated in the coefficients, and the terms are collected in the form:

$$\begin{aligned} n\delta z-[n\delta z]=\Sigma a\sin\chi+\Sigma b\cos\chi&+(\vartheta-\vartheta_0)\,(\Sigma a'\sin\chi+\Sigma b'\cos\chi)\\ &+(\vartheta-\vartheta_0)^2(\Sigma a''\sin\chi+\Sigma b''\cos\chi)\\ &+\ldots\ldots\ldots \end{aligned} \tag{24}$$

where

$$\chi=i\tfrac{1}{2}\varepsilon+j\vartheta \tag{25}$$

Let

$$\begin{array}{llll} a=k\cos K & & b=k\sin K\\ a'=-k'\sin K' & & b'=k'\cos K'\\ a''=k''\cos K'' & & b''=k''\sin K'' \end{array} \tag{26}$$

Then

$$\begin{aligned} n\delta z-[n\delta z]=\Sigma k\sin(\chi+K)&+(\vartheta-\vartheta_0)\,\Sigma k'\cos(\chi+K')\\ &+(\vartheta-\vartheta_0)^2\Sigma k''\sin(\chi+K'')+\ldots\ldots \end{aligned} \tag{27}$$

The tabulation of $n\delta z-[n\delta z]$ for (*10*) *Hygiea* is given on page 27.

Finally, the complete perturbation in the mean anomaly is:

$$n\delta z=[n\delta z]+(n\delta z-[n\delta z]) \tag{28}$$

It is now possible to determine c by successive approximations from equations (20), (19), (18), (21), (22), (27), (28).

From equation (1), which holds for any time t,

$$\begin{matrix} c=\varepsilon_1-e\sin\varepsilon_1-n\delta z\\ t=o\\ \varepsilon=\varepsilon_1 \end{matrix} \tag{29}$$

As a first approximation

$$n\delta z=0 \qquad c=\varepsilon_1-e\sin\varepsilon_1$$

Introducing this value of c in equation (19), a first approximation for $n\delta z$ is made. For $t=0$,

$$\begin{matrix}(\zeta-\zeta_0)=0\\ (\vartheta-\vartheta_0)=0\end{matrix} \tag{30}$$

Substituting the value of $n\delta z$ in equation (29), and computing a new value of c, the process of solution by trials is repeated until a satisfactory agreement is reached.

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(10) *Hygiea.*

Below is the last approximation for the constant c.

(See tabulation of $n\delta z-[n\delta\epsilon]$ on pg. 27.)

		$x=i\frac{\epsilon}{2}+j\vartheta$		$x+K$	$\log\sin(x+K)$	$k\sin(x+K)$
Approx. $n\delta z$	+0°.6124					
$\epsilon_1-e\sin\epsilon_1$	122. 5578	$\frac{1}{2}\epsilon+\vartheta$	285°.683	323°.619	9.7732_n	− 282″
Approx. c, equ. (1), p. 10	121. 9454	$\frac{1}{2}\epsilon+3\vartheta$	9. 443	291. 021	9.9701_n	− 680
$\frac{1-w}{2}c$, p. 13	57. 240	$\frac{1}{2}\epsilon+5\vartheta$	93. 203	258. 21	9.991_n	− 260
$\frac{1-w}{2}c-c'$, p. 12	217. 278	$-\frac{1}{2}\epsilon+\vartheta$	158. 077	183. 00	8.719_n	− 6
		$-\frac{1}{2}\epsilon+3\vartheta$	241. 837	335. 37	9.620_n	− 17
		ϵ	127. 606	135. 14	9. 848	+ 25″
$\frac{w}{2}\epsilon_1$, p. 13	+3. 9053	$\epsilon+2\vartheta$	211. 366	288. 414	9.9772_n	−3403
$[n'\delta z']$, equ. (20), p. 16	+0. 3003	$\epsilon+4\vartheta$	295. 126	256. 179	9.9872_n	− 723
$(9.99572)\left(\frac{w}{2}\epsilon_1-[n'\delta z']\right)$	+3. 5697	$\epsilon+6\vartheta$	18. 886	223. 38	9.837_n	− 168
		$\epsilon+8\vartheta$	102. 646	186. 8	9.073_n	− 5
$(8.3192_n)\left(\frac{w}{2}\epsilon_1-[n'\delta z']\right)$	−0. 0752	$-\epsilon+2\vartheta$	316. 154	14. 11	9. 387	+ 23
		$-\epsilon+4\vartheta$	39. 914	129. 91	9. 885	+ 3
ζ, equ. (19), p. 16	220. 848	$\frac{3}{2}\epsilon+3\vartheta$	137. 049	74. 51	9. 984	+ 121
2ζ	81. 696	$\frac{3}{2}\epsilon+5\vartheta$	220. 809	39. 53	9. 804	+ 44
4ζ	163. 392	$\frac{3}{2}\epsilon+7\vartheta$	304. 569	3. 25	8. 754	+ 1
6ζ	245. 088	$2\epsilon+2\vartheta$	338. 972	236. 180	9.9195_n	− 86
		$2\epsilon+4\vartheta$	62. 732	209. 15	9.688_n	− 19
$2\zeta+72°.525$, p. 16	154. 221	$2\epsilon+6\vartheta$	146. 492	183. 0	8.72_n	− 1
$4\zeta+305.627$	109. 019	$\frac{5}{2}\epsilon+5\vartheta$	348. 415	2. 4	8. 62	0
$6\zeta+186.48$	71. 57	$\frac{5}{2}\epsilon+7\vartheta$	72. 175	327. 9	9.72_n	− 4
						+ 217″−5654″
					$n\delta z-[n\delta z]$	− 5437″
						− 1°.5103
log sin	9. 6384				$(8.3192_n)\left(\frac{w}{2}\epsilon_1-[n'\delta z']\right)$	− 0. 0752
log sin	9. 9756				$[n\delta z]_1$	+ 2. 1994
log sin	9. 9771				$n\delta z$, equ. (21)	+ 0. 6139
	+ 5712″				$c=c_1$	121. 9439
	+ 1944					
	+ 262				$(6.8050_n)c_1$	− 0. 0778
					c_2, p. 19	121. 8661
$[n\delta z]_1$	+7918″					
	+2°.1994					
$(9.67154)[n\delta z]_1$	+1. 032				$\frac{1-w}{2}c_1$	+57. 240
ϑ, equ. (22), p. 16	221°.880				$\frac{1-w}{2}c_1-c'$	217. 278
2ϑ	83. 760					
3ϑ	305. 640				$(9.6715)[n\delta z]_1$	+ 1. 032
4ϑ	167. 520				ϑ_0, equ. (22)	221. 880
5ϑ	29. 400					
6ϑ	251. 280					
7ϑ	113. 160					
8ϑ	335. 040					
$\frac{1}{2}\epsilon$, p. 14	63. 803					
ϵ	127. 606					
$\frac{3}{2}\epsilon$	191. 409					
2ϵ	255. 212					
$\frac{5}{2}\epsilon$	319. 015					

Collecting the elements, and adopting a change of notation, introduced at this point by v. Zeipel, namely, the addition of the subscript unity to the elements just now determined,

$$n_1 = 637''.2633 = 0°.17701758$$
$$\varphi_1 = 6°.3858$$
$$\pi_1 = 230.7971$$
$$c_1 = 121.9439$$

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These elements are constants; they differ from constant osculating elements only by the constants of integration in $n\delta z$ and ν. They are to be used in the same manner as Hansen uses constant osculating elements.

It is possible, in a similar manner, to absorb the constants of integration in the third coordinate in the elements i_0 and Ω_0, but this transformation will be omitted.

It is a convenience to the computer to have n_1 and c_1 transformed to mean elements. The last term in equation (21) increases in magnitude, progressively with the time. The computation of this term of large magnitude may be avoided by modifications of the elements n_1 and c_1.

The method of transformation can be clearly shown from the example (*10*) *Hygiea*,

$$\frac{2}{1-w}\frac{\sigma-\frac{1}{2}(A_2^2+B_2^2)}{1+\frac{1}{2}(A_2^2+B_2^2)}\left(\frac{w}{2}\varepsilon-[n'\delta z']\right)=(6.80497_n)\varepsilon+(8.3192)\,[n'\delta z'] \tag{31}$$

By equation (1)

$$\begin{aligned}(6.80497_n)\varepsilon+(8.3192)\,[n'\delta z'] &= (6.80497_n)\,c_1-0''.4067\,t-14''.6\sin\varepsilon \\ &\quad+(6.80497_n)n\delta z+(8.3192)[n'\delta z']\end{aligned} \tag{32}$$

It is evident from equations (1), (21), and (23) that the first term on the right-hand side of equation (32) may be combined with the mean anomaly at the epoch to form a mean mean anomaly, given by

$$c_2=c_1+(6.80497_n)c_1$$

Furthermore, the second term on the right-hand side of equation (32) may be combined with nt in equation (1). A mean mean motion is thereby introduced, which is given by

$$n_2=n_1-0''.4067=636''.8566$$

Again, the third term on the right-hand side of equation (32) may be combined with a term in $(n\delta z-[n\delta z])$ which has the argument ε. In the construction of $(n\delta z-[n\delta z])$ there occurred the terms

$$+34''.8\sin\varepsilon+4''.6\cos\varepsilon=(1.545)\sin(\varepsilon+7°.53)$$

The addition of $-14''.6\sin\varepsilon$ from equation (32) gives

$$+20''.2\sin\varepsilon+4''.6\cos\varepsilon=(1.320)\sin(\varepsilon+12°.74)$$

These two values for the argument $\chi=\varepsilon$ are tabulated in the body of the table given on p. 27.

Further, since it is intended to improve the perturbations by the use of Newcomb's value for the mass of *Jupiter*, $n\delta z$ must be multiplied by the factor 1.00050. The combination of the correction for the mass of *Jupiter* with the term of the same form in equation (32) gives

$$(+0.00050-0.00064)n\delta z=-0.00014\;n\delta z$$

This correction is the last step in the determination of $n\delta z$, since it depends upon the perturbation itself.

Without change of notation for $n\delta z$, the collected results are:

$$\varepsilon-e\sin\varepsilon=c_2+n\delta z+n_2t \tag{33}$$

where

$$n\delta z=[n\delta z]_1+(n\delta z-[n\delta z])-0.00014\;n\delta z+(8.319)\,[n'\delta z'] \tag{34}$$

It must be remembered that $[n\delta z]_1$ and $(n\delta z-[n\delta z])$ are numerically different from their original values, but there should be no confusion if this transformation is not made before the constant c has been determined.

The constant elements are now:

Epoch and Osculation, 1851, Sept. 17.0, Ber. M. T.

$$\begin{aligned}n_2&=636''.8566=0°.17690461\\ c_2&=121°.8661\\ \varphi_1&=\;\;6.3858\\ \pi_1&=230.7971\\ \Omega_0&=287.6198\\ i_0&=\;\;3.7857\end{aligned}$$

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Equinox and ecliptic, 1850.0

$$a_2^3 = \frac{k^2}{n_2^2} \qquad e_1 = \sin \varphi_1 \qquad p_2 = a_2(1-e_1^2)$$

$$\log k^2 = 9.98741 \qquad \log e_1 = 9.04620 \qquad \log p_2 = 0.49191$$

$$\log n_2^2 = 8.49548 \qquad \log\sqrt{\frac{1-e_1}{1+e_1}} = 9.95150$$

$$\log a_2^3 = 1.49193 \qquad \log a_2 = 0.49731$$

Certain other transformations of the elements which v. Zeipel makes are omitted. Those terms of the perturbations which have the argument ϵ have the same period as the planet and can, therefore, be absorbed in the elements. It would be necessary to set up formulae for this transformation to mean elements, and it is not profitable to do so.

PERTURBATIONS OF THE RADIUS VECTOR.

The perturbations in the radius vector are computed in a manner similar to that for $(n\delta z - [n\delta z])$. In Table XLIII the numerical coefficients are multiplied by their respective factors w^p, η^p, η'^q, ζ^{pq}, the terms are collected, the known parts of the arguments are incorporated in the coefficients, and the terms are grouped in the form:

$$\begin{aligned} \nu = {} & \Sigma a \sin\chi + \Sigma b \cos\chi + \cdots \\ & + (\vartheta - \vartheta_0)\{\Sigma a' \sin\chi + \Sigma b' \cos\chi + \cdots\} \\ & + (\vartheta - \vartheta_0)^2\{\Sigma a'' \sin\chi + \Sigma b'' \cos\chi + \cdots\} + \cdots \end{aligned} \tag{35}$$

Let

$$\begin{aligned} a &= -k \sin K & b &= k \cos K \\ a' &= k' \cos K' & b' &= k' \sin K' \\ a'' &= -k'' \sin K'' & b'' &= k'' \cos K'' \end{aligned} \tag{36}$$

Then

$$\nu = \Sigma k \cos(\chi + K) + (\vartheta - \vartheta_0)\Sigma k' \sin(\chi + K') + (\vartheta - \vartheta_0)^2 \Sigma k'' \cos(\chi + K'') + \cdots \tag{37}$$

and to correct the perturbation for the use of the improved value of the mass, ν should be multiplied by 1.00050.

If the mean motion n_2 is adopted, the constant in ν must be corrected by

$$+\frac{2}{3}\,\frac{n_2 - n_1}{n_1}\,\frac{1}{\sin 1''} \tag{38}$$

This correction of the constant in ν permits the use of the relation

$$n_2^2 a_2^3 = k^2$$

in the computation of a geocentric place; without this correction it would be necessary to use the relation

$$n_1^2 a_1^3 = k^2$$

in the determination of the parameter p. In the computation of the eccentric anomaly it is permissible to use either n_1 or n_2, for the difference is taken up in the modification of $n\delta z$, but the theory of Hansen demands the use of constant elements. Hence, strictly speaking, n_1 must be used in computing a geocentric place. The modification of the constant in ν renders the employment of n_2 equivalent to the use of n_1.

---

(10) *Hygiea.*

$$\frac{2}{3}\,\frac{n_2 - n_1}{n_1}\,\frac{1}{\sin 1''} = -\frac{2}{3}\,\frac{0''.4067}{637''.3}\,\frac{1}{\sin 1''} = -87''.8$$

The constant in Table XLIII or C is $+47''.6$. Therefore, the new constant is:

$$+47''.6 - 87''.8 = -40''.2 = (1.604)\cos 180°.00$$

where the coefficient is logarithmic in seconds of arc.

The perturbation is tabulated on page 27.

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PERTURBATIONS OF THE THIRD COORDINATE.

The perturbations of the third coordinate are derived from Tables LIV, LV_I, LV_{II} or D, E_1, E_2. The first of these is of the same form as the tables for $(n\delta z - [n\delta z])$ and ν. After making analogous transformations and multiplying by the factor $\iota \cos i$, (ι is defined by equation (6)),

$$\iota \cos i \Sigma U_{p \cdot q}\, \eta^p \eta'^q \sin A = \Sigma k \sin (\chi + K) \tag{39}$$

Both Table LV_I or E_1 and Table LV_{II} or E_2 lead to a single numerical quantity, since all the factors and arguments are known constants.

The perturbation u is given by

$$u = \iota \cos i \,[\Sigma U_{p \cdot q}\, \eta^p \eta'^q \sin A + n_2 t\{K_1 (\cos \varepsilon - e_1) + K_2 \sin \varepsilon\} + c_1 (\cos \varepsilon - e_1) + c_2 \sin \varepsilon] \tag{40}$$

in which c_1, c_2, the constants of integration, have not been determined.

The constants c_1 and c_2 are determined by Hansen's conditions:

$$\left.\begin{aligned} u &= 0 \\ \frac{du}{d\varepsilon} &= 0 \end{aligned}\right\} t = 0 \tag{41}$$

Substituting these relations and equation (39) in equation (40), the determination of c_1 and c_2 is given by the solution of

$$C_1 (\cos \varepsilon - e_1) + C_2 \sin \varepsilon = -\Sigma k \sin (\chi + K);\; C_1 \sin \varepsilon - C_2 \cos \varepsilon = \Sigma k \frac{d\chi}{d\varepsilon} \cos (\chi + K) \tag{42}$$

where

$$\begin{aligned} C_1 &= \iota \cos i . c_1 \\ C_2 &= \iota \cos i . c_2 \end{aligned} \tag{43}$$

and

$$\frac{d\chi}{d\varepsilon} = \frac{i}{2} + j \frac{d\vartheta}{d\varepsilon}$$

where

$$\frac{d\vartheta}{d\varepsilon} = \frac{1+\sigma}{1+\frac{1}{2}(A_2^2 + B_2^2)} \frac{w}{2}\left(1 + \frac{1-w}{2} \frac{d[n\delta z]_1}{d\zeta}\right) \tag{44}$$

A double notation is used here, for $\cos i$ is the cosine of the inclination of the orbit, and $\frac{i}{2}$ is the numerical coefficient of ε in the argument χ, but this should cause no confusion.

Dividing and multiplying the factor

$$\iota \cos i \cdot n_2 t$$

by 365.25

$$\iota \cos i \cdot n_2 t = \frac{\iota \cos i \cdot n_2}{365.25} T \tag{45}$$

where T is the interval in Julian years, measured from the date of osculation.

It is evident that

$$C_1 (\cos \varepsilon - e_1) + C_2 \sin \varepsilon$$

can be incorporated in

$$\Sigma k \sin (\chi + K)$$

in the same manner as similar terms were treated in $(n\delta z - [n\delta z])$.

For symmetry of form, let

$$\iota \cos i \cdot n_2 t\{K_1 (\cos \varepsilon - e_1) + K_2 \sin \varepsilon\} = \Sigma k' \cos (\chi + K') \tag{46}$$

Finally, then, without change of notation,

$$u = \Sigma k \sin (\chi + K) + T \Sigma k' \cos (\chi + K') \tag{47}$$

in which the constants of integration are absorbed in the first term. The perturbation u is tabulated on page 27.

The perturbations in the heliocentric coordinates are computed from equations (3) The signs of $\cos a$, $\cos b$, $\cos c$ are determined as follows:

$\cos a > 0$ if $0 < \Omega < 180°$
$\cos b < 0$ if $-90° < \Omega < +90°$
$\cos b < 0$ in any case, if $\varepsilon > i$
$\cos c > 0$ if $\sin i \cos \Omega < \cos i$
$\cos c > 0$ in any case if $i < 45°$

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(10) *Hygiea.*

$$t=0$$

$$\frac{d}{d\zeta}[n\delta z]_1 = [(4.41940)\cos(2\zeta_0 + 72^\circ.5246) + (3.9150)\cos(4\zeta_0 + 305^\circ.627) + (3.220)\cos(6\zeta_0 + 186^\circ.48)]\sin 1''$$

$$\log \frac{w}{2}\frac{1+\sigma}{1+\frac{1}{2}(A_2^2+B_2^2)} = 8.48150 \qquad \frac{d\vartheta}{d\varepsilon} = 0.0285$$

$$\log \frac{1-w}{2} = 9.67154 \qquad \frac{d\chi}{d\varepsilon} = \frac{i}{2} + 0.0285j$$

$$\Sigma k \sin(\chi + K) = -\ 70''.5$$

$$\Sigma k \frac{d\chi}{d\varepsilon}\cos(\chi + K) = +101''.6$$

$$C_1 = +\ 35''.9$$
$$C_2 = +120''.8$$

From Table LIV, multiplied by $\iota \cos i$ we have three terms in

$$\Sigma k \sin(\chi + K) = -4''.2 - 1''.9 \sin\varepsilon + 2''.7 \cos\varepsilon$$

which, added to

$$C_1(\cos\varepsilon - e_1) + C_2 \sin\varepsilon = +120''.8 \sin\varepsilon + 35''.9\ (\cos\varepsilon - e_1)$$

gives for two terms in $\Sigma k \sin(\chi + K)$

$$-7''.8 + 118''.9 \sin\varepsilon + 38''.6 \cos\varepsilon = (0.89)\sin 270^\circ.0 + (2.0970)\sin(\varepsilon + 17^\circ.99)$$

CHECK COMPUTATION.

After the elements have been determined and the final tabulation of the perturbations is ready, the following checks should be performed, even if the computation has been duplicated.

$$t=0$$

$$\theta_0 = \tfrac{1}{2}(\varepsilon_1 - e_1 \sin\varepsilon_1) - g'$$

$$g' = c' + [n'\delta z'] \qquad \theta_0 = \vartheta_0 + \frac{1-w}{2}(n\delta z - [n\delta z]) - \eta w \sin\varepsilon$$

where the necessary quantities are to be taken from the last approximation for c.

Secondly, the heliocentric coordinates

$$x - \Delta x,\ y - \Delta y,\ z - \Delta z$$

for $t=0$ must check when computed by the usual formulae for two body motion and osculating elements, and when computed with the final set of elements and the corresponding perturbations, $n\delta z$ and ν, taken from the final tabulation.

The final tabulation of the perturbation in the third coordinate is checked by the test

$$t=0 \quad ; \quad u=0$$

COMPUTATION OF THE PERTURBATIONS FOR THE TIME t.

It is well to emphasize here the distinction between the elements n_1 and c_1 and the elements n_2 and c_2 in their relation to the perturbations. Let $n\delta z_1$ denote the perturbation in the mean anomaly computed according to equations (20), (19), (18), (21), (22), (27), (28), and let $n\delta z_2$ signify the perturbation computed according to equations (20), (19), (18), (22), the final tabulation of $(n\delta z - [n\delta z])$, and an equation analogous to (34). (It must be remembered that equation (34) is for (*10*) *Hygiea* only. The numerical coefficients are determined for each planet individually:)

Before the determination of c there can be no confusion, for there is but one way to compute the perturbation $n\delta z$. Later, when both c_1 and c_2 are given, the computation may be performed in either manner. The latter method is, of course, adopted. The question then arises, what values of ε and c are to be used in equation (19)?

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Clearly, there is only one value of ϵ, for

$$\epsilon - e_1 \sin \epsilon = c_1 + n_1 t + n\delta z_1 = c_2 + n_2 t + n\delta z_2$$

and both $n\delta z$ and ϵ must be found by trials. Further, since the introduction of n_2 and c_2 arises merely from a transfer of certain terms in the perturbation, the argument of the perturbation is independent of this transformation. Therefore c_1 is the constant in equation (19).

For any time t the order of computation is: equation (33), neglecting $n\delta z$, (20), (19), (18), (22), final tabulation of $n\delta z - [n\delta z]$, and the equation analogous to (34). Since the perturbations are large, the argument ϵ is not sufficiently accurate when $n\delta z$ is neglected. It is, therefore, always necessary to make a second approximation for $n\delta z$. In the first trial the small terms may be omitted.

(10) *Hygiea.*

PERTURBATIONS $n\delta z$, ν, u, FOR 1873, SEPT. 20.4491, BER. M. T.

$\log e_1$ (degrees)	0. 80432	log sin	9.9387_n		
$\log \frac{w}{2}$	8. 48578	log sin	9.9918_n		
$\log \frac{1}{57.30} \cdot \frac{2}{w}$	9. 7561	log sin	9.703_n		
$\frac{1-w}{2} \cdot e - e'$	217°.278				
ζ_0	220°.848	log cos	9.741_n	$x = i \cdot \frac{\epsilon}{2} + j\vartheta$	
ϑ_0	221°.880	log cos	9. 419		
c_2	121°.8661	$\log (\zeta - \zeta_0)$	1. 6337		
		$\log \frac{2}{w}(\zeta - \zeta_0)$	1. 3898	$\frac{1}{2}\epsilon + \vartheta$	315°.350
				$\frac{1}{2}\epsilon + 3\vartheta$	119. 524
				$\frac{1}{2}\epsilon + 5\vartheta$	283. 698
	1873	$\log \frac{2}{w}(\zeta - \zeta_0) \cos$	1.131_n	$-\frac{1}{2}\epsilon + \vartheta$	208. 824
				$-\frac{1}{2}\epsilon + 3\vartheta$	12. 998
Ber. M. T.	Sept.[1] 20. 4491	$\log \frac{2}{w}(\zeta - \zeta_0) \cos$	0. 809		
t	+ 8039d.4491			ϵ	106. 526
$n_2 t$	+ 1422°.2156		− 11405″	$\epsilon + 2\vartheta$	270. 700
$c_2 + n_2 t$	1544. 0817		− 2017	$\epsilon + 4\vartheta$	74. 874
	104. 0817 + 1440°		− 140	$\epsilon + 6\vartheta$	239. 048
$n\delta z$	− [2] 3. 666		− 124	$\epsilon + 8\vartheta$	43. 222
$M = c_2 + n_2 t + n\delta z$	100. 416		+10	$-\epsilon + 2\vartheta$	57. 648
				$-\epsilon + 4\vartheta$	221. 822
ϵ	[3] 106. 526 + 1440°	$[n\delta z]_1$	− 13676″	$\frac{1}{2}\epsilon + \vartheta$	61. 876
	1546. 526			$\frac{1}{2}\epsilon + 3\vartheta$	226. 050
		$\log [n\delta z]_1$ (secs)	4.13596_n	$\frac{1}{2}\epsilon + 5\vartheta$	30. 224
$\log \epsilon$	3. 18935	$\log [n\delta z]_1$ (degrees)	0.57966_n	$\frac{1}{2}\epsilon + 7\vartheta$	194. 398
$\log \frac{w}{2}\epsilon$	1. 67513	$[n\delta z]_1$	−3°.7989	$-\frac{1}{2}\epsilon + \vartheta$	102. 298
$\frac{w}{2}\epsilon$	+ 47°.329			2ϵ	213. 052
				$2\epsilon + 2\vartheta$	17. 226
$\frac{w}{2}\epsilon - [n'\delta z']$	+ 47°.053	log (9.6715) $[n\delta z]_1$	0.2512_n	$2\epsilon + 4\vartheta$	181. 400
				$2\epsilon + 6\vartheta$	345. 574
$\log \left(\frac{w}{2}\epsilon - [n'\delta z']\right)$	1. 67259	(9.6715) $[n\delta z]_1$	−1°.783	$\frac{1}{2}\epsilon + 5\vartheta$	136. 750
				$\frac{1}{2}\epsilon + 7\vartheta$	300. 924
$\log (9.99572)\left(\frac{w}{2}\epsilon - [n'\delta z']\right)$	1. 66831	ϑ	262°.087		
$(9.99572)\left(\frac{w}{2}\epsilon - [n'\delta z']\right)$	+ 46°.592	$\vartheta - \vartheta_0$	40°.207		
ζ	263°.870				
		ϑ	262°.087		
2ζ	167°.740	2ϑ	164. 174		
4ζ	335. 480	3ϑ	66. 261		
6ζ	143. 220	4ϑ	328. 348		
		5ϑ	230. 435		
$\zeta - \zeta_0$	43. 022	6ϑ	132. 522		
		7ϑ	34. 609		
$2\zeta + 72°.5246$	240°.265	8ϑ	296. 696		
$4\zeta + 305.627$	281. 107	(4)			
$6\zeta + 186.48$	329. 70	$\frac{1}{2}\epsilon$	53°. 263		
		ϵ	106. 526		
$2\zeta + 68°.83$	236. 57	$\frac{3}{2}\epsilon$	159. 789		
$4\zeta + 309.75$	285. 23	2ϵ	213. 052		
		$\frac{1}{2}\epsilon$	266. 315		

[1] Corr. for aberr. [2] From previous approx. [3] From Astrand's table. [4] See eq. (1), page 16.

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PERTURBATIONS $n\delta z$, ν, u, FOR 1873, SEPT. 20.4491, BER. M. T.—Continued.

$\chi=i\frac{e}{2}+j\vartheta$	$n\delta z-[n\delta s]$			ν			u		
i j	$x+K$	log sin $(x+K)$	k sin $(x+K)$	$x+K$	log cos $(x+K)$	k cos $(x+K)$	$x+K$	log sin $(x+K)$	k sin $(x+K)$
				°			°		
0 0				180.00	0.000_n	− 40″	270.00	0.000_n	− 8″
0 2				58.608	9.7167	+375″	296.33	9.952_n	−12
0 4				98.841	9.1867_n	− 33	238	9.928_n	0
0 6	°			146.28	9.920_n	− 52			
1 1	353.286	9.0679_n	− 56″	173.425	9.9971_n	− 137	80.40	9.994	+ 11″
1 3	41.102	9.8178	+ 479″	221.824	9.8723_n	− 193	110.78	9.971	+ 14
1 5	88.71	0.000	+ 265	268.86	8.2988_n	− 2	156.04	9.609	+ 3
−1 1	233.74	9.907_n	− 85	192.38	9.9898_n	− 3	122.28	9.927	+ 11
−1 3	106.53	9.982	+ 41	111.41	9.562_n	− 13	172.10	9.138	+ 1
2 0	119.27	9.941	+ 18	300.02	9.699	+ 3	124.52	9.916	+103
2 2	347.748	9.3268_n	− 761	167.726	9.9900_n	−1852	79.94	9.993	+ 59
2 4	35.927	9.7686	+ 437	215.194	9.9123_n	− 329	104.99	9.985	+ 25
2 6	83.53	9.997	+ 243	263.148	9.0767_n	− 15	150.50	9.692	+ 5
2 8	127.4	9.90	+ 35	309.92	9.807	+ 27			
−2 2	115.61	9.955	+ 84	88.6	8.39	0	0.51	7.948	+ 1
−2 4	311.82	9.872_n	− 3	349.03	9.992	+ 6			
3 1				276.65	9.064	+ 1	225.83	9.856_n	− 5
3 3	163.51	9.453	+ 36	341.94	9.978	+ 85	257.49	9.990_n	− 3
3 5	208.94	9.685_n	− 34	28.42	9.944	+ 34	278.56	9.995_n	− 1
3 7	253.08	9.981_n	− 13	54.02	9.769	+ 1			
−3 1				136.98	9.864_n	− 2	4.98	8.939	0
4 0							348.8	9.288_n	0
4 2	274.434	9.9987_n	− 104				40.38	9.811	+ 1
4 4	327.82	9.726_n	− 21	156.79	9.963_n	− 9	85.1	9.998	+ 1
4 6	22.1	9.58	+ 5	198.96	9.976_n	− 4	92.97	9.999	+ 1
5 5	150.8	9.69	+ 5	330.79	9.941	+ 10			
5 7	196.6	9.46_n	− 2	16.85	9.981	+ 8			
			+1648″ −1079″			+550″ −2684″			+236″ −29″

$(\vartheta-\vartheta_0)$[1]orT	$x+K'$	log cos $(x+K')$	k' cos $(x+K')$	$x+K'$	log sin $(x+K')$	k' sin $(x+K')$	$(x+K')$	log cos $(x+K')$	k' cos $(x+K')$
				°					
0 0				270.00	0.000_n	− 6″	180.00	0.000_n	0
0 2				232.94	9.902_n	− 8			
0 4	°			281.51	9.991_n	− 7			
2 0	292.53	9.583	+ 371″	292.573	9.965_n	− 447	47.67	9.828	+ 6″
2 2	4.7	0.00	+ 2	351.93	9.147_n	0			
2 4	41.3	9.88	+ 6	41.29	9.819	+ 3″			
−2 2	123.85	9.746_n	− 2″	305.02	9.913_n	− 1			
4 0	219.90	9.885_n	− 20						
4 2	104.1	9.39_n	− 1	104.23	9.986	+ 4			
4 4	154.82	9.957_n	− 1	147	9.736	+ 1			
			+ 379″ − 24″			+ 8″ − 469″			+ 6″

$(\vartheta-\vartheta_0)^2$	$x+K''$	log sin $(x+K'')$	k'' sin $(x+K'')$	$x+K''$	log cos $(x+K'')$	k'' cos $(x+K'')$
	°			°		
2 0	296.23	9.953_n	− 3″	112.79	9.588_n	− 1″
4 0	227.15	9.865_n	− 1	47	9.834	0
			− 4″			− 1″

[1] For perturbation u use factor T.

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PERTURBATIONS $n\partial z$, ν, u, FOR 1873, SEPT. 20.4491, BER. M. T.—Continued.

$\log(\vartheta-\vartheta_0)$ rad.	9.8462				
$\log(\vartheta-\vartheta_0)^2$	9.692				
$\log T$	1.3426				
$\log \frac{a}{\cos i}\sin 1''$	5.183				
	$n\partial z$		ν		u
$\Sigma k \sin(\chi+K)$	+ 569″	$\Sigma k \cos(\chi+K)$	− 2134″	$\Sigma k \sin(\chi+K)$	+ 207″
$\Sigma k' \cos(\chi+K')$	+ 355	$\Sigma k' \sin(\chi+K')$	− 461″	$\Sigma k' \cos(\chi+K')$	+ 6
$\Sigma k'' \sin(\chi+K'')$	− 4	$\Sigma k'' \cos(\chi+K'')$	− 1″		
$\log \Sigma k' \cos(\chi+K')$	2.5502	$\log \Sigma k' \sin(\chi+K')$	2.664_n	$\log \Sigma k' \cos(\chi+K')$	0.778
$\log(\vartheta-\vartheta_0)\Sigma k' \cos(\chi+K')$	2.396	$\log(\vartheta-\vartheta_0)\Sigma k' \sin(\chi+K')$	2.510_n	$\log T.\Sigma k' \cos(\chi+K')$	2.121
$(\vartheta-\vartheta_0)\Sigma k' \cos(\chi+K')$	+ 249″	$(\vartheta-\vartheta_0)\Sigma k' \sin(\chi+K')$	− 324″	$T.\Sigma k' \cos(\chi+K')$	+ 132″
$\log \Sigma k'' \sin(\chi+K'')$	0.602_n				
$\log(\vartheta-\vartheta_0)^2\Sigma k'' \sin(\chi+K'')$	0.294_n	ν	− 2458″	u	+ 339″
$(\vartheta-\vartheta_0)^2\Sigma$	− 2″	$\log \nu$ (secs)	3.3906_n	$\log u$	2.530
		$\log \nu$ (rad)	8.0762_n	$\log d\beta$	7.713
		$\log(1+\nu)$	9.99480	$\log \cos a$	8.798_n
				$\log \cos b$	9.619_n
$n\partial z-[n\partial z]$	+ 816″			$\log \cos c$	9.958
	+0°.2267				
$(8.3192)\,[n'\partial z']$	+0°.0058			$\log \Delta x$	6.511_n
$[n\partial z]_1$	−3°.7989			$\log \Delta y$	7.332_n
$n\partial z$	− 3.5664			$\log \Delta z$	7.671
$-0.00014\, n\partial z$	+ 5				
$n\partial z$	− 3°.566			Δx	−0.00032
				Δy	−0.00215
				Δz	+0.00469

The computation of the geocentric place on page 26 is analogous to the usual method for two body motion, the fundamental equations being (1), (2), (3). A complete set of formulae and an example of the computation is also given in Memoirs of the National Academy of Sciences, Vol. X, Seventh Memoir, p. 215.

CONSTANTS FOR THE EQUATOR.

	A′ yearly var.	B′ yearly var.	C′ yearly var.	log sin a log cos a	log sin b log cos b	log sin c log cos c
1850.0	220°833+0°01399	229°182+0°01404	238°657+0°01310	9.99914 8.799_n	9.95884 9.619_n	9.62355 9.958
1900.0	321.532+0.01399	229.885+0.01405	239.312+0.01308	9.99914 8.797_n	9.95888 9.619_n	9.62423 9.958
1950.0	322.232+0.01399	230.587+0.01406	239.965+0.01306	9.99915 8.795_n	9.95893 9.620_n	9.62490 9.958

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COMPARISON, OBSERVATION—COMPUTATION, 1873, SEPT. 20.4491, BER. M. T.

	1873	x	+3.0709
Ber. M. T.	Sept. 20.4491	X	−1.00281
c_2+n_2t	104°.0817	Δx	−0.00032
$n\delta z$	− 3.5660	ξ	+2.0678
$M=c_2+n_2t+n\delta z$	100.5157		
		y	−0.89314
dM^0	− 0°.4843	Y	+0.03260
dM'	− 29′.06	Δy	−0.00215
$d\varphi'$	+ 3′.15	η	−0.86269
$\frac{dv}{d\varphi}$	+ 1.8124		
$\left(\frac{Ddv}{d\varphi}\right)dM'$	+ 68		
$\Delta\varphi.\frac{d\varphi}{20}$	+ 8	z	−0.19677
$\Sigma.d\varphi'$	+ 5′.73	Z	+0.01415
$d(v-M)/dM$	− 0.0674	Δz	+0.00469
$\frac{1}{2}D_m.dM^0$	+ 8	ζ	−0.17793
$\Sigma.\delta M'$	+ 1′.94		
$v-M$	+ 12° 0′.14	$\log\rho\cos\delta\cos\alpha$	0.31551
	+ 12° 7′.81	$\cos\alpha$	9.96515
v_1-M_1	+ 12°.1302	$\sin\alpha$	9.58550_n
$\bar{f}=v_1$	112°.6459	$\log\rho\cos\delta\sin\alpha$	9.93586_n
		$\log tg\,\alpha$	9.62035_n
$\log\cos\bar{f}$	9.58550_n	α	337° 21′ 14″ = 22h 29m 24s.9
$\log e_1\cos\bar{f}$	8.63170_n	Red to True α	+1.5
$\log(1+e_1\cos\bar{f})$	9.98099	True α	22h 29m 26s.4
$\log\bar{r}$	0.51092	Obs. α (A. N. 2029)	22h 29m 07s.1
$\log(1+\nu)$	9.99480		
$\log r$	0.50572		
		$\log\rho\cos\delta$	0.35036
A'	321°.1548	$\cos\delta$	9.99864
B'	229.5058	$\sin\delta$	8.89852_n
C'	238.9584	$\log\rho\sin\delta$	9.25025_n
		$\log tg\,\delta$	8.89989_n
$A'+\bar{f}$	73.8007	δ	−4° 32′ 26″
$B'+\bar{f}$	342.1517	Red to True δ	+6″
$C'+\bar{f}$	351.6043	True δ	−4° 32′ 20″
		Obs. δ (A. N. 2029)	−4° 33′ 27″
$\log\sin a$	9.99914		
$\log\sin(A'+\bar{f})$	9.98240		
$\log x$	0.48726	$\log\rho$	0.35172
$\log\sin b$	9.95877		
$\log\sin(B'+\bar{f})$	9.48643_n		
$\log y$	9.95092_n	$(O-C)$	
		$\Delta\alpha\cos\delta$	−19s.3
$\log\sin c$	9.62387	$\Delta\delta$	−1′ 7″
$\log\sin(C'+\bar{f})$	9.16438_n		
$\log z$	9.29397_n		

Given a series of observations well distributed around the orbit and extending over as long an interval as is available, the elements can be corrected by the method of least squares.

For this purpose the formulae by Bauschinger [2] are convenient. The equations of condition are set up for the residuals in the plane of the orbit and perpendicular to the plane, as seen from the earth. This resolution of the residuals is convenient because it keeps the same resolution into components as is used in the theory of Hansen.

It is to be noticed that the elements to be used in computing the differential coefficients are the finally adopted constant elements referred to the equator by the proper transformation. The value of r to be used is

$$r=\bar{r}(1+\nu)$$

except in the equation

$$\sin\bar{\varepsilon}=\frac{\bar{r}}{a_2\sqrt{1-e_1^2}}\sin\bar{f} \qquad \text{(Hansen's notation)}$$

[1] Tafel zur Berechnung der wahren Anomalie, Veröffentlichungen des Rechen-Instituts der Königlichen Sternwarte zu Berlin No. 1.

[2] Über das Problem der Bahnverbesserung, Veröffentlichungen des Königlichen Astronomischen Rechen-Instituts zu Berlin, No. 23, Berlin, 1903.

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The use of $\bar{\epsilon}, \bar{f}, r$ and constant elements is equivalent to the use of osculating elements for the given date of observation.

(10) *Hygiea*

Unit of $k=1''$.

$\chi=i\frac{\epsilon}{2}+j\vartheta$	$n\partial z-[n\partial z]$		ν		u		
i j	log k	K	log k	K	log k	K	
0 0			1.604	180.00	0.89	270.00	
0 2			2.8570	254.434	1.118	132.16	
0 4			2.3364	130.493	8.25	270	
0 6			1.800	13.76			$n\partial z-[n\partial z]=\Sigma k \sin(\chi+K)$
1 1	2.6771	37.936	2.1397	218.075	1.057	125.05	$+(\vartheta-\vartheta_0)\Sigma k' \cos(\chi+K')$
1 3	2.8627	281.578	2.4135	102.300	1.161	351.26	$+(\vartheta-\vartheta_0)^2\Sigma k'' \sin(\chi+K'')$
1 5	2.4238	165.01	1.965	345.16	0.930	232.34	
−1 1	2.022	24.92	0.55	343.56	1.119	273.46	
−1 3	1.628	93.53	1.543	98.41	0.981	159.10	
2 0	[1.545][1]	[7.53][1]					$\nu=\Sigma k \cos(\chi+K)$
	1.320	12.74	0.711	193.49	2.097	17.99	$+(\vartheta-\vartheta_0)\Sigma k' \sin(\chi+K')$
2 2	3.5546	77.048	3.2776	257.026	1.777	169.24	$+(\vartheta-\vartheta_0)^2\Sigma k'' \cos(\chi+K'')$
2 4	2.8719	321.053	2.6054	140.320	1.412	30.12	
2 6	2.389	204.49	2.1033	24.100	1.034	271.45	
2 8	1.64	84.2	1.62	266.70			
−2 2	1.970	57.96	1.27	31.0	1.824	302.86	$u=\Sigma k \sin(\chi+K)$
−2 4	0.602	90.00	0.80	127.21			$+T\Sigma k' \cos(\chi+K')$
3 1			0.90	214.77	0.826	163.95	
3 3	2.100	297.46	1.95	115.89	0.446	31.44	Where T is expressed in Julian years from date of osculation.
3 5	1.841	178.72	1.583	358.20	0.171	248.34	
3 7	1.12	58.68	0.34	219.62			
−3 1			0.42	34.68	0.673	262.68	
4 0					0.00	135.7	$\chi=i\frac{\epsilon}{2}+j\vartheta$ where in ϵ the multiples of 2π must be retained.
4 2	2.0170	257.208			0.270	23.15	
4 4	1.589	146.42	0.97	335.39	9.91	263.7	$\vartheta_0=221.811$
4 6	1.14	36.5	0.66	213.39	9.73	107.40	
5 5	1.038	14.0	1.062	194.04			
5 7	0.88	255.7	0.94	75.93			

$(\vartheta-\vartheta_0)$ or T	log k'	K'	log k'	K'	log k'	K'	
0 0			0.799	270.00	9.690	180.00	
0 2			1.021	68.77			
0 4			0.86	313.16			
2 0	2.9862	186.00	2.6850	186.047	0.957	301.14	
2 2	0.18	94	0.12	81.23			
2 4	0.88	326.4	0.60	326.42			
−2 2	0.60	66.20	0.11	247.37			
4 0	1.414	6.85					
4 2	0.68	86.9	0.580	87.00			
4 4	0.11	333.42	0.09	326			

$(\vartheta-\vartheta_0)^2$	log k''	K''	log k''	K''			
2 0	0.58	189.70	0.26	6.26			
4 0	9.91	14.10	9.6	194			

COMPARISON OF THE REVISED WITH V. ZEIPEL'S ORIGINAL TABLES.

It was originally planned to conclude the example with a least squares solution of the orbit on the basis of the observations used by v. Zeipel for the same purpose, and to test conclusively the relative value of the revised and v. Zeipel's original tables by representing recent observations with both sets of elements and tables.

In the course of the computation doubt arose regarding the accuracy of some of the observations selected by v. Zeipel, which led us to reject them and substitute other observa-

[1] In the determination of the constant c use quantities in brackets.

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tions. This substitution produced an unfavorable distribution of the observed places in the orbit and the resulting least squares solution was not satisfactory.

In the meantime, pending a resumption of the least squares solution on the basis of a more favorable distribution of observed places,[1] the following conclusions may be drawn regarding the revised and v. Zeipel's original tables:

1. v. Zeipel's tables have been slightly improved by the correction of some numerical errors.

2. A moderate further improvement has been accomplished by an extension of the tables in so far as seemed practicable without a more exhaustive and unwarranted study of the practical convergence of the auxiliary series, by including certain terms of higher order and degree.

With reference to the correction of the orbit and the representation of observations by a least squares solution, it should be observed that

(1) A symmetrical distribution of the observed positions in the orbit is essential to counteract the effect of neglected perturbations of higher order and degree and of major planets other than *Jupiter*. For the *Hecuba* Group, in general, the mean motions of the minor planets may be nearly commensurable with those of *Saturn*, *Mars*, or the *Earth* in the ratios 3/2, 3/1, or 3/5.

(2) However accurate the initial osculating elements may be, comparatively large residuals may remain on account of neglected perturbations.

TABLE A (XXXV).

Logarithmic. $n\delta z - [n\delta z]$ Unit = 1″

	Sin	w^{-2}	w^{-3}	w^{-1}	w^0	w	w^2
$\eta\eta'$	$\frac{1}{2}\varepsilon + \vartheta$				4.1570	4.8741_n	
	$\frac{1}{2}\varepsilon + \vartheta + \Delta$				2.7684_n	3.3827	3.7172_n
η^2	$\frac{1}{2}\varepsilon + \vartheta + \Delta$				4.0056_n	4.7686	
η'^2	$\frac{1}{2}\varepsilon + \vartheta + \Delta$				4.0766_n	4.8295	
f^2	$\frac{1}{2}\varepsilon + \vartheta + \Delta$				4.1365	4.8738_n	
$\eta\eta'$	$\frac{1}{2}\varepsilon + \vartheta + 2\Delta$				3.3345	4.5162_n	
η'	$\frac{1}{2}\varepsilon + 3\vartheta + 2\Delta$				4.2240_n	4.9611	5.6685_n
η	$\frac{1}{2}\varepsilon + 3\vartheta + 3\Delta$				4.0671	4.8483_n	5.5636
η'^2	$\frac{1}{2}\varepsilon + 5\vartheta + 3\Delta$				5.0926_n	6.0018	
$\eta\eta'$	$\frac{1}{2}\varepsilon + 5\vartheta + 4\Delta$				5.2325	6.1714_n	
η^2	$\frac{1}{2}\varepsilon + 5\vartheta + 5\Delta$				4.7675_n	5.7344	
f^2	$\frac{1}{2}\varepsilon + 5\vartheta + 4\Delta - \Sigma$				3.8050_n	4.7998	
η'	$-\frac{1}{2}\varepsilon + \vartheta$				3.3112	3.8350_n	4.1355
η	$-\frac{1}{2}\varepsilon + \vartheta + \Delta$				3.2065_n	3.7910	4.0833
η'^2	$-\frac{1}{2}\varepsilon + 3\vartheta + \Delta$				3.5338	4.6236_n	
$\eta\eta'$	$-\frac{1}{2}\varepsilon + 3\vartheta + 2\Delta$				4.0879	5.0382	
η^2	$-\frac{1}{2}\varepsilon + 3\vartheta + 3\Delta$				3.6012_n	4.5318_n	
f^2	$-\frac{1}{2}\varepsilon + 3\vartheta + 2\Delta - \Sigma$				3.2074	4.1925_n	
η	ε		9.868_n	0.5689	2.922	3.4600_n	3.3670
η'	$\varepsilon + \Delta$		9.482	0.2533_n	2.673_n	3.2959	3.1772_n
$\eta\eta'$	$\varepsilon + 2\vartheta + \Delta$	0.746_n	1.384	3.2927_n	4.14906	4.6990_n	
	$\varepsilon + 2\vartheta + 2\Delta$		9.788_n	2.47560	3.16847_n	3.4540	3.3960_n
η^2	$\varepsilon + 2\vartheta + 2\Delta$	0.645	1.342_n	2.305_n	3.6179_n	4.4018	
η'^2	$\varepsilon + 2\vartheta + 2\Delta$	0.326	1.119_n	2.935_n	3.3017_n	4.39206	
f^2	$\varepsilon + 2\vartheta + 2\Delta$			3.4276_n	4.23764	4.76933_n	
$\eta\eta'$	$\varepsilon + 2\vartheta + 3\Delta$	0.28_n	1.102	3.1738	3.5449_n	3.8446_n	
$\eta\eta'^2$	$\varepsilon + 4\vartheta + 2\Delta$			3.6004	4.27485		
η'	$\varepsilon + 4\vartheta + 3\Delta$	9.057	0.692_n	3.10161	3.9302_n	4.52415	4.78162_n
$\eta^2\eta'$	$\varepsilon + 4\vartheta + 3\Delta$			4.0519_n	3.7975		
η'^3	$\varepsilon + 4\vartheta + 3\Delta$			4.1385_n	4.6961		
$f^2\eta'$	$\varepsilon + 4\vartheta + 3\Delta$			4.2431_n	5.1290		
η	$\varepsilon + 4\vartheta + 4\Delta$	9.500_n	0.522	2.9351_n	3.8035	4.41616_n	4.63017
η^3	$\varepsilon + 4\vartheta + 4\Delta$			3.7714	4.2108_n		
$\eta\eta'^2$	$\varepsilon + 4\vartheta + 4\Delta$			4.4165	5.0931_n		
$f^2\eta$	$\varepsilon + 4\vartheta + 4\Delta$			4.1524	5.0661_n		
$\eta^2\eta'$	$\varepsilon + 4\vartheta + 5\Delta$			4.0588_n	4.8136		

[1] Since 1913, when the revision of the tables was concluded, Miss Glancy has continued the problem of (10) *Hygiea* independently at the Observatorio Nacional, Córdoba, with the following highly satisfactory results, which substantiate further the increased accuracy of the revised tables (1) The original osculating elements and the revised tables resulted in a greatly improved representation of the selected observations (1849–1884) over the representation obtained with the original tables. (2) After the correction of the original osculating elements by least squares solution (*a*) on the basis of v. Zeipel's tables and residuals, (*b*) on the basis of the residuals resulting from the revised tables, the representation of the selected observations was equally satisfactory; but 3 later observations, taken in 1910, 1914, and 1917, are represented far better by the revised tables and corresponding elements than by the original tables and corresponding elements. (cf. Astronomical Journal, Vol. 32, p. 27, No. 748, January 1919) A. O. Leuschner.

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Logarithmic. TABLE A (XXXV)—Continued. Unit=1″

	Sin	w^{-3}	w^{-2}	w^{-1}	w^0	w	w^2
$j^2\eta$	$\varepsilon+4\vartheta+3\Delta-\Sigma$			3.2322$_n$	4.2342		
$j^2\ \eta'$	$\varepsilon+4\vartheta+4\Delta-\Sigma$			2.744$_n$	3.0962		
η'^2	$\varepsilon+6\vartheta+4\Delta$	0.28	0.64$_n$	3.8027	4.77998$_n$	5.52852	
$\eta\ \eta'$	$\varepsilon+6\vartheta+5\Delta$	0.596$_n$	1.070	3.9374$_n$	4.94342	5.70347$_n$	
η^2	$\varepsilon+6\vartheta+6\Delta$	0.255	0.8$_n$	3.4684	4.50125$_n$	5.27451	
j^2	$\varepsilon+6\vartheta+5\Delta-\Sigma$	8.8	9.3$_n$	2.415	3.4823$_n$	4.2931	
η'^3	$\varepsilon+8\vartheta+5\Delta$			4.5564	5.4999$_n$		
$\eta\ \eta'^2$	$\varepsilon+8\vartheta+6\Delta$			4.8668$_n$	5.8416		
$\eta^2\eta'$	$\varepsilon+8\vartheta+7\Delta$			4.6990	5.7030$_n$		
η^3	$\varepsilon+8\vartheta+8\Delta$			4.0531$_n$	5.0844		
$j^2\ \eta'$	$\varepsilon+8\vartheta+6\Delta-\Sigma$			3.5829	4.6352$_n$		
$j^2\eta$	$\varepsilon+8\vartheta+7\Delta-\Sigma$			3.3768$_n$	4.4540		
η'^2	$-\varepsilon+2\vartheta$	0.606	1.422$_n$	3.2132	3.6657$_n$	3.9260	
$\eta\ \eta'$	$-\varepsilon+2\vartheta+\Delta$	0.791$_n$	1.690	3.3777$_n$	3.8866	4.72168	
η^2	$-\varepsilon+2\vartheta+2\Delta$	0.418	1.365$_n$	2.894	3.4616$_n$	3.8078	
j^2	$-\varepsilon+2\vartheta+\Delta-\Sigma$	9.34	0.28$_n$	2.938	3.4714$_n$	3.7862	
η'^3	$-\varepsilon+4\vartheta+\Delta$			3.5208	4.07255		
$\eta\ \eta'^2$	$-\varepsilon+4\vartheta+2\Delta$			3.4965$_n$	4.59582		
$\eta^2\eta'$	$-\varepsilon+4\vartheta+3\Delta$			3.2416	4.5467$_n$		
η^3	$-\varepsilon+4\vartheta+4\Delta$			2.430$_n$	3.9848		
$j^2\ \eta'$	$-\varepsilon+4\vartheta+2\Delta-\Sigma$			3.5496	4.19852$_n$		
$j^2\eta$	$-\varepsilon+4\vartheta+3\Delta-\Sigma$			3.3247$_n$	4.05994		
$\eta\ \eta'$	$\frac{1}{2}\varepsilon+3\vartheta+2\Delta$				3.6731	4.0029$_n$	
	$\frac{1}{2}\varepsilon+3\vartheta+3\Delta$				2.3528	3.2475$_n$	3.9005
η^2	$\frac{1}{2}\varepsilon+3\vartheta+3\Delta$				3.6181$_n$	4.2122	
j^2	$\frac{1}{2}\varepsilon+3\vartheta+3\Delta$				3.4072$_n$	4.4000	
$\eta\ \eta'$	$\frac{1}{2}\varepsilon+3\vartheta+4\Delta$				3.5244	4.4012$_n$	
η'	$\frac{1}{2}\varepsilon+5\vartheta+4\Delta$				3.3533	4.4231$_n$	5.2725
η	$\frac{1}{2}\varepsilon+5\vartheta+5\Delta$				3.1780$_n$	4.2730	5.1359$_n$
η'^2	$\frac{1}{2}\varepsilon+7\vartheta+5\Delta$				4.2775	5.4708$_n$	
$\eta\ \eta'$	$\frac{1}{2}\varepsilon+7\vartheta+6\Delta$				4.4051$_n$	5.6177	
η^2	$\frac{1}{2}\varepsilon+7\vartheta+7\Delta$				3.9296	5.1605$_n$	
η	$2\varepsilon+2\vartheta+2\Delta$		9.486	2.1744$_n$	2.708	2.889$_n$	2.599$_n$
η'	$2\varepsilon+2\vartheta+3\Delta$				1.946$_n$	2.501	2.516$_n$
$\eta\ \eta'$	$2\varepsilon+4\vartheta+3\Delta$	8.8$_n$	0.561	2.789$_n$	3.5813	4.1074$_n$	
	$2\varepsilon+4\vartheta+4\Delta$		8.90$_n$	9.599	1.711	2.5795$_n$	3.1726
η^2	$2\varepsilon+4\vartheta+4\Delta$	9.2	0.34$_n$	2.618	3.4962$_n$	4.0890	
η'	$2\varepsilon+6\vartheta+5\Delta$		9.819$_n$	0.5840	2.7821	3.7794$_n$	4.51865
η	$2\varepsilon+6\vartheta+6\Delta$		9.653	0.4645$_n$	2.5979$_n$	3.6265	4.38424$_n$
	$\frac{1}{2}\varepsilon+5\vartheta+5\Delta$				1.2340	2.1166$_n$	2.7076
η'	$\frac{1}{2}\varepsilon+7\vartheta+6\Delta$				2.3679	3.3518$_n$	4.0587
η	$\frac{1}{2}\varepsilon+7\vartheta+7\Delta$				2.1758$_n$	3.1926	3.9204$_n$
	$(\vartheta-\vartheta_0)$ cos						
η	ε	0.1021$_n$	0.728	2.8978$_n$	3.4504	3.7168$_n$	
η^2	ε	1.377$_n$	2.346	3.8211$_n$	4.6762		
$\eta\ \eta'^2$	ε	1.941$_n$	2.815	4.4076$_n$	5.1971		
$j^2\eta$	ε	1.364	2.220$_n$	4.4076	5.1971$_n$	5.7086	
η'	$\varepsilon+\Delta$	9.658	0.774$_n$	2.7836	3.3840$_n$	3.6946	
$\eta^2\ \eta'$	$\varepsilon+\Delta$	1.863	2.755$_n$	4.2546	5.0814$_n$		
η'^3	$\varepsilon+\Delta$	1.844	2.642$_n$	4.1953	4.9770$_n$		
$j^2\ \eta'$	$\varepsilon+\Delta$	1.170$_n$	2.049	4.3715$_n$	5.1770	5.6975$_n$	
$\eta\ \eta'^2$	$\varepsilon+2\Delta$	1.742$_n$	2.574	4.0203$_n$	4.8466		
$j^2\ \eta'$	$\varepsilon+\Sigma$	0.716	1.65$_n$	4.0809	4.8829$_n$	5.4008	
$j^2\eta$	$\varepsilon+\Delta+\Sigma$	1.00$_n$	1.89	4.3427$_n$	5.0837	5.5553$_n$	
$\eta^2\eta'$	$-\varepsilon+\Delta$	1.562	2.455$_n$	3.9535	4.7803$_n$		
η^2	2ε	9.801	0.43$_n$	2.5842	3.1493$_n$	3.4158	
$\eta\ \eta'$	$2\varepsilon+\Delta$	9.357$_n$	0.473	2.4548$_n$	3.0830	3.3936$_n$	
	$(\vartheta-\vartheta_0)^2$ sin						
η	ε		9.56$_n$	0.42			
η'	$\varepsilon+\Delta$		9.43	0.32$_n$			

$n\delta z-[n\delta z]=\Sigma w^s\eta^p\eta'^q j^{2t}C_1$ sin Arg.$+(\vartheta-\vartheta_0)\Sigma w^s\eta^p\eta'^q j^{2t}C_2$ cos Arg.$+(\vartheta-\vartheta_0)^2\Sigma w^s\eta^p\eta'^q j^{2t}C_3$ sin Arg.
where C_1, C_2, C_3 represent the respective coefficients.

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TABLE B (XXXVIII).

Logarithmic. $\phi(\vartheta)$ Unit—1 radian.

	Cos	w^{-4}	w^{-3}	w^{-2}	w^{-1}	w^{0}	w^{1}	w^{2}	w^{3}	w^{4}
				1.5	3.909$_n$	4.960	6.6748$_n$	7.2764	7.540$_n$	7.31
η^2			2.0	4.644$_n$	5.160	6.150	8.048$_n$	8.838	8.655$_n$	8.100$_n$
η'^2			1.9	3.41$_n$	4.75$_n$	6.509	8.2077$_n$	8.994	8.919$_n$	
j^2				2.83$_n$	5.146	6.299$_n$	7.994	8.740$_n$	8.656	
$\eta\,\eta'$	Δ		2.34$_n$	4.446	4.57	6.728$_n$	8.4022	9.1999$_n$	9.0854	8.079
$\eta\,\eta'^2$	2ϑ	1.6	2.6$_n$	5.744	6.535$_n$	8.3811	9.1031$_n$	9.0128		
η'	$2\vartheta+\Delta$		0.8$_n$	3.068	5.2988	7.2212$_n$	7.3772	8.0372	8.764$_n$	8.668
$\eta^2\eta'$	$2\vartheta+\Delta$	2.32$_n$	3.30	5.886$_n$	6.718	8.5059$_n$	9.2804	9.2017$_n$		
η'^2	$2\vartheta+\Delta$			5.301$_n$	6.149	8.2302$_n$	9.0154	8.938$_n$		
$j^2\ \eta'$	$2\vartheta+\Delta$					8.5592	9.3245$_n$	9.2428		
η^3	$2\vartheta+2\Delta$	2.48	3.40$_n$	5.422	6.292$_n$	7.476	8.664$_n$	8.636		
η	$2\vartheta+2\Delta$		1.22	2.94$_n$	5.1206$_n$	7.6416	7.9638$_n$	7.083$_n$	8.645	8.582$_n$
$\eta\,\eta'^2$	$2\vartheta+2\Delta$	1.9	3.0$_n$	5.442	6.328$_n$	8.0915$_n$	8.630$_n$	8.742		
$j^2\eta$	$2\vartheta+2\Delta$					8.5904$_n$	9.3489	9.8024$_n$	9.6532	
$\eta^2\eta'$	$2\vartheta+3\Delta$	2.04$_n$	3.00	4.98$_n$	5.89	8.0326	8.1973$_n$	7.69		
$j^2\eta$	$2\vartheta+\Delta-\Sigma$			4.51	5.42$_n$	8.1011	8.873$_n$	8.792		
$j^2\ \eta'$	$2\vartheta+2\Delta-\Sigma$			4.04$_n$	5.00	6.89$_n$	8.182	8.158$_n$		
η'^2	$4\vartheta+2\Delta$		2.66$_n$	2.7	6.1031	8.4188$_n$	8.5297	6.0	7.90$_n$	
$\eta\,\eta'$	$4\vartheta+3\Delta$		2.72	4.369	6.2526$_n$	8.5594	8.7988$_n$	7.94$_n$	8.287	8.210
η^2	$4\vartheta+4\Delta$		2.20$_n$	4.624$_n$	5.824	8.0924$_n$	8.4333	7.24	7.74$_n$	8.044$_n$
j^2	$4\vartheta+3\Delta-\Sigma$		1.5$_n$	2.45	4.68	7.1747$_n$	7.301	8.111	8.127$_n$	
η'^2	$6\vartheta+3\Delta$			5.301$_n$	6.149	9.1294$_n$	9.7728	9.6609$_n$		
$\eta\,\eta'^2$	$6\vartheta+4\Delta$			5.92	6.74$_n$	9.4432	0.14644$_n$	0.05077		
$\eta^2\eta'$	$6\vartheta+5\Delta$	2.0$_n$	3.0	5.93$_n$	6.79	9.2774$_n$	0.03298	9.9494$_n$		
η^3	$6\vartheta+6\Delta$	2.0	3.0$_n$	5.420	6.292$_n$	8.634	9.4351$_n$	9.3608		
$j^2\ \eta'$	$6\vartheta+4\Delta-\Sigma$			4.04$_n$	5.00	8.272$_n$	9.1028	9.0334$_n$		
$j^2\eta$	$6\vartheta+5\Delta-\Sigma$			4.51	5.42$_n$	8.0554	8.926$_n$	8.864		
	$(\vartheta-\vartheta_0)$ sin									
$\eta\,\eta'$	Δ			2.60$_n$	4.71	5.94$_n$	6.507$_n$	6.606		
η'	$2\vartheta+\Delta$		1.36	2.48	4.49	5.255$_n$	5.51	5.25$_n$		
η	$2\vartheta+2\Delta$		1.82$_n$	2.42	4.64$_n$	5.350	5.51$_n$	5.16		
η'^2	$4\vartheta+2\Delta$		2.34	3.00	5.392	6.179$_n$	6.528$_n$	6.665		
$\eta\,\eta'$	$4\vartheta+3\Delta$		2.89$_n$	3.46	5.702$_n$	6.467	6.851	6.979$_n$		
η^2	$4\vartheta+4\Delta$		2.66	3.459$_n$	5.357	6.127$_n$	6.530$_n$	6.653		
	$(\vartheta-\vartheta_0)^2$ cos									
η^2				2.08$_n$	2.08		5.546$_n$	5.546		
η'^2				2.54	2.54$_n$		5.396$_n$	5.396		
$\eta\,\eta'$	Δ			2.5$_n$	2.5		5.776	5.776$_n$		
		m'^3	m'^3	m'^3, m'^2	m'^3, m'^2	m'^2, m'	m'^2, m'	m'^2, m'	m'^2, m'	m'^2, m

$\phi(\vartheta)=\Sigma w^q.\eta^p.\eta'^q.j^{2t}.C_1$ cos Arg.$+(\vartheta-\vartheta_0)\Sigma w^q.\eta^p.\eta'^q.j^{2t}.C_2$ sin Arg.$+(\vartheta-\vartheta_0)^2\Sigma w^q.\eta^p.\eta'^q.j^{2t}.C_3$ cos Arg.
where C_1, C_2, C_3 represent the respective coefficients.

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TABLE C (XLIII).

Logarithmic. p Unit=1″

	Cos	10^{-3}	10^{-2}	10^{-1}	10^{0}	10^{1}	10^{2}
			8.72	9.88$_n$	1.6349	2.1070$_n$	2.2333
η^2		9.80	0.212$_n$		2.759	3.4922$_n$	
η'^2		8.9	9.23		2.937	3.6295$_n$	
f^2					2.937$_n$	3.6295	
$\eta\ \eta'$	Δ	9.66$_n$	9.78		3.1136$_n$	3.8440	
$\eta\ \eta'^2$	2ϑ	0.556$_n$	1.204	3.2111$_n$	3.7970		
η'	$2\vartheta+\Delta$		0.504$_n$	2.3472	2.456$_n$	2.686$_n$	3.4735
$\eta^2\eta'$	$2\vartheta+\Delta$	0.997	1.711$_n$	3.6559	4.3103$_n$		
η'^2	$2\vartheta+\Delta$	0.438	1.220$_n$	3.3654	4.0763$_n$		
$f^2\ \eta'$	$2\vartheta+\Delta$			3.6975$_n$	4.3810		
η	$2\vartheta+2\Delta$		0.438	2.952$_n$	3.2529	3.0689$_n$	3.3979$_n$
η^3	$2\vartheta+2\Delta$	0.732$_n$	1.497	3.2410$_n$	4.0643		
$\eta\ \eta'^2$	$2\vartheta+2\Delta$	0.772$_n$	1.589	3.4136	4.0723		
$f^2\eta$	$2\vartheta+2\Delta$			3.9048	4.5649$_n$	4.9303	
$\eta^2\eta'$	$2\vartheta+3\Delta$	0.505	1.344$_n$	3.4757$_n$	2.783		
$f^2\eta$	$2\vartheta+\Delta-\Sigma$	9.33$_n$	0.15	2.938$_n$	3.5830		
$f^2\ \eta'$	$2\vartheta+2\Delta-\Sigma$	9.20	0.10$_n$	2.0251	3.2961$_n$		
η'^2	$4\vartheta+2\Delta$	8.9	1.2819$_n$	3.5514	3.6173$_n$	3.8147	
$\eta\ \eta'$	$4\vartheta+3\Delta$	9.75$_n$	1.5024	3.7885$_n$	4.1394	4.3110$_n$	
η^2	$4\vartheta+4\Delta$	9.98	1.1342$_n$	3.4007	3.9091$_n$	4.1480	
f^2	$4\vartheta+3\Delta-\Sigma$		9.64$_n$	2.305	2.542$_n$	2.749$_n$	
η'^2	$6\vartheta+3\Delta$	0.438	1.220$_n$	4.2675	4.7993$_n$		
$\eta\ \eta'^2$	$6\vartheta+4\Delta$	1.125$_n$	1.862	4.6479$_n$	5.2324		
$\eta^2\eta'$	$6\vartheta+5\Delta$	1.198	1.947$_n$	4.5397	5.1768$_n$		
η^3	$6\vartheta+6\Delta$	0.732$_n$	1.508	3.9457$_n$	4.6328		
$f^2\ \eta'$	$6\vartheta+4\Delta-\Sigma$	9.20	0.10$_n$	3.4099	4.1710$_n$		
$f^2\eta$	$6\vartheta+5\Delta-\Sigma$	9.70$_n$	0.56	3.2601$_n$	4.0542		
$\eta\ \eta'$	$\varepsilon+\vartheta$				3.4878$_n$	4.1106	
	$\varepsilon+\vartheta+\Delta$			8.3$_n$	2.2106	2.7179$_n$	2.919
f^2	$\varepsilon+\vartheta+\Delta$				3.5709$_n$	4.2261	
η^2	$\varepsilon+\vartheta+\Delta$				3.4507	4.1296$_n$	
η'^2	$\varepsilon+\vartheta+\Delta$				3.5100	4.1837$_n$	
$\eta\ \eta'$	$\varepsilon+\vartheta+2\Delta$				2.579$_n$	3.9270	
η'	$\varepsilon+3\vartheta+2\Delta$			0.08	3.6873	4.1471$_n$	4.7839
η	$\varepsilon+3\vartheta+3\Delta$			9.5	3.5727$_n$	4.1511	4.7545$_n$
η'^2	$\varepsilon+5\vartheta+3\Delta$				4.5568	5.1414$_n$	
$\eta\ \eta'$	$\varepsilon+5\vartheta+4\Delta$				4.7261$_n$	5.4067	
η^3	$\varepsilon+5\vartheta+5\Delta$				4.2862	5.0418$_n$	
f^2	$\varepsilon+5\vartheta+4\Delta-\Sigma$				3.2570	4.0005$_n$	
η'	$-\varepsilon+\vartheta$			1.086$_n$	2.7090	3.3467$_n$	3.7098
η	$-\varepsilon+\vartheta+\Delta$			0.88	2.1967$_n$	3.0952	3.5636$_n$
η'^2	$-\varepsilon+3\vartheta+\Delta$				2.514	4.1049$_n$	
$\eta\ \eta'$	$-\varepsilon+3\vartheta+2\Delta$				4.0853	3.9122	
η^2	$-\varepsilon+3\vartheta+3\Delta$				3.8341$_n$	3.8118	
f^2	$-\varepsilon+3\vartheta+2\Delta-\Sigma$				2.416	3.6926$_n$	
η	ε		9.62	0.58$_n$	2.143$_n$	2.682	2.9151$_n$
η'	$\varepsilon+\Delta$		9.04$_n$	9.9	2.061	2.666$_n$	2.9477
$\eta\ \eta'$	$\varepsilon+2\vartheta+\Delta$	0.444	1.1661$_n$	3.0588	3.8035$_n$	4.2554	
	$\varepsilon+2\vartheta+2\Delta$		9.487	2.1744$_n$	2.7280	2.972$_n$	2.976
η^2	$\varepsilon+2\vartheta+2\Delta$	0.344$_n$	1.1143	2.692$_n$	3.5334	4.0772$_n$	
η'^2	$\varepsilon+2\vartheta+2\Delta$	0.025$_n$	0.828	2.634	3.0726	4.0416$_n$	
f^2	$\varepsilon+2\vartheta+2\Delta$			3.1265	3.8806$_n$	4.3473	
$\eta\ \eta'$	$\varepsilon+2\vartheta+3\Delta$	9.98	0.811$_n$	2.873$_n$	3.1697	3.5856	
$\eta\ \eta'^2$	$\varepsilon+4\vartheta+2\Delta$	1.105	1.89$_n$	2.864	4.3477$_n$		
η'	$\varepsilon+4\vartheta+3\Delta$	8.8$_n$	0.398	2.8000$_n$	3.5327	4.0065$_n$	4.3207
$\eta^2\eta'$	$\varepsilon+4\vartheta+3\Delta$	1.260$_n$	2.083	3.0931	4.4160		
η'^2	$\varepsilon+4\vartheta+3\Delta$			3.8375	4.0446$_n$		
$f^2\ \eta'$	$\varepsilon+4\vartheta+3\Delta$	0.267	1.15$_n$	3.9421	4.6972$_n$		
η	$\varepsilon+4\vartheta+4\Delta$	9.19	0.248$_n$	2.6356	3.4317$_n$	3.9469	4.2558$_n$
η^3	$\varepsilon+4\vartheta+4\Delta$	0.774	1.66$_n$	3.0934$_n$	3.7866$_n$		
$\eta\ \eta'^2$	$\varepsilon+4\vartheta+4\Delta$			4.1154$_n$	4.5547		
$f^2\eta$	$\varepsilon+4\vartheta+4\Delta$	0.455$_n$	1.32	3.8518$_n$	4.6436		
$\eta^2\eta'$	$\varepsilon+4\vartheta+5\Delta$			3.7579	4.3244$_n$		
$f^2\eta$	$\varepsilon+4\vartheta+3\Delta-\Sigma$			3.0030	3.8869$_n$		

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TABLE C (XLIII)—Continued.

Logarithmic. ν Unit=1″

	Cos	w^{-2}	w^{-3}	w^{-1}	w^{0}	w	w^{2}
$f^2\ \eta'$	$\varepsilon+4\vartheta+4\Delta-\Sigma$			2.4425	1.85_n		
η'^2	$\varepsilon+6\vartheta+4\Delta$	9.98_n	0.480	3.5016_n	4.3723	4.9952_n	
$\eta\ \eta'$	$\varepsilon+6\vartheta+5\Delta$	0.296	0.823_n	3.6369	4.5582_n	5.2093	
η^2	$\varepsilon+6\vartheta+6\Delta$	9.95_n	0.538	3.1685_n	4.1334	4.8131_n	
f^2	$\varepsilon+6\vartheta+5\Delta-\Sigma$	8.5_n	9.15	2.114_n	3.0881	3.7886_n	
η'^3	$\varepsilon+8\vartheta+5\Delta$			4.2554_n	4.9349		
$\eta\ \eta'^2$	$\varepsilon+8\vartheta+6\Delta$	1.320	2.152_n	4.5657	5.3010_n		
$\eta^2\eta'$	$\varepsilon+8\vartheta+7\Delta$	1.228_n	2.093	4.3995_n	5.1827		
η^3	$\varepsilon+8\vartheta+8\Delta$	0.648	1.54_n	3.7543	4.5812_n		
$f^2\ \eta'$	$\varepsilon+8\vartheta+6\Delta-\Sigma$			3.2818_n	4.1442		
$f^2\eta$	$\varepsilon+8\vartheta+7\Delta-\Sigma$			3.0763	3.9759_n		
η'^2	$-\varepsilon+2\vartheta$	0.305	1.1007_n	2.912	3.4958_n	3.8151	
$\eta\ \eta'$	$-\varepsilon+2\vartheta+\Delta$	0.490_n	1.3330	3.0166_n	3.7273	4.3119	
η^2	$-\varepsilon+2\vartheta+2\Delta$	0.117	0.982_n	2.288_n	3.2375_n	3.7892	
f^2	$-\varepsilon+2\vartheta+\Delta-\Sigma$	9.04	9.96_n	2.636	3.2817_n	3.6568	
η'^3	$-\varepsilon+4\vartheta+\Delta$			3.2197	3.9650_n		
$\eta\ \eta'^2$	$-\varepsilon+4\vartheta+2\Delta$	1.146_n	1.89	3.0204	4.2441		
$\eta^2\eta'$	$-\varepsilon+4\vartheta+3\Delta$	1.005	1.78_n	3.5247_n	4.0012_n		
η^3	$-\varepsilon+4\vartheta+4\Delta$	0.290_n	1.15	3.1793	2.982		
$f^2\ \eta'$	$-\varepsilon+4\vartheta+2\Delta-\Sigma$			3.2486	4.0585_n		
$f^2\eta$	$-\varepsilon+4\vartheta+3\Delta-\Sigma$	9.98_n	0.8	2.957_n	3.8580		
η	$\frac{1}{2}\varepsilon+\vartheta+\Delta$			9.0	2.3363	3.0704_n	3.5111
η'	$\frac{1}{2}\varepsilon+\vartheta+2\Delta$			9.5	1.500	2.3585	3.1842_n
$\eta\ \eta'$	$\frac{1}{2}\varepsilon+3\vartheta+2\Delta$				2.779	3.7820_n	
	$\frac{1}{2}\varepsilon+3\vartheta+3\Delta$			9.28	2.1614_n	3.0257	3.6491_n
η^2	$\frac{1}{2}\varepsilon+3\vartheta+3\Delta$				1.32	2.966	
η'^2	$\frac{1}{2}\varepsilon+3\vartheta+3\Delta$				3.3450	4.1111_n	
f^2	$\frac{1}{2}\varepsilon+3\vartheta+3\Delta$				3.2309	4.1965_n	
$\eta\ \eta'$	$\frac{1}{2}\varepsilon+3\vartheta+4\Delta$				3.2994_n	4.1520	
η'	$\frac{1}{2}\varepsilon+5\vartheta+4\Delta$			1.017	3.1617_n	4.1967	5.0160_n
η	$\frac{1}{2}\varepsilon+5\vartheta+5\Delta$			0.88_n	2.9688	4.0380_n	4.8781
η'^2	$\frac{1}{2}\varepsilon+7\vartheta+5\Delta$				4.0855_n	5.2422	
$\eta\ \eta'$	$\frac{1}{2}\varepsilon+7\vartheta+6\Delta$				4.1991	5.3823_n	
η^2	$\frac{1}{2}\varepsilon+7\vartheta+7\Delta$				3.7114_n	4.9188	
f^2	$\frac{1}{2}\varepsilon+7\vartheta+6\Delta-\Sigma$				2.615_n	3.8317	
$\eta\ \eta'$	$-\frac{1}{2}\varepsilon+\vartheta$				3.2411	3.7872_n	
η^2	$-\frac{1}{2}\varepsilon+\vartheta+\Delta$				2.819_n	3.4476	
f^2	$-\frac{1}{2}\varepsilon+\vartheta-\Sigma$				2.9181_n	3.4813	
η^2	2ε				2.364_n	3.0737	
$\eta\ \eta'$	$2\varepsilon+\Delta$				2.624	3.3489_n	
η'^2	$2\varepsilon+2\Delta$				2.207_n	2.976	
f^2	$2\varepsilon+\Delta+\Sigma$				2.620_n	3.2765	
η	$2\varepsilon+2\vartheta+2\Delta$			9.8_n	1.63	2.362_n	2.873
η'	$2\varepsilon+2\vartheta+3\Delta$			9.5	1.796	2.303_n	2.1007
$\eta\ \eta'$	$2\varepsilon+4\vartheta+3\Delta$				1.92_n	2.700	
	$2\varepsilon+4\vartheta+4\Delta$		8.7	8.8	1.5802_n	2.4158	2.9867_n
η^2	$2\varepsilon+4\vartheta+4\Delta$				2.330	3.1764_n	
η'^2	$2\varepsilon+4\vartheta+4\Delta$				3.1079	3.9008_n	
f^2	$2\varepsilon+4\vartheta+4\Delta$				2.736	3.6809_n	
$\eta\ \eta'$	$2\varepsilon+4\vartheta+5\Delta$				2.9881_n	3.8425	
η'	$2\varepsilon+6\vartheta+5\Delta$		9.64	0.53	2.652_n	3.6204	4.3279_n
η	$2\varepsilon+6\vartheta+6\Delta$		9.48	0.36_n	2.4419	3.4512_n	4.1892
η'^2	$2\varepsilon+8\vartheta+6\Delta$				3.6135_n	4.6784	
$\eta\ \eta'$	$2\varepsilon+8\vartheta+7\Delta$				3.7124	4.8075_n	
η^2	$2\varepsilon+8\vartheta+8\Delta$				3.2109_n	4.3338	
f^2	$2\varepsilon+8\vartheta+7\Delta-\Sigma$				2.068_n	3.2092	
	$\frac{1}{2}\varepsilon+5\vartheta+5\Delta$			9.3_n	1.140_n	2.0056	2.5727_n
η'	$\frac{1}{2}\varepsilon+7\vartheta+6\Delta$			0.5_n	2.2749_n	3.2377	3.9184_n
η	$\frac{1}{2}\varepsilon+7\vartheta+7\Delta$			0.3	2.0542	3.0565_n	3.7710
	$\frac{1}{2}\varepsilon+7\vartheta+7\Delta$			8.1	0.43_n	1.346	1.959_n
	$(\vartheta-\vartheta_0)\sin$						
$\eta\ \eta'$	Δ	9.66	0.810_n	2.7559	3.3840_n	3.6946	
η'	$2\vartheta+\Delta$		9.79_n	0.54			
η	$2\vartheta+2\Delta$		9.92	0.63_n			

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TABLE C (XLIII)—Continued.

Logarithmic. ν Unit=1″

	$(\vartheta-\vartheta_0)$ sin	w^{-2}	w^{-3}	w^{-4}	w^0	w	w^2
η	ε	9.801_n	0.425	2.5970_n	3.1493	3.4158_n	
η^3	ε	1.075_n	2.045	3.5201_n	4.3751		
$\eta\ \eta'^2$	ε	1.640_n	2.514	4.1066_n	4.8961		
$j^2\eta$	ε	1.063	1.916_n	4.1066	4.8961_n	5.4076	
η'	$\varepsilon+\quad\Delta$	9.36	0.471_n	2.4824	3.0830_n	3.3936	
$\eta^2\eta'$	$\varepsilon+\quad\Delta$	1.565	2.456_n	3.9671	4.7890_n		
$\eta\eta'^2$	$\varepsilon+\quad\Delta$	1.543	2.341_n	3.8942	4.6760_n		
$j^2\eta'$	$\varepsilon+\quad\Delta$	0.87_n	1.75	4.0705_n	4.8759	5.3965_n	
$\eta\eta'^2$	$\varepsilon+\quad 2\Delta$	1.441_n	2.273	3.7192_n	4.5456		
$j^2\eta'$	$\varepsilon+\qquad\Sigma$	0.42	1.36_n	3.7799	4.5819_n	5.0998	
$j^2\eta$	$\varepsilon+\quad\Delta+\Sigma$	0.695_n	1.585	4.0417_n	4.7827	5.2543_n	
η	$\varepsilon+4\vartheta+4\Delta$		9.59_n	0.45			
η'	$\varepsilon+4\vartheta+3\Delta$		9.46	0.34_n			
η	$2\varepsilon+2\vartheta+2\Delta$		9.45	0.11_n			
η'	$2\varepsilon+2\vartheta+3\Delta$		9.32_n	0.04			
$\eta^2\eta'$	$-\varepsilon+\quad\Delta$	1.255_n	2.149	3.6240_n	4.4615		
	$(\vartheta-\vartheta_0)^2$ cos						
η	ε		9.25	0.117_n			
η'	$\varepsilon+\quad\Delta$		9.12_n	0.02			
		m'^2	m'^2	m'^2, m'	m'	m'	m'

$\nu=\Sigma w^2\eta p_\eta' q_j^{2i} C_1 \cos \text{Arg.}+(\vartheta-\vartheta_0)\Sigma w^2\eta p_\eta' q_j^{2i} C_2 \sin \text{Arg.}+(\vartheta-\vartheta_0)^2\Sigma w^2\eta p_\eta' q_j^{2i}\ C_3 \cos \text{Arg.}$
where C_1, C_2, C_3 represent the respective coefficients.

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TABLE D (LIV).

Logarithmic. $\Sigma U_{p\cdot q\eta' p\eta' q}$ sin Arg Unit=1″.

	Sin	w^{-1}	w^0	w
η	$-A-\Pi'$		3.0628_m	3.7258
η'	$-\Pi'$		2.8235	3.5528_m
	$2\theta+A-\Pi'$		2.2831	2.8483_m
η	$4\theta+3A-\Pi'$	1.705	3.1591_m	3.9166_m
η'	$4\theta+2A-\Pi'$		3.2462	3.8608
η	$\frac{1}{2}\varepsilon+\theta\quad-\Pi'$		3.2112	3.8544
η'	$\frac{1}{2}\varepsilon+\theta+A-\Pi'$		2.5875	3.4153_m
	$\frac{1}{2}\varepsilon+3\theta+2A-\Pi'$		2.2787	2.6304_m
η'	$\frac{1}{2}\varepsilon+5\theta+3A-\Pi'$		3.3155	3.5865_m
η	$\frac{1}{2}\varepsilon+5\theta+4A-\Pi'$		3.0779_m	3.3972
η	$-\frac{1}{2}\varepsilon-\theta-2A-\Pi'$		3.1158_m	3.7378
η'	$-\frac{1}{2}\varepsilon-\theta-A-\Pi'$		3.1493	3.7544_m
	$-\frac{1}{2}\varepsilon+\theta\quad-\Pi'$		2.3242	3.0060_m
η'	$-\frac{1}{2}\varepsilon+3\theta+A-\Pi'$		3.3863	4.1833_m
η	$-\frac{1}{2}\varepsilon+3\theta+2A-\Pi'$		3.3532_m	4.1452
η	$\varepsilon+2\theta+A-\Pi'$	2.6364	3.3704_m	3.8423
η'	$\varepsilon+2\theta+2A-\Pi'$	1.423_m	2.706	3.4014_m
	$\varepsilon+4\theta+3A-\Pi'$	1.4042_m	2.1720	2.6339_m
η'	$\varepsilon+6\theta+4A-\Pi'$	2.3306_m	3.1922	3.7582_m
η	$\varepsilon+6\theta+5A-\Pi'$	2.1137	3.0138_m	3.6101
η	$-\varepsilon-2\theta-3A-\Pi'$	2.7175	3.4858_m	3.9484
η'	$-\varepsilon-2\theta-2A-\Pi'$	2.7756_m	3.5070	3.9456_m
	$-\varepsilon\quad-A-\Pi'$		1.6810	2.2463_m
η'	$-\varepsilon+2\theta\quad-\Pi'$	2.8125	3.4427_m	3.7846
η	$-\varepsilon+2\theta+A-\Pi'$	2.9121_m	3.4958	3.8338_m
η	$\frac{1}{2}\varepsilon+3\theta+2A-\Pi'$		2.6058	3.5312_m
η'	$\frac{1}{2}\varepsilon+3\theta+3A-\Pi'$		1.760	1.82_m
	$\frac{1}{2}\varepsilon+5\theta+4A-\Pi'$		1.7510_m	2.8113
η'	$\frac{1}{2}\varepsilon+7\theta+5A-\Pi'$		2.9120_m	4.0813
η	$-\frac{1}{2}\varepsilon-3\theta-4A-\Pi'$		2.8673	3.8458_m
η'	$-\frac{1}{2}\varepsilon-3\theta-3A-\Pi'$		2.9620_m	3.9124
	$-\frac{1}{2}\varepsilon-\theta-2A-\Pi'$		2.0569_m	2.7932
η'	$-\frac{1}{2}\varepsilon+\theta-A-\Pi'$		2.9275_m	3.4708
η	$-\frac{1}{2}\varepsilon+\theta\quad-\Pi'$		2.9702	3.5487_m
η	$2\varepsilon+4\theta+3A-\Pi'$		1.640	2.781_m
η'	$2\varepsilon+4\theta+4A-\Pi'$		1.617	2.340_m
	$2\varepsilon+6\theta+5A-\Pi'$		1.206_m	2.2110
η	$-2\varepsilon-4\theta-5A-\Pi'$		2.4012	3.3634_m
η'	$-2\varepsilon-4\theta-4A-\Pi'$		2.5241_m	3.4544
	$-2\varepsilon-2\theta-3A-\Pi'$		1.5290_m	2.3210
η'	$-2\varepsilon\quad-2A-\Pi'$		2.3174_m	3.0558
η	$-2\varepsilon\quad-A-\Pi'$		2.3514	3.0737_m
		m'		

$$\frac{u}{\varepsilon\cos i}=\Sigma U_{p\cdot q\eta' p\eta' q}\ \sin\ \text{Arg.}+n_2 t\left\{K_1(\cos\varepsilon-e_1)+K_2\sin\varepsilon\right\}+c_1(\cos\varepsilon-e_1)+c_2\sin\varepsilon.$$

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TABLE E_1 (LV_I).

Logarithmic. K_1 Unit=1″.

	Cos	w^0	w	w^2
η'^2	$\Delta-\Pi'$	2.9180_n	3.7732	2.8138
	$\Delta+\Pi'$	1.9821	2.5473_n	
η^2	$\Delta+\Pi'$	2.8035	3.7182_n	
η'^2	$\Delta+\Pi'$	3.5175	4.3017_n	
j^2	$\Delta+\Pi'$	3.1764_n	3.9772	
$\eta\,\eta'$	$2\Delta+\Pi'$	3.4580_n	4.2668	
		m'		

$$K_1=\Sigma w^s \eta^p \eta'^q j^{2k} \cos \text{Arg.}$$

TABLE E_2 (LV_{II}).

Logarithmic. K_2 Unit=1″.

	Sin	w^0	w	w^2
η'^2	$\Delta-\Pi'$	2.9180	3.7732_n	2.8138$_n$
$\eta\,\eta'$	Π'	3.7799	4.5819_n	
	$\Delta+\Pi'$	1.9821_n	2.5473	
η^2	$\Delta+\Pi'$	3.7744_n	4.5420	
η'^2	$\Delta+\Pi'$	3.5175_n	4.3017	
$\eta\,\eta'$	$2\Delta+\Pi'$	3.4580	4.2668_n	
j^2	$\Delta+\Pi'$	3.1764	3.9772_n	
		m'		

$$K_2=\Sigma w^s \eta^p \eta'^q j^{2k} \sin \text{Arg.}$$

$$\frac{u}{\epsilon \cos i}=\Sigma U_{p,q} \eta^p \eta'^q \sin \text{Arg.}+nt\left\{K_1(\cos \epsilon-\epsilon)+K_2 \sin \epsilon\right\}+c_1(\cos \epsilon-\epsilon)+c_2 \sin \epsilon$$

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TABLE F (LVI)

Logarithmic. $w - w_0$ Unit = 1 radian.

	Cos	w^{-3}	w^{-2}	w^{-1}	w^0	w	w^2
			4. 360	5. 1966_n	5. 7767		
	Γ			4. 766	6. 6599	7. 3732_n	7. 7492
	2Γ			4. 446	7. 1194	7. 7572_n	8. 0553
	3Γ			4. 412	6. 8442	7. 5458_n	7. 9060
	4Γ			4. 484	6. 5883	7. 3450_n	7. 7602
	5Γ				6. 3437	7. 1490_n	7. 6136
	7Γ				5. 875	6. 7632_n	7. 3134
η_0	$-5\Gamma+2\theta_0+2\Delta_0$				6. 5090	6. 6325_n	7. 4746_n
	$-4\Gamma+2\theta_0+2\Delta_0$			4. 161_n	6. 169	7. 0658	7. 8698_n
	$-3\Gamma+2\theta_0+2\Delta_0$			3. 19	6. 8821_n	7. 6078	7. 9975_n
	$-2\Gamma+2\theta_0+2\Delta_0$			3. 52	7. 0986_n	7. 6970	7. 9394_n
	$-\Gamma+2\theta_0+2\Delta_0$			5. 1420	6. 359	7. 0722_n	7. 4480
	$2\theta_0+2\Delta_0$		4. 379	7. 6355_n	8. 2144	8. 4125_n	
	$\Gamma+2\theta_0+2\Delta_0$			4. 856_n	8. 0894_n	8. 9548	9. 5668_n
	$2\Gamma+2\theta_0+2\Delta_0$			4. 92_n	7. 8150_n	8. 6561	9. 2006_n
	$3\Gamma+2\theta_0+2\Delta_0$			5. 5174_n	7. 6056_n	8. 4650	9. 0111_n
	$4\Gamma+2\theta_0+2\Delta_0$			5. 4248_n	7. 4128_n	8. 2958	8. 8561_n
	$5\Gamma+2\theta_0+2\Delta_0$				7. 2254_n	8. 1426	8. 7346_n
	$7\Gamma+2\theta_0+2\Delta_0$				6. 8746_n	7. 8484	8. 4936_n
η'	$-5\Gamma+2\theta_0+\Delta_0$				6. 8776_n	7. 5604	7. 8425_n
	$-4\Gamma+2\theta_0+\Delta_0$			4. 582	6. 8815_n	7. 4536	7. 5238_n
	$-3\Gamma+2\theta_0+\Delta_0$			4. 674	6. 6271_n	6. 7816	7. 3174
	$-2\Gamma+2\theta_0+\Delta_0$			4. 99	6. 7985	7. 4732_n	7. 7966
	$-\Gamma+2\theta_0+\Delta_0$			5. 4623_n			
	$2\theta_0+\Delta_0$		4. 605_n	7. 1987	7. 8314_n	8. 1061	
	$\Gamma+2\theta_0+\Delta_0$			5. 0056	8. 2964	9. 1086_n	9. 6833
	$2\Gamma+2\theta_0+\Delta_0$			4. 38	8. 0434	8. 8316_n	9. 3296
	$3\Gamma+2\theta_0+\Delta_0$			5. 6251	7. 8458	8. 6564_n	9. 1558
	$4\Gamma+2\theta_0+\Delta_0$			5. 5812	7. 6603	8. 5030_n	9. 0248
	$5\Gamma+2\theta_0+\Delta_0$				7. 4778	8. 3544_n	8. 9050
	$7\Gamma+2\theta_0+\Delta_0$				7. 1130	8. 0545_n	8. 6668
η_0^2		4. 664	4. 71	5. 83			
	Γ				7. 8102	8. 6250_n	
	2Γ				7. 7520_n	8. 1242	
	3Γ				7. 6172_n	6. 6043_n	
	4Γ				7. 7135_n	8. 2308	
η_0^2	$-4\Gamma+4\theta_0+4\Delta_0$				7. 1862	7. 9072_n	
	$-3\Gamma+4\theta_0+4\Delta_0$				7. 1804	7. 8679_n	
	$-2\Gamma+4\theta_0+4\Delta_0$				6. 817	7. 456_n	
	$-\Gamma+4\theta_0+4\Delta_0$				8. 4680_n	8. 8822	
	$4\theta_0+4\Delta_0$	4. 666	5. 807_n	8. 0913	8. 8270_n	9. 2073	
	$\Gamma+4\theta_0+4\Delta_0$				8. 7850	9. 8236_n	
	$2\Gamma+4\theta_0+4\Delta_0$				8. 5144	9. 4910_n	
	$3\Gamma+4\theta_0+4\Delta_0$				8. 3274	9. 3006_n	
	$4\Gamma+4\theta_0+4\Delta_0$				8. 1627	9. 1494_n	
	$5\Gamma+4\theta_0+4\Delta_0$				8. 0050	9. 0105_n	
$\eta_0\,\eta'$	$-4\Gamma+4\theta_0+3\Delta_0$				7. 354_n	8. 1083	
	$-3\Gamma+4\theta_0+3\Delta_0$				7. 5708_n	8. 2084	
	$-2\Gamma+4\theta_0+3\Delta_0$						
	$-\Gamma+4\theta_0+3\Delta_0$				8. 8838	9. 0548_n	
	$4\theta_0+3\Delta_0$	4. 516_n	6. 2084	8. 5565_n	9. 2180	9. 5174_n	
	$\Gamma+4\theta_0+3\Delta_0$				9. 2783_n	0. 2833	
	$2\Gamma+4\theta_0+3\Delta_0$				9. 0241_n	9. 9635	
	$3\Gamma+4\theta_0+3\Delta_0$				8. 8480_n	9. 7850	
	$4\Gamma+4\theta_0+3\Delta_0$				8. 6916_n	9. 6434	
	$5\Gamma+4\theta_0+3\Delta_0$				8. 5401_n	9. 5128	
		m'^2	m'^2	m'^2, m'	m'^2, m'	m'	m'

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TABLE F (LVI)—Continued.

Logarithmic. $w - w_0$ Unit = 1 radian.

	Cos	w^{-3}	w^{-2}	w^{-1}	w^0	w	w^2
$\eta_0\,\eta'$	$-4\Gamma + \Delta_0$				7.7640	7.8364_n	
	$-3\Gamma + \Delta_0$				7.4203	8.3915	
	$-2\Gamma + \Delta_0$				7.8104_n	8.6268	
	$-\Gamma + \Delta_0$				8.0479_n	8.8018	
	$+\Delta_0$	4.518_n	5.886_n	5.70_n			
	$\Gamma + \Delta_0$				7.1339	7.8500	
	$2\Gamma + \Delta_0$				7.8421	8.4293_n	
	$3\Gamma + \Delta_0$				7.9669	8.6796_n	
	$4\Gamma + \Delta_0$				7.9760	8.7576_n	
η'^2	$-4\Gamma + 4\theta_0 + 2\Delta_0$				6.9002	7.6938_n	
	$-3\Gamma + 4\theta_0 + 2\Delta_0$				7.1638	7.8502_n	
	$-2\Gamma + 4\theta_0 + 2\Delta_0$						
	$-\Gamma + 4\theta_0 + 2\Delta_0$				8.1860_n	8.4016	
	$4\theta_0 + 2\Delta_0$	3.76	6.0608_n	8.4157	8.9760_n	9.1661	
	$\Gamma + 4\theta_0 + 2\Delta_0$				9.1714	0.1382_n	
	$2\Gamma + 4\theta_0 + 2\Delta_0$				8.9358	9.8333_n	
	$3\Gamma + 4\theta_0 + 2\Delta_0$				8.7718	9.6681_n	
	$4\Gamma + 4\theta_0 + 2\Delta_0$				8.6236	9.5372_n	
η'^3		3.76	5.7516	4.7			
	Γ				7.8677	8.6727_n	
	2Γ				7.8610_n	8.2228	
	3Γ				8.1026_n	8.7296	
	4Γ				8.1538_n	8.8728	
f^2	Γ				7.9418_n	8.7337	
	2Γ				7.9312_n	8.7154	
	3Γ				7.7920_n	8.6154	
	4Γ				7.639_n	8.5001	
f^2	$-4\Gamma + 4\theta_0 + 3\Delta_0 - \Sigma_0$				7.446	8.1156_n	
	$-3\Gamma + 4\theta_0 + 3\Delta_0 - \Sigma_0$				7.1858	7.8677_n	
	$-2\Gamma + 4\theta_0 + 3\Delta_0 - \Sigma_0$						
	$-\Gamma + 4\theta_0 + 3\Delta_0 - \Sigma_0$				7.6176_n	7.9693	
	$4\theta_0 + 3\Delta_0 - \Sigma_0$		4.804_n	7.168	7.9368_n	8.3724	
	$\Gamma + 4\theta_0 + 3\Delta_0 - \Sigma_0$				7.7887	8.8492_n	
	$2\Gamma + 4\theta_0 + 3\Delta_0 - \Sigma_0$				7.448	8.4531_n	
	$3\Gamma + 4\theta_0 + 3\Delta_0 - \Sigma_0$				7.1976	8.2026_n	
	$4\Gamma + 4\theta_0 + 3\Delta_0 - \Sigma_0$				6.978	7.9963_n	
η_0^3	$2\theta_0 + 2\Delta_0$	5.418_n	6.292	7.4754_n	8.6636		
η_0^3	$6\theta_0 + 6\Delta_0$	5.418_n	6.292	8.6328_n	9.4351		
$\eta_0^2\eta'$	$2\theta_0 + \Delta_0$	5.885	6.719_n	8.5059	9.2804_n		
$\eta_0^2\eta'$	$2\theta_0 + 3\Delta_0$	4.974	5.896_n	8.0326_n	8.1975		
$\eta_0^2\eta'$	$6\theta_0 + 5\Delta_0$	5.935	6.780_n	9.2774	0.0330_n		
$\eta_0\,\eta'^2$	$2\theta_0$	5.744_n	6.535	8.3811_n	9.1030		
$\eta_0\,\eta'^2$	$2\theta_0 + 2\Delta_0$	5.441_n	6.327	8.0917	8.6300		
$\eta_0\,\eta'^2$	$6\theta_0 + 4\Delta_0$	5.919_n	6.744	9.4432_n	0.1464		
η'^3	$2\theta_0 + \Delta_0$	5.301	6.149_n	8.2302	9.0152_n		
η'^3	$6\theta_0 + 3\Delta_0$	5.301	6.149_n	9.1294	9.7729_n		
$f^2\eta_0$	$2\theta_0 + 2\Delta_0$			8.5904	9.3492_n	9.8022	
$f^2\eta_0$	$2\theta_0 + \Delta_0 - \Sigma_0$	4.502_n	5.41	8.1011_n	8.8726		
$f^2\eta_0$	$6\theta_0 + 5\Delta_0 - \Sigma_0$	4.502_n	5.41	8.0554_n	8.9263		
$f^2\ \eta'$	$2\theta_0 + \Delta_0$			8.5592_n	9.3245		
$f^2\ \eta'$	$2\theta_0 + 2\Delta_0 - \Sigma_0$	4.057	5.021_n	6.887	8.1804_n		
$f^2\ \eta'$	$6\theta_0 + 4\Delta_0 - \Sigma_0$	4.057	5.021_n	8.2718	9.1021_n		

$$w - w_0 = \Sigma C w^s \eta^p \eta'^q f^{2z} \cos \text{Arg.}$$

where C represents the coefficient.

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TABLE G (LVII).

Logarithmic. $S \sin \phi + C \cos \phi$ Unit=1″.

	Cos	w^{-3}	w^{-2}	w^{-1}	w^{0}	w	w^{2}
	$\phi-5\Gamma+2\theta_0+2\Delta_0$			8.81	1.082$_n$	1.5710	1.612$_n$
	$\phi-4\Gamma+2\theta_0+2\Delta_0$			9.009	1.2314$_n$	1.5493	0.989$_n$
	$\phi-3\Gamma+2\theta_0+2\Delta_0$			9.318	0.931	1.604$_n$	1.916
	$\phi-2\Gamma+2\theta_0+2\Delta_0$			9.207	1.6478	2.1070$_n$	2.2333
	$\phi-\Gamma+2\theta_0+2\Delta_0$			9.711	1.950	2.3426$_n$	2.3713
	$\phi+2\theta_0+2\Delta_0$		9.196	2.1712$_n$	2.5678	2.565$_n$	
	$\phi+\Gamma+2\theta_0+2\Delta_0$			9.230$_n$	2.3541$_n$	3.1493	3.7107$_n$
	$\phi+2\Gamma+2\theta_0+2\Delta_0$			9.220$_n$	1.9114$_n$	2.6867	3.1657$_n$
	$\phi+3\Gamma+2\theta_0+2\Delta_0$			9.724$_n$	1.5372$_n$	2.3831	2.8623$_n$
	$\phi+4\Gamma+2\theta_0+2\Delta_0$			9.494$_n$	1.2544$_n$	2.1315	2.6333$_n$
	$\phi+5\Gamma+2\theta_0+2\Delta_0$			9.100$_n$	1.018$_n$	1.9034	2.4248$_n$
η_0	$\phi-5\Gamma+4\theta_0+4\Delta_0$			9.771$_n$	1.042$_n$	1.868	2.357$_n$
	$\phi-4\Gamma+4\theta_0+4\Delta_0$			0.064$_n$	1.723$_n$	2.3515	2.6814$_n$
	$\phi-3\Gamma+4\theta_0+4\Delta_0$			0.3185$_n$	2.1626$_n$	2.6961	2.9214$_n$
	$\phi-2\Gamma+4\theta_0+4\Delta_0$			0.497$_n$	2.7787$_n$	3.0649	3.0993$_n$
	$\phi-\Gamma+4\theta_0+4\Delta_0$			1.0286$_n$	3.2379$_n$	3.1223	3.9385$_n$
	$\phi+4\theta_0+4\Delta_0$	9.199	9.04$_n$	2.6172	3.2511$_n$	3.4930	
	$\phi+\Gamma+4\theta_0+4\Delta_0$			0.7226	3.1702	4.1580$_n$	4.9365
	$\phi+2\Gamma+4\theta_0+4\Delta_0$			0.669	2.7877	3.7083$_n$	4.3605
	$\phi+3\Gamma+4\theta_0+4\Delta_0$			0.9435	2.5117	3.4261$_n$	4.0450
	$\phi+4\Gamma+4\theta_0+4\Delta_0$			0.5122	2.2732	3.2042$_n$	
η_0	$\phi-5\Gamma$			9.814$_n$	1.925	2.634$_n$	2.984
	$\phi-4\Gamma$			0.0434$_n$	2.0527	2.6896$_n$	2.9432
	$\phi-3\Gamma$			0.3541$_n$	2.145	2.675$_n$	2.744
	$\phi-2\Gamma$		9.140	0.362$_n$	2.1351	2.3850$_n$	2.4864$_n$
	$\phi-\Gamma$			0.4164$_n$	2.3504$_n$	3.0929	3.5397$_n$
	ϕ		9.274$_n$	0.1436$_n$			
	$\phi+\Gamma$			0.3102$_n$	2.497	3.1875$_n$	3.5978
	$\phi+2\Gamma$		9.137$_n$	9.918	1.9006$_n$	1.0453	2.8834
	$\phi+3\Gamma$			9.465	0.812$_n$	2.5218$_n$	3.3564
	$\phi+4\Gamma$			9.20$_n$	1.406$_n$	1.729	
η'	$\phi-5\Gamma+4\theta_0+3\Delta_0$			9.476	1.327	1.889$_n$	2.2299
	$\phi-4\Gamma+4\theta_0+3\Delta_0$			9.781	1.447	2.1506$_n$	2.5419
	$\phi-3\Gamma+4\theta_0+3\Delta_0$			9.811	2.1070	2.6309$_n$	2.8608
	$\phi-2\Gamma+4\theta_0+3\Delta_0$			0.3489	2.5095	2.9557$_n$	3.0952
	$\phi-\Gamma+4\theta_0+3\Delta_0$			0.9511	3.3599	2.7758	3.9726
	$\phi+4\theta_0+3\Delta_0$	8.76$_n$	0.158	2.7932$_n$	3.3085	3.4526$_n$	
	$\phi+\Gamma+4\theta_0+3\Delta_0$			9.961$_n$	3.3609$_n$	4.3114	5.0691$_n$
	$\phi+2\Gamma+4\theta_0+3\Delta_0$			0.491$_n$	2.9943$_n$	3.8728	4.4922$_n$
	$\phi+3\Gamma+4\theta_0+3\Delta_0$			1.0464$_n$	2.7293$_n$	3.6067	4.1945$_n$
	$\phi+4\Gamma+4\theta_0+3\Delta_0$			0.678$_n$	2.4992$_n$	3.3946	
η'	$\phi-5\Gamma+\Delta_0$			9.848	2.0766$_n$	2.712	2.9697$_n$
	$\phi-4\Gamma+\Delta_0$			0.0792	2.1609$_n$	2.6968	2.7976$_n$
	$\phi-3\Gamma+\Delta_0$			0.3941	2.157$_n$	2.491	1.51
	$\phi-2\Gamma+\Delta_0$		9.013$_n$	0.248	2.0455	2.7898$_n$	3.2380
	$\phi-\Gamma+\Delta_0$			9.901	2.584	3.2539$_n$	3.6434
	ϕ		9.885$_n$	0.8518			
	$\phi+\Gamma+\Delta_0$			0.1664	1.836	2.448	3.3029$_n$
	$\phi+2\Gamma+\Delta_0$		9.009	9.76$_n$	2.1633	2.6170$_n$	2.2433
	$\phi+3\Gamma+\Delta_0$			9.38$_n$	2.1064	2.7194$_n$	2.9212
	$\phi+4\Gamma+\Delta_0$				1.9892	2.6870$_n$	
η_0^2	$\phi-5\Gamma+6\theta_0+6\Delta_0$				2.3144	2.9730$_n$	
	$\phi-4\Gamma+6\theta_0+6\Delta_0$				2.9538	3.3785$_n$	
	$\phi-3\Gamma+6\theta_0+6\Delta_0$				3.3102	3.5843$_n$	
	$\phi-2\Gamma+6\theta_0+6\Delta_0$				3.4970	3.8423$_n$	
	$\phi-\Gamma+6\theta_0+6\Delta_0$				3.9455	3.7269$_n$	
	$\phi+6\theta_0+6\Delta_0$	9.95$_n$	1.1109$_n$	3.1673$_n$	3.9296	4.3377$_n$	
	$\phi+\Gamma+6\theta_0+6\Delta_0$				3.9144$_n$	5.0372	
	$\phi+2\Gamma+6\theta_0+6\Delta_0$				3.5594$_n$	4.5942	
	$\phi+3\Gamma+6\theta_0+6\Delta_0$				3.3121$_n$	4.3236	
η_0^2	$\phi-5\Gamma+2\theta_0+2\Delta_0$				2.1657	2.7221$_n$	
	$\phi-4\Gamma+2\theta_0+2\Delta_0$				2.1255	2.8004$_n$	
	$\phi-3\Gamma+2\theta_0+2\Delta_0$				2.234	3.1304$_n$	
	$\phi-2\Gamma+2\theta_0+2\Delta_0$				2.576	3.3804$_n$	
	$\phi-\Gamma+2\theta_0+2\Delta_0$				3.1995	3.8325$_n$	
	$\phi+2\theta_0+2\Delta_0$	0.344$_n$	1.017	2.689$_n$	3.4822	3.9938$_n$	
	$\phi+\Gamma+2\theta_0+2\Delta_0$				2.2480	3.2839$_n$	
	$\phi+2\Gamma+2\theta_0+2\Delta_0$		9.45		3.1612	3.8424$_n$	

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TABLE G (LVII)—Continued.

Logarithmic. $S \sin \phi + C \cos \phi$ Unit=1″.

	Cos	10^{-3}	10^{-2}	10^{-1}	10^{0}	10	10^{2}
η_0^2	$\phi-5\Gamma-2\theta_0-2\Delta_0$				2.700_n	3.5481	
	$\phi-4\Gamma-2\theta_0-2\Delta_0$				2.817_n	3.6251	
	$\phi-3\Gamma-2\theta_0-2\Delta_0$				2.9247_n	3.6905	
	$\phi-2\Gamma-2\theta_0-2\Delta_0$		9.59_n		3.0241_n	3.7470	
	$\phi-\Gamma-2\theta_0-2\Delta_0$				3.1364_n	3.8346	
	$\phi-2\theta_0-2\Delta_0$	0.117	0.95_n	2.297_n	2.7856_n	3.6614	
	$\phi+\Gamma-2\theta_0-2\Delta_0$				2.8942_n	3.5604	
	$\phi+2\Gamma-2\theta_0-2\Delta_0$				2.297_n	3.1129	
$\eta_0\,\eta'$	$\phi-5\Gamma+6\theta_0+5\Delta_0$				2.4885_n	3.1691	
	$\phi-4\Gamma+6\theta_0+5\Delta_0$				2.976_n	3.5560	
	$\phi-3\Gamma+6\theta_0+5\Delta_0$				3.6541_n	3.8829	
	$\phi-2\Gamma+6\theta_0+5\Delta_0$				3.9514_n	4.1632	
	$\phi-\Gamma+6\theta_0+5\Delta_0$				4.3903_n	4.0037_n	
	$\phi+6\theta_0+5\Delta_0$	0.295	1.366	3.6364	4.3301_n	4.6662	
	$\phi+\Gamma+6\theta_0+5\Delta_0$				4.4005	5.4966_n	
	$\phi+2\Gamma+6\theta_0+5\Delta_0$				4.0582	5.0612_n	
	$\phi+3\Gamma+6\theta_0+5\Delta_0$				3.8204	4.8027_n	
$\eta_0\,\eta'$	$\phi-5\Gamma+2\theta_0+\Delta_0$				2.426_n	3.0684	
	$\phi-4\Gamma+2\theta_0+\Delta_0$				2.399_n	3.0310	
	$\phi-3\Gamma+2\theta_0+\Delta_0$				2.410_n	3.1305	
	$\phi-2\Gamma+2\theta_0+\Delta_0$				2.701_n	3.4602	
	$\phi-\Gamma+2\theta_0+\Delta_0$				3.2842_n	3.8558	
	$\phi+2\theta_0+\Delta_0$	0.444	1.188_n	3.0569	3.7266_n	4.1122	
	$\phi+\Gamma+2\theta_0+\Delta_0$				2.8541	3.5823_n	
	$\phi+2\Gamma+2\theta_0+\Delta_0$				3.2191_n	3.7635	
$\eta_0\,\eta'$	$\phi-5\Gamma-2\theta_0-\Delta_0$				3.1551	3.9530_n	
	$\phi-4\Gamma-2\theta_0-\Delta_0$				3.2454	3.9948_n	
	$\phi-3\Gamma-2\theta_0-\Delta_0$				3.3100	4.0023_n	
	$\phi-2\Gamma-2\theta_0-\Delta_0$		9.93		3.3277	3.9401_n	
	$\phi-\Gamma-2\theta_0-\Delta_0$				3.1976	3.4598_n	
	$\phi-2\theta_0-\Delta_0$	0.490_n	1.324	3.0145_n	3.7326	4.2787	
	$\phi+\Gamma-2\theta_0-\Delta_0$				3.3632	3.9402_n	
	$\phi+2\Gamma-2\theta_0-\Delta_0$				2.7792	3.5224_n	
$\eta_0\,\eta'$	$\phi-5\Gamma+2\theta_0+3\Delta_0$				2.2738_n	2.847	
	$\phi-4\Gamma+2\theta_0+3\Delta_0$				2.116_n	3.0290	
	$\phi-3\Gamma+2\theta_0+3\Delta_0$				2.5858_n	3.3787	
	$\phi-2\Gamma+2\theta_0+3\Delta_0$				2.809_n	3.5429	
	$\phi-\Gamma+2\theta_0+3\Delta_0$				2.650_n	3.7297	
	$\phi+2\theta_0+3\Delta_0$	9.98	0.60_n	2.873_n	2.685	3.7980	
	$\phi+\Gamma+2\theta_0+3\Delta_0$				3.5126_n	4.2856	
	$\phi+2\Gamma+2\theta_0+3\Delta_0$		9.46_n		3.3438_n	4.1208	
η'^2	$\phi-5\Gamma+6\theta_0+4\Delta_0$				1.9950	2.7422_n	
	$\phi-4\Gamma+6\theta_0+4\Delta_0$				2.6112	3.1949_n	
	$\phi-3\Gamma+6\theta_0+4\Delta_0$				3.0556	3.5583_n	
	$\phi-2\Gamma+6\theta_0+4\Delta_0$				3.7934	3.7947_n	
	$\phi-\Gamma+6\theta_0+4\Delta_0$				4.2260	4.4064	
	$\phi+6\theta_0+4\Delta_0$	9.98_n	0.76_n	3.5017_n	4.1098	4.3552_n	
	$\phi+\Gamma+6\theta_0+4\Delta_0$				4.2852_n	5.3521	
	$\phi+2\Gamma+6\theta_0+4\Delta_0$				3.9567_n	4.9249	
η'^2	$\phi-5\Gamma+2\theta_0+2\Delta_0$				2.5018	3.0963_n	
	$\phi-4\Gamma+2\theta_0+2\Delta_0$				2.453	3.0935_n	
	$\phi-3\Gamma+2\theta_0+2\Delta_0$				2.4799	3.2779_n	
	$\phi-2\Gamma+2\theta_0+2\Delta_0$				2.9375	3.6294_n	
	$\phi-\Gamma+2\theta_0+2\Delta_0$				3.2833	3.8982_n	
	$\phi+2\theta_0+2\Delta_0$	0.025_n	0.60	2.634	3.2781	4.0439_n	
	$\phi+\Gamma+2\theta_0+2\Delta_0$				3.5607	4.2381_n	
	$\phi+2\Gamma+2\theta_0+2\Delta_0$				3.4629	4.1704_n	
η'^2	$\phi-5\Gamma-2\theta_0$				3.0090_n	3.7477	
	$\phi-4\Gamma-2\theta_0$				3.0676_n	3.7445	
	$\phi-3\Gamma-2\theta_0$				3.0764_n	3.6664	
	$\phi-2\Gamma-2\theta_0$				2.958_n	3.3121	
	$\phi-\Gamma-2\theta_0$				3.1140	4.0201_n	
	$\phi-2\theta_0$	0.305	1.127_n	2.912	3.5491_n	3.9085	
	$\phi+\Gamma-2\theta_0$				3.0396_n	3.6320	
	$\phi+2\Gamma-2\theta_0$				2.4706_n	3.2330	

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TABLE G (LVII)—Continued.

Logarithmic. $S \sin \psi + C \cos \psi$ Unit=1″.

	Cos	w^{-3}	w^{-2}	w^{-1}	w^0	w	w^2
j^3	$\psi - 5\Gamma + 6\theta_0 + 5\Delta_0 - \Sigma_0$				2. 006	2.7505_n	
	$\psi - 4\Gamma + 6\theta_0 + 5\Delta_0 - \Sigma_0$				2. 335	2.981_n	
	$\psi - 3\Gamma + 6\theta_0 + 5\Delta_0 - \Sigma_0$				2. 544	3.1436_n	
	$\psi - 2\Gamma + 6\theta_0 + 5\Delta_0 - \Sigma_0$				2. 718	3.2445_n	
	$\psi - \Gamma + 6\theta_0 + 5\Delta_0 - \Sigma_0$				2. 970	2.9116_n	
	$\psi + 6\theta_0 + 5\Delta_0 - \Sigma_0$	8.6_n	9. 7	2.114_n	2. 923	3.4067_n	
	$\psi + \Gamma + 6\theta_0 + 5\Delta_0 - \Sigma_0$				2.7948_n	3. 9420	
	$\psi + 2\Gamma + 6\theta_0 + 5\Delta_0 - \Sigma_0$				2.3824_n	3. 4488	
j^2	$\psi - 5\Gamma + 2\theta_0 + 2\Delta_0$				9. 6	2. 387	
	$\psi - 4\Gamma + 2\theta_0 + 2\Delta_0$				1.916_n	2. 911	
	$\psi - 3\Gamma + 2\theta_0 + 2\Delta_0$				2.5178_n	3. 3047	
	$\psi - 2\Gamma + 2\theta_0 + 2\Delta_0$				2.938_n	3. 6294	
	$\psi - \Gamma + 2\theta_0 + 2\Delta_0$				3.3406_n	3. 9330	
	$\psi + 2\theta_0 + 2\Delta_0$		0. 5910	3. 1266	3.8021_n	4. 1894	
	$\psi + \Gamma + 2\theta_0 + 2\Delta_0$				3. 4070	4.3178_n	
	$\psi + 2\Gamma + 2\theta_0 + 2\Delta_0$				3. 0472	3.9308_n	
j^3	$\psi - 5\Gamma - 2\theta_0 - \Delta_0 + \Sigma_0$				0.732_n	1. 085	
	$\psi - 4\Gamma - 2\theta_0 - \Delta_0 + \Sigma_0$				0. 35	1.895_n	
	$\psi - 3\Gamma - 2\theta_0 - \Delta_0 + \Sigma_0$				1. 463	2.5146_n	
	$\psi - 2\Gamma - 2\theta_0 - \Delta_0 + \Sigma_0$				2. 064	3.0255_n	
	$\psi - \Gamma - 2\theta_0 - \Delta_0 + \Sigma_0$				2. 6816	3.6280_n	
	$\psi - 2\theta_0 - \Delta_0 + \Sigma_0$	9. 04	0.11_n	2. 636	3.3284_n	3. 7399	
	$\psi + \Gamma - 2\theta_0 - \Delta_0 + \Sigma_0$				3.0572_n	3. 6490	
	$\psi + 2\Gamma - 2\theta_0 - \Delta_0 + \Sigma_0$				2.9121_n	3. 5491	
η_0^3	$\psi + 4\theta_0 + 4\Delta_0$	0. 775	1.65_n	3.1052_n	3.0342_n		
	$\psi - 4\theta_0 - 4\Delta_0$	0.29_n	1. 10	3. 1888	3.6104_n		
	$\psi + 8\theta_0 + 8\Delta_0$	0. 65	1.54_n	3. 7520	4.5812_n		
$\eta_0^2\eta'$	$\psi + 4\theta_0 + 5\Delta_0$			3. 7577	4.3244_n		
	$\psi + 4\theta_0 + 3\Delta_0$	1.260_n	2. 081	3. 1240	4. 1388		
	$\psi - 4\theta_0 - 3\Delta_0$	1. 005	1.77_n	3.5356_n	3. 3560		
	$\psi + 8\theta_0 + 7\Delta_0$	1.228_n	2. 093	4.3980_n	5. 1827		
$\eta_0\eta'^2$	$\psi + 4\theta_0 + 4\Delta_0$			4.1155_n	4. 5547		
	$\psi + 4\theta_0 + 2\Delta_0$	1. 106	1.88_n	2. 831	4.1803_n		
	$\psi - 4\theta_0 - 2\Delta_0$	1.146_n	1. 88	3. 0422	4. 0180		
	$\psi + 8\theta_0 + 6\Delta_0$	1. 321	2.152_n	4. 5658	5.3010_n		
η'^3	$\psi + 4\theta_0 + 3\Delta_0$			3. 8375	4.0446_n		
	$\psi - 4\theta_0 - \Delta_0$			3. 2197	3.9650_n		
	$\psi + 8\theta_0 + 5\Delta_0$			4.2553_n	4. 9349		
$j^2\ \eta_0$	$\psi + 4\theta_0 + 3\Delta_0 - \Sigma_0$			3. 0024	3.8634_n		
	$\psi - 4\theta_0 - 3\Delta_0 + \Sigma_0$	9.98_n	0. 8	2.956_n	3. 8331		
	$\psi + 8\theta + 7\Delta_0 - \Sigma_0$			3. 0757	3.9759_n		
	$\psi + 4\theta_0 + 4\Delta_0$	0.46_n	1. 32	3.8514_n	4. 6436		
$j^2\ \eta'$	$\psi + 4\theta_0 + 4\Delta_0 - \Sigma_0$			2. 442	1.846_n		
	$\psi - 4\theta_0 - 2\Delta_0 + \Sigma_0$			3. 2486	4.0585_n		
	$\psi + 8\theta_0 + 6\Delta_0 - \Sigma_0$			3.2818_n	4. 1441		
	$\psi + 4\theta_0 + 3\Delta_0$	0. 27	1.15_n	3. 9421	4.6972_n		

$S \sin \psi + C \cos \psi = \Sigma C w^a \eta^p \eta'^q j^{2s} \cos \text{Arg.}$

where C represents the coefficient.

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II. TABLES FOR THE DETERMINATION OF THE PERTURBATIONS OF THE HECUBA GROUP OF MINOR PLANETS.

DEVELOPMENT OF THE DIFFERENTIAL EQUATIONS FOR W AND FOR THE THIRD COORDINATE.

It would be futile to attempt to give a brief but comprehensive outline of the fundamental developments in the theory of Bohlin-v. Zeipel which would assist the reader to an understanding of the construction of the tables. In broad outlines, the problem is the integration of Hansen's differential equations for $n\delta z$, ν, and $\frac{u}{\cos i}$, by means of the method developed by Bohlin and according to the modifications introduced by v. Zeipel for purposes of numerical computation. The first division of the problem is the development of functions of the partial derivatives of the perturbative function; the second division of the problem is the integration of the Hansen equations in the form of infinite series.

For the theory the reader is referred to the original works of Hansen[1], Bohlin[2], and v. Zeipel[3]. As indicated in the introduction to the first section, unless otherwise stated, the references to Bohlin refer to the French edition and are designated by B; references to v. Zeipel are designated by Z. Although duplication of material which can be found in either reference is to be avoided, our experience in attempting to reproduce v. Zeipel's tables led us to fill in certain gaps which are troublesome to the reader and the computer.

The first section of v. Zeipel's theory is concerned with an independent development of Hansen's differential equations for $n\delta z$ and ν and a repetition of the differential equation for $\frac{u}{\cos i}$, and the introduction of Bohlin's argument θ. In passing, it is well to emphasize two facts: First, the variables ϵ and f used throughout the theory are *analogous* to Hansen's $\bar{\epsilon}$ and $\bar{f}$; the dash is unnecessary, for the physically real values do not appear. Second, the constant elements a, e, π, c, Ω, i are neither osculating nor mean elements; they are defined in the section on constants of integration.

The perturbative function and its partial derivatives are developed in Fourier's series, in which the arguments depend upon the relative positions of the disturbed and disturbing bodies and in which the coefficients are infinite series in ascending powers of the eccentricities and the inclination of the orbits. The coefficients in the latter are elliptic integrals depending upon the ratio of the semi-major axes.

Since these elliptic integrals are functions of the ratio of the semi-major axes, or of the mean daily motions, they can be developed in Taylor's series, in which the given function and its successive partial derivatives are expressed for exact commensurability and the series proceeds according to a small quantity w, defined by $w=1-2\mu$, where μ is the ratio of *Jupiter's* mean motion to that of the planet and where μ differs but little from $\frac{1}{2}$. These elliptic integrals enter the coefficients in all of the subsequent trigonometric series. Hence all the coefficients are series in w. With some exceptions the terms in w^0, w, and w^2 have been used. The development of all functions in powers of w is the essential principle underlying the group method of determining perturbations.

The following pages contain the tables which are, in general, parallel to those of v. Zeipel. At the end of sections 2, 3, 4, 5 there are brief written comparisons. To facilitate comparisons

[1] Auseinandersetzung einer zweckmässigen Methode zur Berechnung der absoluten Störungen der kleinen Planeten.

[2] Formeln und Tafeln zur gruppenweisen Berechnung der allgemeinen Störungen benachbarter Planeten. Nova Acta Reg. Soc. Sc. Upsaliensis, Ser. III, Band XVII, 1896.
Sur le Développement des Perturbations Planétaires. Application aux Petites Planètes. Stockholm, 1902.

[3] Angenäherte Jupiterstörungen für die Hecuba-Gruppe. St. Pétersbourg, 1902.

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with v. Zeipel's tables, those numerical quantities which are in disagreement are inclosed in brackets. There are also certain mathematical developments useful to the reader. These relations are sometimes taken from v. Zeipel and sometimes supplement his text.

Certain simple functions of the elliptic integrals $\gamma_i^{m,n}$, defined by *Z* 19, eqs. (73), (74), (75), are tabulated in Table I (cf. Z 23).

Tables II–IVw^2 (cf. Z 26–32), giving the partial derivatives of the perturbative function, are computed according to *Z* 24, eq. (77), by means of Table I and *B* 184, Tables XVI–XVIII and *B* (Ger.) 182, Tables XII–XIV.

The elimination of *Jupiter's* mean anomaly from the argument gives *Z* 25, eq. (78), in which the coefficients are derived from Table II–IV w^2 by the formulae given in *B* 61. These coefficients are tabulated in Tables V–VII w^2 (cf. *Z* 33–39).

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Logarithmic. TABLE I[1]. Unit=1".

n	0	1	2	3	4	5	6	7	8	9	10
$\omega_0^{1.n}$	2. 1464086	{1. 196164}	1. 655608	1. 380429	1. 124592	0. 880042	0. 642905	0. 411123	0. 183046	9. 95811	9. 73559
$\omega_1^{1.n}$	1. 333861	1. 334774	1. 068171	0. 817690	0. 576825	0. 34328	0. 11256	9. 88623	9. 66263	9. 44121	9. 22161
$\omega_2^{1.n}$	0. 892830	0. 934567	0. 688725	0. 45110	0. 21906	9. 99110	9. 76625	9. 54386	9. 32346	9. 10470	8. 88734
$\omega_3^{1.n}$	0. 54795	0. 60716	0. 37239	0. 14249	9. 91620	9. 69269	9. 47141	9. 25194	9. 03398	8. 81730	8. 60172
$\omega_4^{1.n}$	0. 24722	0. 31599	0. 08801	9. 86323	9. 64095	9. 4207	9. 2021	8. 9849	8. 7688		
$\omega_5^{1.n}$	9. 97185	0. 04660	9. 82322	9. 60211	9. 38289	9. 1650	8. 9485				
$\omega_0^{2.n}$	{1. 898161}	2. 283141	2. 153770	2. 006171	1. 847875	1. 68250	1. 51210	1. 33802	1. 16081	0. 98134	0. 79996
$\omega_1^{2.n}$	2. 221528	2. 382049	2. 227850	2. 065161	1. 896645	1. 72387	1. 54784	1. 36930	1. 18855	1. 00614	0. 82229
$\omega_2^{2.n}$	2. 226015	2. 367601	2. 201284	2. 03009	1. 85527	1. 67762	1. 49771	1. 31598			
$\omega_3^{2.n}$	2. 17409	2. 30577	2. 13230	1. 95568	1. 77661	1. 5964	1. 4127				
$\omega_0^{5.n}$	2. 726380	3. 001680	2. 941123	2. 856672	2. 75505	2. 64058	2. 51617	2. 38392			
$\omega_1^{5.n}$	3. 22034	3. 44169	3. 34392	3. 23252	3. 11061						
$\varepsilon\vert\varepsilon\vert\varepsilon\vert\varepsilon\vert\varepsilon\vert\varepsilon_0^{1.n}$	2. 086910$_n$	{1. 696947$_n$}	2. 025704$_n$	1. 862100$_n$	1. 694089$_n$	1. 522172$_n$	1. 347043$_n$	1. 16938$_n$	0. 98938$_n$	0. 80767$_n$	0. 62442$_n$
$\varepsilon\vert\varepsilon\vert\varepsilon\vert\varepsilon\vert\varepsilon\vert\varepsilon_1^{1.n}$	1. 806020$_n$	1. 906225$_n$	1. 716180$_n$	1. 529203$_n$	1. 342993$_n$	1. 15669$_n$	0. 96997$_n$	0. 78270$_n$	0. 59483$_n$	0. 40635$_n$	0. 21732$_n$
$\varepsilon\vert\varepsilon\vert\varepsilon\vert\varepsilon\vert\varepsilon\vert\varepsilon_2^{1.n}$	1. 603899$_n$	1. 704102$_n$	1. 508098$_n$	1. 31435$_n$	1. 12169$_n$	0. 92956$_n$	0. 73762$_n$	0. 54570$_n$	0. 35368$_n$	0. 16150$_n$	9. 96912$_n$
$\varepsilon\vert\varepsilon\vert\varepsilon\vert\varepsilon\vert\varepsilon\vert\varepsilon_3^{1.n}$	1. 41336$_n$	1. 51356$_n$	1. 31543$_n$	1. 11886$_n$	0. 92322$_n$	0. 72817$_n$	0. 53345$_n$	0. 33892$_n$	0. 1444$_n$	9. 9500$_n$	
$\varepsilon\vert\varepsilon\vert\varepsilon\vert\varepsilon\vert\varepsilon\vert\varepsilon_4^{1.n}$	1. 22659$_n$	1. 32679$_n$	1. 12769$_n$	0. 92971$_n$	0. 73250$_n$	0. 53579$_n$	0. 33942$_n$	0. 14337$_n$	9. 9476$_n$		
$\varepsilon\vert\varepsilon\vert\varepsilon\vert\varepsilon\vert\varepsilon_0^{3.n}$	{2. 51524$_n$}	2. 84832$_n$	2. 78195$_n$	2. 69286$_n$	2. 58759$_n$	2. 47019$_n$	2. 34339$_n$	2. 20917$_n$	2. 06865$_n$	1. 92310$_n$	1. 77326$_n$
$\varepsilon\vert\varepsilon\vert\varepsilon\vert\varepsilon_1^{3.n}$	2. 95077$_n$	3. 15383$_n$	3. 04132$_n$	2. 91803$_n$	2. 78638$_n$	2. 64800$_n$	2. 50413$_n$	2. 35570$_n$	2. 20324$_n$	2. 04749$_n$	1. 88881$_n$
$\varepsilon\vert\varepsilon\vert\varepsilon_2^{3.n}$	3. 10392$_n$	3. 27800$_n$	3. 14309$_n$	3. 00185$_n$	2. 85545$_n$	2. 70474$_n$	2. 55037$_n$	2. 39290$_n$			
$\varepsilon\vert\varepsilon\vert\varepsilon_0^{5.n}$	3. 51583$_n$	3. 80371$_n$	3. 76747$_n$	3. 71207$_n$	3. 64099$_n$	3. 55696$_n$	3. 46222$_n$	3. 35851$_n$			
$\varepsilon\vert\varepsilon_1^{5.n}$	4. 13087$_n$	4. 37108$_n$	4. 29565$_n$	4. 20810$_n$	4. 11042$_n$						
$\varepsilon\vert\varepsilon\vert\varepsilon\vert\varepsilon\vert\varepsilon_0^{1.n}$	1. 34814	{1. 79250}	1. 94314	1. 92018	1. 85954	1. 77438	1. 67197	1. 55683	1. 43166	1. 29874	1. 15943
$\varepsilon\vert\varepsilon\vert\varepsilon\vert\varepsilon\vert\varepsilon_1^{1.n}$	1. 93221	2. 12908	2. 01556	1. 89337	1. 76366	1. 62754	1. 48597	1. 33971	1. 18943	1. 03563	0. 87878
$\varepsilon\vert\varepsilon\vert\varepsilon\vert\varepsilon_2^{1.n}$	1. 99420	2. 14965	2. 00180	1. 85113	1. 69761	1. 54142	1. 38270	1. 22170	1. 05858	0. 89355	
$\varepsilon\vert\varepsilon\vert\varepsilon_3^{1.n}$	1. 96578	2. 10476	1. 94185	1. 77768	1. 61213	1. 44520	1. 27690	1. 10730			
$\varepsilon\vert\varepsilon\vert\varepsilon\vert_0^{3.n}$	{2. 81277}	3. 11481	3. 09258	3. 05198	2. 99474	2. 92310	2. 83919	2. 74499	2. 64179	2. 53103	2. 41372
$\varepsilon\vert\varepsilon\vert\varepsilon\vert_1^{3.n}$	3. 38698	3. 62237	3. 54448	3. 45591	3. 35805	3. 25216	3. 13926	3. 02025			
$\varepsilon\vert\varepsilon\vert\varepsilon\vert_2^{3.n}$	3. 68273	3. 88445	3. 77729								

[1] For $n=0$ the table gives $\log \frac{1}{2}\omega$. For explanation of brackets, { }, see [ ], Z 14, eq. (63). (v.Zeipel) $\overline{\omega}_i^{m\cdot n} = -\overline{\omega}_i^{s\cdot n}$ (Bohlin).

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Logarithmic. TABLE II. Unit=1″.

n	0	1	2	3	4	5	6	7	8	9	10
$P_{0\cdot0}[n.-n]$	———	1.196164	1.956638	1.857550	1.726652	1.579012	1.421056	1.256221	1.08614	0.91235	0.73559
$P_{1\cdot0}[n+1.-n]$	1.634891_n	1.936834_n	1.846322_n	1.720780_n	1.576825_n	1.42156_n	1.25869_n	1.09035_n	0.91790_n	0.74224_n	0.56403_n
$P_{1\cdot0}[n-1.-n]$	1.634891	———	2.310470_n	2.497527_n	2.534520_n	2.506584_n	2.441893_n	2.353544_n	2.24835_n	2.13091_n	2.00396_n
$P_{0\cdot1}[n.-n+1]$	———	1.635604	1.640475_n	2.019701_n	2.112868_n	2.113048_n	2.065516_n	1.988837_n	1.89209_n	1.78110_n	1.65922_n
$P_{0\cdot1}[n.-n-1]$	———	2.025593	2.832690	2.880695	2.859122	2.798595	2.713086	2.610276	2.49444	2.36884	2.23544
$P_{2\cdot0}[n+2.-n]$	2.023856	2.225475	2.096678	1.95119	1.79511	1.63176	1.46311	1.29047	1.11470	0.93639	0.75603
$P_{2\cdot0}[n.-n]$	———	2.261969	2.117706	2.10627	2.04755	1.96143	1.85717	1.73972	1.61291	1.47809	1.33716
$P_{2\cdot0}[n-2.-n]$	2.023856_n	2.199192_n	———	2.659837	2.948238	3.069862	3.113630	3.110973	3.07683	3.02020	2.94643
$P_{1\cdot1}[n+1.-n+1]$	2.283141_n	2.492130_n	2.272769_n	1.95625_n	1.45482_n	0.76106	1.33715	1.42913	1.41871	1.36100	1.27562
$P_{1\cdot1}[n-1.-n+1]$	2.283141	———	1.396759	2.572122	2.881369	3.017744	3.071278	3.075436	3.04626	2.99338	2.92258
$P_{1\cdot1}[n+1.-n-1]$	2.283141_n	2.817171_n	2.874471_n	2.858127_n	2.800971_n	2.71782_n	2.61666_n	2.50228_n	2.37777_n	2.24526_n	2.10626_n
$P_{1\cdot1}[n-1.-n-1]$	2.283141	———	3.206887_n	3.531457_n	3.673689_n	3.730739_n	3.737245_n	3.710125_n	3.65862_n	3.58899_n	3.50313_n
$P_{0\cdot2}[n.-n+2]$	———	2.177073	2.038481	1.952501	2.071811	2.196710	2.269672	2.295743	2.28582	2.24911	2.19176
$P_{0\cdot2}[n.-n]$	———	2.404066	2.698322_n	3.116000_n	3.292184_n	3.367379_n	3.385271_n	3.366005_n	3.32023_n	3.25499_n	3.17458_n
$P_{0\cdot2}[n.-n-2]$	———	2.722215	3.551521	3.707664	3.773067	3.785150	3.761920	3.713611	3.64622	3.56422	3.47045
$P_{0\cdot0}[n+1.-n+1]+\sigma$	1.898161	2.584171	2.630891	2.608231	2.546845	2.46065	2.35720	2.24111	2.11505	1.98134	1.84135
$P_{0\cdot0}[n-1.-n-1]-\sigma$	1.898161_n	———	2.153770	2.307201	2.324996	2.28456	2.21107	2.11617	2.00591	1.88443	1.75420
$P_{0\cdot0}[n+1.-n-1]+\delta$	1.898161_n	2.584171_n	2.630891_n	2.608231_n	2.546845_n	2.46065_n	2.35720_n	2.24111_n	2.11505_n	1.98134_n	1.84135_n
$P_{0\cdot0}[n-1.-n+1]-\delta$	1.898161	———	2.153770_n	2.307201_n	2.324996_n	2.28456_n	2.21107_n	2.11617_n	2.00591_n	1.88443_n	1.75420_n
$P_{3\cdot0}[n+1.-n]$	2.25167_n										
$P_{3\cdot0}[n-1.-n]$	2.25167		2.44348_n		2.30607_n						
$P_{3\cdot0}[n-3.-n]$					3.02278_n		3.55125_n		3.69307_n		
$P_{2\cdot1}[n.-n+1]$		2.95041									
$P_{2\cdot1}[n-2.-n+1]$		2.82358_n		2.50563_n		3.54376_n					
$P_{2\cdot1}[n.-n-1]$		3.21039		3.42436							
$P_{2\cdot1}[n-2.-n-1]$				3.71288		4.30087		4.47100			
$P_{1\cdot2}[n-1.-n+2]$	2.91212		2.68308_n								
$P_{1\cdot2}[n+1.-n]$	2.93746_n										
$P_{1\cdot2}[n-1.-n]$	2.93746		2.55874		4.06225						
$P_{1\cdot2}[n+1.-n-2]$	2.91212_n		3.71917_n								
$P_{1\cdot2}[n-1.-n-2]$			3.94779_n		4.59492_n		4.78966_n				
$P_{0\cdot3}[n.-n+1]$		2.95804									
$P_{0\cdot3}[n.-n-1]$		3.06432		3.99638							
$P_{0\cdot3}[n.-n-3]$		3.36346		4.44360		4.65468					
$P_{1\cdot0}[n.-n+1]+\sigma$		3.09682_n									
$P_{1\cdot0}[n.-n-1]-\sigma$		2.93746_n		3.11584							
$P_{1\cdot0}[n-2.-n-1]-\sigma$				2.92477		3.24597		3.24515			

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$P_{1\cdot0}[n.-n-1]+\partial$		3.09682		3.49574							
$P_{1\cdot0}[n.-n+1]-\partial$		2.93746									
$P_{1\cdot0}[n-2.-n+1]-\partial$		2.93746n		2.92477		3.24597					
$P_{0\cdot1}[n+1.-n]+\sigma$	2.52256										
$P_{0\cdot1}[n-1.-n]-\sigma$	2.52256n		1.72430		2.57107n						
$P_{0\cdot1}[n-1.-n-2]-\sigma$			3.24621		3.58597		3.59437				
$P_{0\cdot1}[n+1.-n]+\delta$	2.52256n										
$P_{0\cdot1}[n+1.-n-2]+\partial$	2.81256n		3.72333n								
$P_{0\cdot1}[n-1.-n+2]-\partial$	2.81256		2.79444n								
$P_{0\cdot1}[n-1.-n]-\delta$	2.52256		3.07659n		3.47840n						

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Logarithmic. TABLE II w. Unit=1".

n	0	1	2	3	4	5	6	7	8	9	10
$P_{0\cdot0}[n.-n]$	———	1.696947$_n$	2.326734$_n$	2.339221$_n$	2.296149$_n$	2.221142$_n$	2.125194$_n$	2.01448$_n$	1.89247$_n$	1.76191$_n$	1.62442$_n$
$P_{1\cdot0}[n+1.-n]$	2.107050	2.508285	2.494331	2.432293	2.342993	2.23587	2.11610	1.98682	1.85010	1.70738	1.55974
$P_{1\cdot0}[n-1.-n]$	2.107050$_n$	———	2.722988	3.003816	3.120166	3.160145	3.154557	3.11841	3.05995	2.98480	2.89638
$P_{0\cdot1}[n.-n+1]$	———	2.207255$_n$	0.613819	2.368909	2.620202	2.717783	2.744353	2.72873	2.68442	2.61961	2.53911
$P_{0\cdot1}[n.-n-1]$	———	2.556580$_n$	3.228771$_n$	3.377812$_n$	3.438885$_n$	3.448053$_n$	3.422723$_n$	3.37281$_n$	3.30420$_n$	3.22121$_n$	3.12662$_n$
$P_{2\cdot0}[n+2.-n]$	2.65255$_n$	2.92884$_n$	2.85948$_n$	2.76460$_n$	2.65301$_n$	2.52955$_n$	2.39716$_n$	2.25778$_n$	2.11278$_n$	1.96311$_n$	1.80954$_n$
$P_{2\cdot0}[n.-n]$	———	2.92348$_n$	3.05366$_n$	3.10049$_n$	3.08583$_n$	3.03584$_n$	2.96268$_n$	2.87290$_n$	2.77085$_n$	2.65893$_n$	2.53921$_n$
$P_{2\cdot0}[n-2.-n]$	2.65255	2.81627	———	3.20793$_n$	3.55914$_n$	3.74023$_n$	3.83827$_n$	3.88477$_n$	3.89537$_n$	3.87961$_n$	3.84335$_n$
$P_{1\cdot1}[n+1.-n+1]$	2.84832	3.16261	3.03557	2.81950	2.49006	1.80523	1.93390$_n$	2.21271$_n$	2.28216$_n$	2.27834$_n$	
$P_{1\cdot1}[n-1.-n+1]$	2.84832$_n$	———	2.54847	2.78202$_n$	3.36491$_n$	3.61642$_n$	3.74911$_n$	3.81610$_n$	3.84000$_n$	3.83352$_n$	
$P_{1\cdot1}[n+1.-n-1]$	2.84832	3.43829	3.55417	3.59183	3.58361	3.54488	3.48425	3.40707	3.31690	3.21626	
$P_{1\cdot1}[n-1.-n-1]$	2.84832$_n$	———	3.66096	4.06072	4.27377	4.39417	4.45708	4.48041	4.47449	4.44629	
$P_{0\cdot2}[n.-n+2]$	———	2.79945$_n$	2.75578$_n$	2.66099$_n$	2.71530$_n$	2.84736$_n$	2.96308$_n$	3.03888$_n$	3.07708$_n$	3.08493$_n$	
$P_{0\cdot2}[n.-n]$	———	3.04572$_n$	2.17570	3.46537	3.80207	3.97510	4.06697	4.10839	4.11475	4.09540	
$P_{0\cdot2}[n.-n-2]$	———	3.28370$_n$	3.97539$_n$	4.22145$_n$	4.36389$_n$	4.44247$_n$	4.47743$_n$	4.48070$_n$	4.45961$_n$	4.41955$_n$	
$P_{0\cdot0}[n+1.-n+1]+\sigma$	2.51524$_n$	3.14934$_n$	3.25907$_n$	3.29492$_n$	3.28656$_n$	3.24834$_n$	3.18849$_n$	3.11226$_n$	3.02289$_n$	2.92310$_n$	2.81465$_n$
$P_{0\cdot0}[n-1.-n-1]-\sigma$	2.51524	———	2.78195$_n$	2.99389$_n$	3.06471$_n$	3.07225$_n$	3.04236$_n$	2.98732$_n$	2.91375$_n$	2.82619$_n$	2.72750$_n$
$P_{0\cdot0}[n+1.-n-1]+\delta$	2.51524	3.14934	3.25907	3.29492	3.28656	3.24834	3.18849	3.11226	3.02289	2.92310	2.81465
$P_{0\cdot0}[n-1.-n+1]-\delta$	2.51524$_n$	———	2.78195	2.99389	3.06471	3.07225	3.04236	2.98732	2.91375	2.82619	2.72750
$P_{3\cdot0}[n+1.-n]$	3.04286										
$P_{3\cdot0}[n-1.-n]$	3.04286$_n$		3.36582		3.63249						
$P_{3\cdot0}[n-3.-n]$					3.67411		4.29317		4.52091		
$P_{2\cdot1}[n.-n+1]$		3.71643$_n$									
$P_{2\cdot1}[n-2.-n+1]$		3.55282		2.58404$_n$		4.12637					
$P_{2\cdot1}[n.-n-1]$		3.93243$_n$		4.34583$_n$							
$P_{2\cdot1}[n-2.-n-1]$				4.29563$_n$		4.98494$_n$		5.25186$_n$			
$P_{1\cdot2}[n-1.-n+2]$	3.54910$_n$		3.45486								
$P_{1\cdot2}[n+1.-n]$	3.62947										
$P_{1\cdot2}[n-1.-n]$	3.62947$_n$		3.59594		4.54375$_n$						
$P_{1\cdot2}[n+1.-n-2]$	3.54910		4.43013								
$P_{1\cdot2}[n-1.-n-2]$			4.44517		5.21067		5.51720				
$P_{0\cdot2}[n.-n+1]$		3.66472$_n$									
$P_{0\cdot2}[n.-n-1]$		3.77533$_n$		4.34178							
$P_{0\cdot2}[n.-n-3]$		3.95620$_n$		4.97543$_n$		5.32048$_n$					
$P_{1\cdot0}[n.-n+1]+\sigma$		3.75365									
$P_{1\cdot0}[n.-n-1]-\sigma$		3.62946		3.89906							
$P_{1\cdot0}[n-2.-n-1]-\sigma$				3.66409		4.06165		4.13352			

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Unit=1″.

TABLE II w.[3]

$P_{1\cdot0}[n.-n-1]+\delta$			3. 75365$_m$		4. 22581$_m$							
$P_{1\cdot0}[n.-n+1]-\delta$			3. 62946$_m$		3. 66409$_m$		4. 06165$_m$					
$P_{1\cdot0}[n-2.-n+1]-\delta$			3. 62946									
$P_{0\cdot1}[n+1.-n]+\sigma$	3. 25180$_m$											
$P_{0\cdot1}[n-1.-n]-\sigma$	3. 25180			2. 99523$_m$		2. 98848						
$P_{0\cdot1}[n-1.-n-2]-\sigma$				3. 91656$_m$		4. 34713$_m$		4. 43848$_m$				
$P_{0\cdot1}[n+1.-n]+\delta$	3. 25180											
$P_{0\cdot1}[n+1.-n-2]+\delta$	3. 49076			4. 39370								
$P_{0\cdot1}[n-1.-n+2]-\delta$	3. 49076$_m$			3. 53278								
$P_{0\cdot1}[n-1.-n]-\delta$	3. 25180$_m$			3. 76576		4. 24543						

$P_{0\cdot0}[n.-n]$	———	1. 79250	2. 24417	2. 39730	2. 46160	2. 47335	2. 45012	2. 40193	2. 33475	2. 25298	2. 15943	
$P_{1\cdot0}[n+1.-n]$	2. 23324$_m$	2. 73114$_m$	2. 78871$_m$	2. 79646$_m$	2. 76366$_m$	2. 70672$_m$	2. 63210$_m$	2. 54383$_m$	2. 44470$_m$	2. 33666$_m$		
$P_{1\cdot0}[n-1.-n]$	2. 23324	———	2. 74649$_m$	3. 11775$_m$	3. 31910$_m$	3. 43434$_m$	3. 49490$_m$	3. 51720$_m$	3. 51090$_m$	3. 48266$_m$		
$P_{0\cdot1}[n.-n+1]$	———	2. 43011	2. 37864	1. 47552$_m$	2. 60654$_m$	2. 88392$_m$	3. 01794$_m$	3. 08198$_m$	3. 10228$_m$	3. 09238$_m$		
$P_{0\cdot1}[n.-n-1]$	———	2. 71373	3. 21558	3. 47213	3. 62606	3. 71456	3. 75768	3. 76766	3. 75213	3. 71674		
$P_{2\cdot0}[n.-n]$	———		3. 52227		3. 80784	4. 04609	4. 20009					
$P_{2\cdot0}[n-2.-n]$			———									
$P_{1\cdot1}[n-1.-n+1]$				3. 00694$_m$								
$P_{1\cdot1}[n+1.-n-1]$		3. 73949$_m$		———								
$P_{1\cdot1}[n-1.-n-1]$		———		4. 22410$_m$	4. 50219$_m$	4. 68741$_m$						
$P_{0\cdot2}[n.-n]$			3. 44001									
$P_{0\cdot2}[n.-n-2]$	———		4. 02969	4. 35355	4. 57425							
$P_{0\cdot0}[n+1.-n+1]+\sigma$		3. 41584										
$P_{0\cdot0}[n-1.-n-1]-\sigma$		———		3. 35301								
$P_{0\cdot0}[n+1.-n-1]+\delta$		3. 41584$_m$		———								
$P_{0\cdot0}[n-1.-n+1]-\delta$		———		3. 35301$_m$								
$P_{1\cdot0}[n.-n+1]+\sigma$		4. 18333$_m$										
$P_{1\cdot0}[n.-n-1]+\delta$		4. 13333										
$P_{1\cdot0}[n.-n+1]-\delta$		4. 04093		4. 09610								
$P_{1\cdot0}[n-2.-n+1]-\delta$		4. 04093$_m$										
$P_{0\cdot1}[n+1.-n]+\sigma$	3. 68801											
$P_{0\cdot1}[n-1.-n]-\sigma$	3. 68801$_m$											
$P_{0\cdot1}[n+1.-n]+\delta$	3. 68801$_m$											
$P_{0\cdot1}[n-1.-n+2]-\delta$				3. 97690$_m$								
$P_{0\cdot1}[n-1.-n]-\delta$	3. 68801											

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Logarithmic. TABLE III. Unit=1″.

n	0	1	2	3	4	5	6	7	8	9	10
$Q_{0\cdot0}[n.-n]$	1. 634891	1. 770420	2. 056512	1. 930336	1. 784183	1. 626670	1. 461775	1. 291778	1. 11765	0. 94075	0. 76140
$Q_{1\cdot0}[n+1.-n]$	2. 172568$_n$	2. 297774$_n$	2. 122754$_n$	1. 946074$_n$	1. 767441$_n$	1. 58698$_n$	1. 40490$_n$	1. 22142$_n$	1. 03671$_n$	0. 85090$_n$	0. 66417$_n$
$Q_{1\cdot0}[n-1.-n]$	2. 172568$_n$	2. 500222$_n$	2. 769564$_n$	2. 777719$_n$	2. 736598$_n$	2. 664600$_n$	2. 571599$_n$	2. 463518$_n$	2. 34380$_n$	2. 21522$_n$	2. 07946$_n$
$Q_{0\cdot1}[n.-n+1]$	2. 283141	2. 297774	1. 273400	1. 914007$_n$	2. 093349$_n$	2. 116256$_n$	2. 076961$_n$	2. 003548$_n$	1. 90802$_n$	1. 79730$_n$	1. 67524$_n$
$Q_{0\cdot1}[n.-n-1]$	2. 283141	2. 637762	2. 968456	2. 973184	2. 929137	2. 854882	2. 760132	2. 650672	2. 52983	2. 40035	2. 26382
$Q_{2\cdot0}[n+2.-n]$	2. 453562	2. 566336	2. 380403	2. 19478	2. 00881	1. 82226	1. 63507	1. 44723	1. 25876	1. 06969	0. 88007
$Q_{2\cdot0}[n.-n]$	2. 754592	3. 031368	2. 893804	2. 769075	2. 632508	2. 48787	2. 33729	2. 18203	2. 02324	1. 86136	1. 69674
$Q_{2\cdot0}[n-2.-n]$	2. 453562	2. 916121	3. 162607	3. 293194	3. 356762	3. 372490	3. 353968	3. 310214	3. 24697	3. 16860	3. 07783
$Q_{1\cdot1}[n+1.-n+1]$	2. 937465$_n$	2. 970967$_n$	2. 681434$_n$	2. 35193$_n$	1. 93963$_n$	1. 22854$_n$	1. 18462	1. 43523	1. 46243	1. 41794	1. 33737
$Q_{1\cdot1}[n-1.-n+1]$	2. 937465$_n$	3. 124618$_n$	2. 744514$_n$	2. 427062	2. 956557	3. 110594	3. 160757	3. 157977	3. 12177	3. 06253	2. 98615
$Q_{1\cdot1}[n+1.-n-1]$	2. 937465$_n$	3. 237886$_n$	3. 187950$_n$	3. 108818$_n$	3. 010146$_n$	2. 89742$_n$	2. 77408$_n$	2. 64243$_n$	2. 50409$_n$	2. 36024$_n$	2. 21178$_n$
$Q_{1\cdot1}[n-1.-n-1]$	2. 937465$_n$	3. 414623$_n$	3. 721093$_n$	3. 840450$_n$	3. 893162$_n$	3. 900319$_n$	3. 875164$_n$	3. 826219$_n$	3. 75883$_n$	3. 67708$_n$	3. 58369$_n$
$Q_{0\cdot2}[n.-n+2]$	2. 912119	2. 900312	2. 571441	2. 299258	2. 217983	2. 254404	2. 298011	2. 314149	2. 30055	2. 26230	2. 20417
$Q_{0\cdot2}[n.-n]$	2. 937465	3. 150133	2. 137444	3. 010528$_n$	3. 275878$_n$	3. 374162$_n$	3. 399983$_n$	3. 383558$_n$	3. 33861$_n$	3. 27329$_n$	3. 19243$_n$
$Q_{0\cdot2}[n.-n-2]$	2. 912119	3. 372021	3. 724815	3. 821212	3. 856471	3. 850676	3. 815716	3. 759151	3. 68567	3. 59900	3. 50152
$Q_{0\cdot0}[n+1.-n+1]+\sigma$	2. 615092	2. 937464	2. 883904	2. 804889	2. 707475	2. 59632	2. 47454	2. 34446	2. 20738	2. 06475	1. 91742
$Q_{0\cdot0}[n-1.-n-1]-\sigma$	2. 615092	2. 937464	2. 883904	2. 804889	2. 707475	2. 59632	2. 47454	2. 34446	2. 20738	2. 06475	1. 91742
$Q_{0\cdot0}[n+1.-n-1]+\delta$	2. 615092$_n$	2. 937464$_n$	2. 883904$_n$	2. 804889$_n$	2. 707475$_n$	2. 59632$_n$	2. 47454$_n$	2. 34446$_n$	2. 20738$_n$	2. 06475$_n$	1. 91742$_n$
$Q_{0\cdot0}[n-1.-n+1]-\delta$	2. 615092$_n$	2. 937464$_n$	2. 883904$_n$	2. 804889$_n$	2. 707475$_n$	2. 59632$_n$	2. 47454$_n$	2. 34446$_n$	2. 20738$_n$	2. 06475$_n$	1. 91742$_n$
$Q_{2\cdot0}[n+1.-n]$	3. 16940$_n$										
$Q_{2\cdot0}[n-1.-n]$	3. 16940$_n$				3. 31760$_n$						
$Q_{2\cdot0}[n-3.-n]$					3. 78025$_n$				3. 95271$_n$		
$Q_{2\cdot1}[n.-n+1]$		[3. 83982]									
$Q_{2\cdot1}[n-2.-n+1]$		3. 67126				3. 74216$_n$					
$Q_{2\cdot1}[n.-n-1]$				4. 00864							
$Q_{2\cdot1}[n-2.-n-1]$				4. 38857				4. 67817			
$Q_{1\cdot2}[n-1.-n+2]$			3. 57901$_n$								
$Q_{1\cdot2}[n+1.-n]$	3. 74233$_n$										
$Q_{1\cdot2}[n-1.-n]$	3. 74233$_n$				4. 13481						
$Q_{1\cdot2}[n+1.-n-2]$			4. 06877$_n$								
$Q_{1\cdot2}[n-1.-n-2]$			4. 51776$_n$				4. 93640$_n$				
$Q_{0\cdot3}[n.-n+1]$		3. 77983									
$Q_{0\cdot3}[n.-n-1]$				3. 88524$_n$							
$Q_{0\cdot3}[n.-n-3]$		4. 05121				4. 73013					
$Q_{1\cdot0}[n.-n+1]+\sigma$		3. 86075$_n$									
$Q_{1\cdot0}[n.-n-1]-\sigma$				3. 54750$_n$							
$Q_{1\cdot0}[n-2.-n-1]-\sigma$				3. 78391$_n$				3. 57149$_n$			

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$Q_{1\cdot0}[n.-n-1]+\delta$										
$Q_{1\cdot0}[n.-n+1]-\delta$	3. 74232						3. 71396			
$Q_{1\cdot0}[n-2.-n+1]-\delta$	3. 74232									
$Q_{0\cdot1}[n+1.-n]+\sigma$		3. 37022								
$Q_{0\cdot1}[n-1.-n]-\sigma$		3. 37022		4. 02594						
$Q_{0\cdot1}[n-1.-n-2]-\sigma$						3. 86667				
$Q_{0\cdot1}[n+1.-n]+\delta$		3. 37022$_n$		4. 02594$_n$				2. 55964$_n$		
$Q_{0\cdot1}[n+1.-n-2]+\delta$				3. 65248$_n$						3. 87205
$Q_{0\cdot1}[n-1.-n+2]-\delta$		3. 37022$_n$								
$Q_{0\cdot1}[n-1.-n]-\delta$								3. 89184$_n$		

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Logarithmic. TABLE III w. Unit=1″.

n	0	1	2	3	4	5	6	7	8	9	10
$Q_{0\cdot0}[n.-n]$	2. 107050$_n$	2. 324123$_n$	2. 500012$_n$	2. 456410$_n$	2. 383497$_n$	2. 290224$_n$	2. 182060$_n$	2. 06264$_n$	1. 93418$_n$	1. 79862$_n$	1. 65717$_n$
$Q_{1\cdot0}[n+1.-n]$	2. 761374	2. 948526	2. 828604	2. 701746	2. 568459	2. 42951	2. 28563	2. 13748	1. 98561	1. 83045	1. 67243
$Q_{1\cdot0}[n-1.-n]$	2. 761374	3. 117300	3. 287547	3. 346232	3. 362633	3. 346294	3. 304908	3. 24416	3. 16781	3. 07910	2. 98013
$Q_{0\cdot1}[n.-n+1]$	2. 848322$_n$	2. 948526$_n$	2. 553490$_n$	1. 83791	2. 55053	2. 70884	2. 75385	2. 74519	2. 70312	2. 63898	2. 55831
$Q_{0\cdot1}[n.-n-1]$	2. 848322$_n$	3. 238533$_n$	3. 460532$_n$	3. 526791$_n$	3. 545790$_n$	3. 530227$_n$	3. 488898$_n$	3. 42791$_n$	3. 35124$_n$	3. 26216$_n$	3. 16280$_n$
$Q_{2\cdot0}[n+2.-n]$	3. 19118$_n$	3. 35005$_n$	3. 20584$_n$	3. 05864$_n$	2. 90832$_n$	2. 75501$_n$	2. 59890$_n$	2. 44023$_n$	2. 27924$_n$	2. 11612$_n$	
$Q_{2\cdot0}[n.-n]$	3. 49221$_n$	3. 78130$_n$	3. 72230$_n$	3. 64840$_n$	3. 55797$_n$	3. 45496$_n$	3. 34206$_n$	3. 22114$_n$	3. 09370$_n$	2. 96069$_n$	
$Q_{2\cdot0}[n-2.-n]$	3. 19118$_n$	3. 62597$_n$	3. 79235$_n$	3. 92707$_n$	4. 02294$_n$	4. 08050$_n$	4. 10554$_n$	4. 10412$_n$	4. 08099$_n$	4. 04028$_n$	
$Q_{1\cdot1}[n+1.-n+1]$	3. 62947	3. 72967	3. 50687	3. 25188	2. 94397	2. 52009	1. 34395	2. 13049$_n$	2. 30111$_n$	2. 32725$_n$	
$Q_{1\cdot1}[n-1.-n+1]$	3. 62947	3. 85386	3. 66684	3. 13763	3. 29292$_n$	3. 67977$_n$	3. 83164$_n$	3. 89857$_n$	3. 91787$_n$	3. 90586$_n$	
$Q_{1\cdot1}[n+1.-n-1]$	3. 62947	3. 95032	3. 93470	3. 89345	3. 83267	3. 75650	3. 66791	3. 56910	3. 46175	3. 34713	
$Q_{1\cdot1}[n-1.-n-1]$	3. 62947	4. 09283	4. 30437	4. 44724	4. 54300	4. 59781	4. 61956	4. 61498	4. 58903	4. 54583	
$Q_{0\cdot2}[n.-n+2]$	3. 54909$_n$	3. 61500$_n$	3. 35799$_n$	3. 10979$_n$	2. 96526$_n$	2. 95859$_n$	3. 01478$_n$	3. 06840$_n$	3. 09806$_n$	3. 10227$_n$	
$Q_{0\cdot2}[n.-n]$	3. 62947$_n$	3. 88133$_n$	3. 61213$_n$	2. 74115	3. 72579	3. 96812	4. 07974	4. 12792	4. 13642	4. 11736	
$Q_{0\cdot2}[n.-n-2]$	3. 54909$_n$	4. 01770$_n$	4. 26538$_n$	4. 40391$_n$	4. 49178$_n$	4. 53874$_n$	4. 55360$_n$	4. 54322$_n$	4. 51236$_n$	4. 46502$_n$	
$Q_{0\cdot0}[n+1.-n+1]+\sigma$	3. 32494$_n$	3. 62946$_n$	3. 60373$_n$	3. 55967$_n$	3. 49933$_n$	3. 42500$_n$	3. 33884$_n$	3. 24270$_n$	3. 13782$_n$	3. 02565$_n$	2. 90711$_n$
$Q_{0\cdot0}[n-1.-n-1]-\sigma$	3. 32494$_n$	3. 62946$_n$	3. 60373$_n$	3. 55967$_n$	3. 49933$_n$	3. 42500$_n$	3. 33884$_n$	3. 24270$_n$	3. 13782$_n$	3. 02565$_n$	2. 90711$_n$
$Q_{0\cdot0}[n+1.-n-1]+\delta$	3. 32494	3. 62946	3. 60373	3. 55967	3. 49933	3. 42500	3. 33884	3. 24270	3. 13782	3. 02565	2. 90711
$Q_{0\cdot0}[n-1.-n+1]-\delta$	3. 32494	3. 62946	3. 60373	3. 55967	3. 49933	3. 42500	3. 33884	3. 24270	3. 13782	3. 02565	2. 90711
$Q_{2\cdot0}[n+1.-n]$	4. 03175										
$Q_{2\cdot0}[n-1.-n]$	4. 03175				4. 32136						
$Q_{2\cdot0}[n-3.-n]$					4. 50550				4. 80030		
$Q_{2\cdot1}[n.-n+1]$		[4. 68736$_n$]									
$Q_{2\cdot1}[n-2.-n+1]$		4. 49191$_n$				4. 24224					
$Q_{2\cdot1}[n.-n-1]$				4. 89559$_n$							
$Q_{2\cdot1}[n-2.-n-1]$				5. 07295$_n$				5. 48264$_n$			
$Q_{1\cdot2}[n-1.-n+2]$			4. 42446								
$Q_{1\cdot2}[n+1.-n]$	4. 54525										
$Q_{1\cdot2}[n-1.-n]$	4. 54525				4. 41262						
$Q_{1\cdot2}[n+1.-n-2]$			4. 85406								
$Q_{1\cdot2}[n-1.-n-2]$			5. 16184				5. 69280				
$Q_{0\cdot2}[n.-n+1]$		4. 58186$_n$									
$Q_{0\cdot2}[n.-n-1]$				2. 59704							
$Q_{0\cdot2}[n.-n-3]$		4. 74024$_n$				5. 43190$_n$					
$Q_{1\cdot0}[n.-n+1]+\sigma$		4. 63965									
$Q_{1\cdot0}[n.-n-1]-\sigma$				4. 40186							
$Q_{1\cdot0}[n-2.-n-1]-\sigma$				4. 59921				[4. 49128] [1]			

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$Q_{1\cdot 0}[n.-n-1]+\delta$				4.67205_n							
$Q_{1\cdot 0}[n.-n+1]-\delta$		4.54525_n									
$Q_{1\cdot 0}[n-2.-n+1]-\delta$		4.54525_n				4.57721_n					
$Q_{0\cdot 1}[n+1.-n]+\sigma$	4.19068_n										
$Q_{0\cdot 1}[n-1.-n]-\sigma$	4.19068_n				3.22170_n						
$Q_{0\cdot 1}[n-1.-n-2]-\sigma$			4.79432_n				4.75273_n				
$Q_{0\cdot 1}[n+1.-n]+\delta$	4.19068										
$Q_{0\cdot 1}[n+1.-n-2]+\delta$			4.79432								
$Q_{0\cdot 1}[n-1.-n+2]-\delta$			4.47930								
$Q_{0\cdot 1}[n-1.-n]-\delta$	4.19068				4.71754						

[1] In v. Zeipel's table this quantity is misplaced. It occurs in the column $n-5$.

Logarithmic. TABLE IIIw^2. Unit=1″.

n	0	1	2	3	4	5	6	7	8	9	10
$Q_{0\cdot0}[n.-n]$	2. 23324	2. 52014	2. 58292	2. 60872	2. 60803	2. 58234	2. 53549	2. 47134	2. 39284	2. 30268	2. 20271
$Q_{1\cdot0}[n+1.-n]$	3. 05368$_n$	3. 28708$_n$	3. 21286$_n$	3. 13038$_n$	3. 03946$_n$	2. 94059$_n$	2. 83441$_n$	2. 72166$_n$	2. 60305$_n$	2. 47917$_n$	
$Q_{1\cdot0}[n-1.-n]$	3. 05368$_n$	3. 41485$_n$	3. 50016$_n$	3. 57826$_n$	3. 63742$_n$	3. 67159$_n$	3. 68131$_n$	3. 66944$_n$	3. 63892$_n$	3. 59276$_n$	
$Q_{0\cdot1}[n.-n+1]$	3. 11484	3. 28708	3. 09682	2. 73074	2. 08444$_n$	2. 81745$_n$	3. 01401$_n$	3. 09669$_n$	3. 12342$_n$	3. 11549$_n$	
$Q_{0\cdot1}[n.-n-1]$	3. 11484	3. 51345	3. 63465	3. 73328	3. 80402	3. 84435	3. 85750	3. 84754	3. 81809	3. 77253	
$Q_{2\cdot0}[n.-n]$			4. 21525								
$Q_{2\cdot0}[n-2.-n]$			4. 15081			4. 45147	4. 51320				
$Q_{1\cdot1}[n-1.-n+1]$				4. 00343$_n$							
$Q_{1\cdot1}[n+1.-n-1]$		4. 36877$_n$									
$Q_{1\cdot1}[n-1.-n-1]$		4. 47566$_n$			4. 86044$_n$	4. 95144$_n$					
$Q_{0\cdot2}[n.-n+2]$	3. 90847										
$Q_{0\cdot2}[n.-n]$			4. 22059								
$Q_{0\cdot2}[n.-n-2]$	3. 90847			4. 66360	4. 78473						
$Q_{0\cdot0}[n+1.-n+1]+\sigma$		4. 04093									
$Q_{0\cdot0}[n-1.-n-1]-\sigma$		4. 04093									
$Q_{0\cdot0}[n+1.-n-1]+\delta$		4. 04093$_n$									
$Q_{0\cdot0}[n-1.-n+1]-\partial$		[4. 04093$_n$]		4. 00956$_n$							
$Q_{1\cdot0}[n.-n+1]+\sigma$		5. 14238$_n$									
$Q_{1\cdot0}[n.-n-1]+\partial$											
$Q_{1\cdot0}[n.-n+1]-\partial$		5. 06752									
$Q_{1\cdot0}[n-2.-n+1]-\partial$		5. 06752									
$Q_{0\cdot1}[n+1.-n]+\sigma$	4. 72557										
$Q_{0\cdot1}[n-1.-n]-\sigma$	4. 72557										
$Q_{0\cdot1}[n+1.-n]+\delta$	4. 72557$_n$										
$Q_{0\cdot1}[n-1.-n+2]-\partial$			5. 01861$_n$								
$Q_{0\cdot1}[n-1.-n]-\delta$	[4. 72557$_n$]										

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Logarithmic. TABLE IV. Unit=1″.

	n	0	1	2	3	4	5	6
	$R_{0\cdot0}[n.-n+1]+\pi'$	1.898161$_n$	2.283141$_n$	2.153770$_n$	2.006171$_n$	1.847875$_n$	1.68250$_n$	1.51210$_n$
	$R_{0\cdot0}[n.-n-1]-\pi'$	1.898161	2.283141	2.153770	2.006171	1.847875	1.68250	1.51210
	$R_{1\cdot0}[n+1.-n+1]+\pi'$	2.522558	2.683079	2.528880	2.366191	2.197675	2.02490	1.84887
	$R_{1\cdot0}[n-1.-n+1]+\pi'$	2.522558	2.937464	2.958044	2.924776	2.858077	2.76886	2.66352
	$R_{1\cdot0}[n+1.-n-1]-\pi'$	2.522558$_n$	2.683079$_n$	2.528880$_n$	2.366191$_n$	2.197675$_n$	2.02490$_n$	1.84887$_n$
	$R_{1\cdot0}[n-1.-n-1]-\pi'$	2.522558$_n$	2.937464$_n$	2.958044$_n$	2.924776$_n$	2.858077$_n$	2.76886$_n$	2.66352$_n$
	$R_{0\cdot1}[n.-n+2]+\pi'$	2.812563$_n$	3.024413$_n$	2.794447$_n$	2.523495$_n$	2.197675$_n$	1.76164$_n$	0.74650$_n$
	$R_{0\cdot1}[n.-n]+\pi'$	2.522558$_n$	3.024413$_n$	3.076598$_n$	3.058902$_n$	3.001314$_n$	2.91803$_n$	2.81684$_n$
	$R_{0\cdot1}[n.-n]-\pi'$	2.522558	2.462558	1.724281	1.856833$_n$	2.093957$_n$	2.12968$_n$	2.09513$_n$
	$R_{0\cdot1}[n.-n-2]-\pi'$	2.812563	3.261391	3.246209	3.190606	3.108845	3.00884	2.89541
	$R_{2\cdot0}[n.-n+1]+\pi'$		3.49405$_n$					
	$R_{2\cdot0}[n-2.-n+1]+\pi'$		3.36728$_n$					
	$R_{1\cdot1}[n-1.-n+2]+\pi'$			3.70912				
	$R_{1\cdot1}[n+1.-n]+\pi'$	3.30370						
	$R_{1\cdot1}[n-1.-n]+\pi'$	3.30370						
	$R_{1\cdot1}[n+1.-n]-\pi'$	3.30370$_n$						
	$R_{1\cdot1}[n-1.-n]-\pi'$	3.30370$_n$						
	$R_{0\cdot2}[n.-n+1]+\pi'$		3.81842$_n$					
	$R_{0\cdot2}[n.-n+1]-\pi'$		3.21895					
	$R_{0\cdot0}[n-1.-n]-\sigma+\pi'$	2.72638$_n$						
	$R_{0\cdot0}[n+1.-n]+\delta+\pi'$	2.72638						
	$R_{0\cdot0}[n-1.-n+2]-\delta+\pi'$			2.94112				
	$R_{0\cdot0}[n+1.-n]+\sigma-\pi'$	2.72638						
	$R_{0\cdot0}[n-1.-n]-\delta-\pi'$	2.72638$_n$						
Factor w.	$R_{0\cdot0}[n.-n+1]+\pi'$	2.51524	2.84832	2.78195	2.69286	2.58759	2.47019	
	$R_{0\cdot0}[n.-n-1]-\pi'$	2.51524$_n$	2.84832$_n$	2.78195$_n$	2.69286$_n$	2.58759$_n$	2.47019$_n$	
	$R_{1\cdot0}[n+1.-n+1]+\pi'$	3.25180$_n$	3.45486$_n$	3.34235$_n$	3.21906$_n$	3.08741$_n$	2.94903$_n$	
	$R_{1\cdot0}[n-1.-n+1]+\pi'$	3.25180$_n$	3.62946$_n$	3.66471$_n$	3.66409$_n$	3.63529$_n$	3.58453$_n$	
	$R_{1\cdot0}[n+1.-n-1]-\pi'$	3.25180	3.45468	3.34235	3.21906	3.08741	2.94903	
	$R_{1\cdot0}[n-1.-n-1]-\pi'$	3.25180	3.62946	3.66471	3.66409	3.63529	3.58453	
	$R_{0\cdot1}[n.-n+2]+\pi'$	3.49076	3.69598	3.53278	3.33224	3.08741	2.77380	
	$R_{0\cdot1}[n.-n]+\pi'$	3.25180	3.69598	3.76576	3.78484	3.76831	3.72575	
	$R_{0\cdot1}[n.-n]-\pi'$	3.25180$_n$	3.33141$_n$	2.99523$_n$	2.24789$_n$	2.51136	2.76863	
	$R_{0\cdot1}[n.-n-2]-\pi'$	3.49076$_n$	3.89134$_n$	3.91658$_n$	3.90661$_n$	3.87001$_n$	3.81284$_n$	
	$R_{2\cdot0}[n.-n+1]+\pi'$		4.36208					
	$R_{2\cdot0}[n-2.-n+1]+\pi'$		4.18801					
	$R_{1\cdot1}[n-1.-n+2]+\pi'$			4.52584$_n$				
	$R_{1\cdot1}[n+1.-n]+\pi'$	4.13780$_n$						
	$R_{1\cdot1}[n-1.-n]+\pi'$	4.13780$_n$						
	$R_{1\cdot1}[n+1.-n]-\pi'$	4.13780						
	$R_{1\cdot1}[n-1.-n]-\pi'$	4.13780						
	$R_{0\cdot2}[n.-n+1]+\pi'$		4.60272					
	$R_{0\cdot2}[n.-n+1]-\pi'$		4.07416$_n$					
	$R_{0\cdot0}[n-1.-n]-\sigma+\pi'$	3.51583						
	$R_{0\cdot0}[n+1.-n]+\delta+\pi'$	3.51583$_n$						
	$R_{0\cdot0}[n-1.-n+2]-\delta+\pi'$			3.76747$_n$				
	$R_{0\cdot0}[n+1.-n]+\sigma-\pi'$	3.51583$_n$						
	$R_{0\cdot0}[n-1.-n]-\delta-\pi'$	3.51583						
Factor w^2.	$R_{0\cdot0}[n.-n+1]+\pi'$		3.1148$_n$		3.0520$_n$			
	$R_{0\cdot0}[n.-n-1]-\pi'$		3.1148		3.0520		2.9231	
	$R_{1\cdot0}[n+1.-n+1]+\pi'$							
	$R_{1\cdot0}[n-1.-n+1]+\pi'$				4.0961			
	$R_{1\cdot0}[n+1.-n-1]-\pi'$		3.9234$_n$					
	$R_{1\cdot0}[n-1.-n-1]-\pi'$		4.0409$_n$				4.0774$_n$	
	$R_{0\cdot1}[n.-n+2]+\pi'$	3.8736$_n$						
	$R_{0\cdot1}[n.-n]+\pi'$			4.1593$_n$				
	$R_{0\cdot1}[n.-n]-\pi'$			3.6562				
	$R_{0\cdot1}[n.-n-2]-\pi'$	3.8736$_n$				4.3090		

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TABLE V. Unit=1″.

n	0	1	2	3	4	5	6	7	8	9	10
$P_{0\cdot0}(n.-n)$	———	+ 15.7096	+ 90.4978	+ 72.0360	+ 53.2917	+ 37.9325	+ 26.3667	+ 18.0394	+ 12.194	+ 8.172	+ 5.440
$P_{1\cdot0}(n+1.-n)$	− 43.141	− 78.609	+ 20.300	+ 55.479	+ 68.840	+ 68.434	+ 60.958	+ 50.825	+ 40.497	+ 31.252	+ 23.535
$P_{1\cdot0}(n-1.-n)$	+ 43.141	− 7.855	− 294.893	− 422.487	− 448.971	− 415.891	− 355.726	− 288.845	− 225.93	− 171.96	− 128.11
$P_{0\cdot1}(n.-n+1)$	———	+ 43.232	− 43.699	− 104.641	− 129.678	− 129.732	− 116.283	− 97.462	− 78.00	− 60.41	− 45.63
$P_{0\cdot1}(n.-n-1)$	———	+ 106.070	+ 680.283	+ 759.792	+ 722.973	+ 628.920	+ 516.529	+ 407.639	+ 312.20	+ 233.80	+ 171.96
$P_{2\cdot0}(n+2.-n)$	+ 105.647	+ 126.796	+ 99.984	+ 91.548	+ 93.487	+ 95.377	+ 93.271	+ 86.916	+ 77.46	+ 66.53	+ 55.38
$P_{2\cdot0}(n.-n)$	———	+ 222.102	− 93.564	− 427.143	− 710.886	− 882.229	− 940.779	− 912.941	− 829.60	− 718.87	− 600.53
$P_{2\cdot0}(n-2.-n)$	− 105.647	− 156.231	+ 249.644	+ 1009.605	+ 1679.002	+ 2095.709	+ 2247.590	+ 2191.601	+ 1999.68	+ 1738.68	+ 1456.57
$P_{1\cdot1}(n+1.-n+1)$	− 191.93	− 310.55	− 209.26	− 195.06	− 223.02	− 253.70	− 268.97	− 265.52	− 246.77	− 218.68	− 186.46
$P_{1\cdot1}(n-1.-n+1)$	+ 191.93	———	+ 46.78	+ 478.00	+ 955.49	+ 1301.17	+ 1469.07	+ 1482.08	+ 1385.39	+ 1226.51	+ 1042.02
$P_{1\cdot1}(n+1.-n-1)$	− 191.93	− 550.33	+ 271.44	+ 798.27	+ 1175.06	+ 1364.58	+ 1394.14	+ 1312.67	+ 1166.27	+ 993.10	+ 818.08
$P_{1\cdot1}(n-1.-n-1)$	+ 191.93	− 106.07	− 2630.66	− 4919.41	− 6524.67	− 7266.23	− 7268.47	− 6760.67	− 5961.29	− 5050.44	− 4145.64
$P_{0\cdot2}(n.-n+2)$	———	+ 150.35	+ 109.26	+ 89.64	+ 117.98	+ 157.29	+ 186.07	+ 197.58	+ 193.12	+ 177.46	+ 155.51
$P_{0\cdot2}(n.-n)$	———	+ 253.55	− 499.25	− 1306.17	− 1959.67	− 2330.12	− 2428.12	− 2322.76	− 2090.40	− 1798.83	− 1494.79
$P_{0\cdot2}(n.-n-2)$	———	+ 527.49	+ 5360.58	+ 5101.10	+ 5930.17	+ 6097.47	+ 5779.90	+ 5171.43	+ 4428.10	+ 3666.25	+ 2954.27
$P_{0\cdot0}(n+1.-n+1)+\sigma$	+ 79.10	+ 383.86	+ 427.46	+ 405.72	+ 352.24	+ 288.84	+ 227.62	+ 174.22	+ 130.33	+ 95.79	+ 69.40
$P_{0\cdot0}(n-1.-n-1)-\sigma$	− 79.10	———	+ 142.49	+ 202.86	+ 211.35	+ 192.56	+ 162.58	+ 130.67	+ 101.37	+ 76.64	+ 56.78
$P_{0\cdot0}(n+1.-n-1)+\partial$	− 79.10	− 383.86	− 427.46	− 406.72	− 352.24	− 288.84	− 227.62	− 174.22	− 130.33	− 95.79	− 69.40
$P_{0\cdot0}(n-1.-n+1)-\partial$	+ 79.10	———	− 142.49	− 202.86	− 211.35	− 192.56	− 162.58	− 130.67	− 101.37	− 76.64	− 56.78
$P_{2\cdot0}(n+1.-n)$	− 179										
$P_{2\cdot0}(n-1.-n)$	+ 179		− 194		+ 2857						
$P_{2\cdot0}(n-3.-n)$					− 3585		− 8819		−11254		
$P_{2\cdot1}(n.-n+1)$		+ 892									
$P_{2\cdot1}(n-2.-n+1)$		− 666		− 746		− 5840					
$P_{2\cdot1}(n.-n-1)$		+ 2174		− 5739							
$P_{2\cdot1}(n-2.-n-1)$				+13482		+38961		+53361			
$P_{1\cdot2}(n-1.-n+2)$	+ 817		− 482								
$P_{1\cdot2}(n+1.-n)$	− 866										
$P_{1\cdot2}(n-1.-n)$	+ 866		+ 861		+15460						
$P_{1\cdot2}(n+1.-n-2)$	− 817		+ 1883								
$P_{1\cdot2}(n-1.-n-2)$			−15988		−57139		−84730				
$P_{0\cdot2}(n.-n+1)$		+ 908									
$P_{0\cdot2}(n.-n-1)$		+ 1160		− 9917							
$P_{0\cdot2}(n.-n-3)$		+ 2909		+27771		+45152					
$P_{1\cdot0}(n.-n+1)+\sigma$		− 1250									
$P_{1\cdot0}(n.-n-1)-\sigma$		− 866		− 900							
$P_{1\cdot0}(n-2.-n-1)-\sigma$				− 1247		− 2340		− 2281			

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$P_{1\cdot 0}(n.-n-1)+\partial$		+ 1634		+ 3942							
$P_{1\cdot 0}(n.-n+1)-\partial$		+ 866									
$P_{1\cdot 0}(n-2.-n+1)-\partial$		− 866		+ 1044		+ 2147					
$P_{0\cdot 1}(n+1.-n)+\sigma$	+ 333										
$P_{0\cdot 1}(n-1.-n)-\sigma$	− 333		+ 53		− 372						
$P_{0\cdot 1}(n-1.-n-2)-\sigma$			+ 1763		+ 3855		+ 3930				
$P_{0\cdot 1}(n+1.-n)+\partial$	− 333										
$P_{0\cdot 1}(n+1.-n-2)+\partial$	− 649		− 5288								
$P_{0\cdot 1}(n-1.-n+2)-\partial$	+ 649		− 623								
$P_{0\cdot 1}(n-1.-n)-\partial$	+ 333		− 1193		− 3009						

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TABLE Vw. Unit=1″.

n	0	1	2	3	4	5	6	7	8	9	10
$P_{0\cdot0}(n.-n)$	———	− 49.77	− 212.19	− 218.38	− 197.76	− 166.40	− 133.41	− 103.39	− 78.07	− 57.80	− 42.11
$P_{1\cdot0}(n+1.-n)$	+ 127.95	+ 297.43	+ 99.93	− 57.00	− 175.24	− 243.85	− 269.59	− 264.86	− 241.46	− 209.11	− 174.28
$P_{1\cdot0}(n-1.-n)$	− 127.95	+ 24.88	+ 740.62	+ 1336.40	+ 1714.29	+ 1861.91	+ 1827.67	+ 1675.31	+ 1460.30	+ 1225.69	+ 998.30
$P_{0\cdot1}(n.-n+1)$	———	− 161.16	+ 4.11	+ 233.84	+ 417.06	+ 522.14	+ 555.08	+ 535.46	+ 483.52	+ 416.49	+ 346.03
$P_{0\cdot1}(n.-n-1)$	———	− 360.23	− 1693.44	− 2386.8	− 2747.2	− 2805.8	− 2646.8	− 2359.4	− 2014.6	− 1664.2	− 1338.5
$P_{2\cdot0}(n+2.-n)$	− 449.3	− 693.9	− 517.5	− 421.4	− 404.7	− 428.1	− 458.0	− 474.8	− 470.9	− 447.7	− 409.5
$P_{2\cdot0}(n.-n)$	———	− 987.2	− 703.1	+ 338.4	+ 1769.5	+ 3138.4	+ 4173.4	+ 4777.7	+ 4968.0	+ 4830.2	+ 4464.0
$P_{2\cdot0}(n-2.-n)$	+ 449.3	+ 648.8	− 634.5	− 3373.0	− 6656.7	− 9633.2	− 11773.5	− 12899.7	−13075.8	− 12509.4	−11436.8
$P_{1\cdot1}(n+1.-n+1)$	+ 705.2	+ 1454.2	+ 1087.4	+ 893.8	+ 934.6	+ 1108.1	+ 1301.8	+ 1443.2	+ 1500.9	+ 1476.1	
$P_{1\cdot1}(n-1.-n+1)$	− 705.2	———	+ 351.5	− 839.2	− 2942.5	− 5178.8	− 6999.5	− 8154.2	− 8610.8	− 8481.8	
$P_{1\cdot1}(n+1.-n-1)$	+ 705.2	+ 2383.2	+ 1042.2	− 866.6	− 3034.2	− 4910.7	− 6214.1	− 6884.8	− 6991.5	− 6675.7	
$P_{1\cdot1}(n-1.-n-1)$	− 705.2	+ 360.2	+ 7121.2	+16274.2	+ 25650.6	+ 33200.8	+ 37910.8	+ 39666.4	+38884.5	+ 36265.0	
$P_{0\cdot2}(n.-n+2)$	———	− 630.2	− 569.9	− 458.1	− 519.2	− 703.7	− 918.5	− 1093.6	− 1194.2	− 1216.0	
$P_{0\cdot2}(n.-n)$	———	− 1111.0	+ 149.9	+ 2919.9	+ 6339.7	+ 9442.8	+ 11667.3	+ 12834.8	+13024.3	+ 12456.5	
$P_{0\cdot2}(n.-n-2)$	———	− 1921.8	− 9449.0	−16651.5	− 23114.8	− 27699.4	− 30021.4	− 30248.4	−28814.6	− 26275.8	
$P_{0\cdot0}(n+1.-n+1)+\sigma$	− 327.52	− 1410.40	− 1815.81	− 1972.04	− 1934.46	− 1771.50	− 1543.43	− 1295.0	− 1054.1	− 837.7	− 652.6
$P_{0\cdot0}(n-1.-n-1)-\sigma$	+ 327.52	———	− 605.27	− 986.02	− 1160.67	− 1181.00	− 1102.45	− 971.2	− 819.9	− 670.2	− 534.0
$P_{0\cdot0}(n+1.-n-1)+\delta$	+ 327.52	+ 1410.40	+ 1815.81	+ 1972.04	+ 1934.46	+ 1771.50	+ 1543.43	+ 1295.0	+ 1054.1	+ 837.7	+ 652.6
$P_{0\cdot0}(n-1.-n+1)-\delta$	− 327.52	———	+ 605.27	+ 986.02	+ 1160.67	+ 1181.00	+ 1102.45	+ 971.2	+ 819.9	+ 670.2	+ 534.0
$P_{3\cdot0}(n+1.-n)$	+1104										
$P_{3\cdot0}(n-1.-n)$	−1104		+ 2975		− 6146						
$P_{3\cdot0}(n-3.-n)$					+ 14870		+ 47337		+74635		
$P_{2\cdot1}(n.-n+1)$		− 5205									
$P_{2\cdot1}(n-2.-n+1)$		+ 3571		+ 339		+ 22690					
$P_{2\cdot1}(n.-n-1)$		−10942		+ 2561							
$P_{2\cdot1}(n-2.-n-1)$				−47527		−183621		−318377			
$P_{1\cdot2}(n-1.-n+2)$	−3541		+ 2850								
$P_{1\cdot2}(n+1.-n)$	+4261										
$P_{1\cdot2}(n-1.-n)$	−4261		+ 3794		− 47653						
$P_{1\cdot2}(n+1.-n-2)$	+3541		+ 8025								
$P_{1\cdot2}(n-1.-n-2)$			+46770		+231778		+449089				
$P_{0\cdot3}(n.-n+1)$		− 4621									
$P_{0\cdot3}(n.-n-1)$		− 5961		+21967							
$P_{0\cdot3}(n.-n-3)$		− 9041		−94500		−209160					
$P_{1\cdot0}(n.-n+1)+\sigma$		+ 5671									
$P_{1\cdot0}(n.-n-1)-\sigma$		+ 4260		+ 5954							
$P_{1\cdot0}(n-2.-n-1)-\sigma$				+ 6586		+ 15068		+ 17484			

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$P_{1\cdot0}(n.-n-1)+\delta$		− 7081		−20744							
$P_{1\cdot0}(n.-n+1)-\delta$		− 4260									
$P_{1\cdot0}(n-2.-n+1)-\delta$		+ 4260		− 5600		− 13887					
$P_{0\cdot1}(n+1.-n)+\sigma$	−1786										
$P_{0\cdot1}(n-1.-n)-\sigma$	+1786		− 989		+ 974						
$P_{0\cdot1}(n-1.-n-2)-\sigma$			− 8252		− 22240		− 27446				
$P_{0\cdot1}(n+1.-n)+\delta$	+1786										
$P_{0\cdot1}(n+1.-n-2)+\delta$	+3096		+24757								
$P_{0\cdot1}(n-1.-n+2)-\delta$	−3096		+ 3410								
$P_{0\cdot1}(n-1.-n)-\delta$	−1786		+ 5831		+ 17597						

TABLE Vw^2. Unit=1″.

$P_{0\cdot0}(n.-n)$	———	+ 62	+ 175	+ 250	+ 290	+ 297	+ 282	+ 252	+ 216	+ 179	+144
$P_{1\cdot0}(n+1.-n)$	− 171	− 507	− 446	− 251	− 1	+ 235	+ 417	+ 533	+ 586	+ 588	
$P_{1\cdot0}(n-1.-n)$	+ 171	− 31	− 734	− 1686	− 2664	− 3462	− 3971	−4173	−4107	−3844	
$P_{0\cdot1}(n.-n+1)$	———	+ 269	+ 239	− 30	− 404	− 765	− 1042	−1208	−1266	−1237	
$P_{0\cdot1}(n.-n-1)$	———	+ 517	+ 1643	+ 2966	+ 4227	+ 5183	+ 5724	+5857	+5651	+5209	
$P_{2\cdot0}(n.-n)$			+ 3217								
$P_{2\cdot0}(n-2.-n)$			+ 646		+11174	+18846	+26496				
$P_{1\cdot1}(n-1.-n+1)$		———		− 986							
$P_{1\cdot1}(n+1.-n-1)$		− 4972									
$P_{1\cdot1}(n-1.-n-1)$		− 517		−22685	−42351	−64234					
$P_{0\cdot2}(n.-n+2)$	———										
$P_{0\cdot2}(n.-n)$	———		+ 2754								
$P_{0\cdot2}(n.-n-2)$	———		+10708	+22571	+37519						
$P_{0\cdot0}(n-1.-n-1)-\sigma$		———		+ 2254		+ 3351					
$P_{0\cdot0}(n+1.-n-1)+\delta$		− 2605									
$P_{0\cdot0}(n-1.-n+1)-\delta$		———		− 2254							
$P_{1\cdot0}(n.-n+1)+\sigma$		−13593									
$P_{1\cdot0}(n.-n-1)+\delta$		+16199									
$P_{1\cdot0}(n.-n+1)-\delta$		+10988									
$P_{1\cdot0}(n-2.-n+1)-\delta$		−10988		+14731							
$P_{0\cdot1}(n+1.-n)+\sigma$	+4875										
$P_{0\cdot1}(n-1.-n)-\sigma$	−4875										
$P_{0\cdot1}(n+1.-n)+\delta$	−4875										
$P_{0\cdot1}(n-1.-n+2)-\delta$			− 9482								
$P_{0\cdot1}(n-1.-n)-\delta$	+4875										

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TABLE VI. Unit=1″.

n	0	1	2	3	4	5	6	7	8	9	10
$Q_{0\cdot0}(n.-n)$	+ 43.141	+ 58.941	+ 113.897	+ 85.180	+ 60.839	+ 42.332	+ 28.958	+ 19.758	+ 13.112	+ 8.725	+ 5.773
$Q_{1\cdot0}(n+1.-n)$	− 148.79	− 169.04	− 18.77	+ 39.45	+ 63.14	+ 67.20	+ 61.47	+ 51.87	+ 41.56	+ 32.17	+ 24.25
$Q_{1\cdot0}(n-1.-n)$	− 148.79	− 345.86	− 702.15	− 727.17	− 666.93	− 567.79	− 459.78	− 359.27	− 273.14	− 203.40	− 148.94
$Q_{0\cdot1}(n.-n+1)$	+ 191.99	+ 198.51	+ 18.77	− 82.04	− 123.98	− 130.69	− 119.39	− 100.82	− 80.91	− 62.70	− 47.34
$Q_{0\cdot1}(n.-n-1)$	+ 191.99	+ 434.27	+ 929.94	+ 940.12	+ 849.45	+ 715.95	+ 575.62	+ 447.38	+ 338.71	+ 251.39	+ 183.58
$Q_{2\cdot0}(n+2.-n)$	+ 284.16	+ 276.53	+ 164.39	+ 119.94	+ 106.65	+ 102.12	+ 97.26	+ 89.65	+ 79.51	+ 68.16	+ 56.68
$Q_{2\cdot0}(n.-n)$	+ 568.32	+ 1001.22	+ 213.59	− 370.68	− 787.74	− 1015.36	− 1085.72	− 1047.12	− 943.56	− 810.74	− 671.88
$Q_{2\cdot0}(n-2.-n)$	+ 284.16	+ 989.93	+ 2099.34	+ 2959.17	+ 3486.04	+ 3644.88	+ 3508.30	+ 3180.28	+ 2753.6	+2301.3	+1868.8
$Q_{1\cdot1}(n+1.-n+1)$	− 961.86	− 935.33	− 470.84	− 306.90	− 272.99	− 278.31	− 283.17	− 275.22	− 254.19	− 224.64	− 191.29
$Q_{1\cdot1}(n-1.-n+1)$	− 769.93	− 1332.35	− 564.67	+ 349.39	+ 1090.78	+ 1551.40	+ 1746.43	+ 1741.18	+ 1606.8	+1405.7	+1181.6
$Q_{1\cdot1}(n+1.-n-1)$	− 769.93	− 1295.09	− 146.61	+ 595.49	+ 1099.98	+ 1358.22	+ 1420.25	+ 1350.54	+ 1205.0	+1027.8	+ 846.8
$Q_{1\cdot1}(n-1.-n-1)$	− 961.86	− 3032.17	− 6656.21	− 8805.72	− 9942.80	−10096.98	− 9516.42	− 8491.72	− 7263.0	−6011.1	−4844.0
$Q_{0\cdot2}(n.-n+2)$	+ 816.81	+ 794.90	+ 372.77	+ 199.19	+ 165.19	+ 179.64	+ 198.61	+ 206.13	+ 199.78	+ 182.94	+ 160.02
$Q_{0\cdot2}(n.-n)$	+ 865.89	+ 1412.97	+ 137.23	− 1024.54	− 1887.44	− 2366.80	− 2511.79	− 2418.56	− 2180.8	−1876.3	−1557.5
$Q_{0\cdot2}(n.-n-2)$	+ 816.81	+ 2355.16	+ 5306.59	+ 6625.40	+ 7185.73	+ 7090.49	+ 6542.08	+ 5743.16	+ 4849.2	+3971.9	+3173.4
$Q_{0\cdot0}(n+1.-n+1)+\sigma$	+ 412.18	+ 865.89	+ 765.43	+ 638.10	+ 509.89	+ 394.75	+ 298.22	+ 221.03	+ 161.20	+ 116.08	+ 82.68
$Q_{0\cdot0}(n-1.-n-1)-\sigma$	+ 412.18	+ 865.89	+ 765.43	+ 638.10	+ 509.89	+ 394.75	+ 298.22	+ 221.03	+ 161.20	+ 116.08	+ 82.68
$Q_{0\cdot0}(n+1.-n-1)+\delta$	− 412.18	− 865.89	− 765.43	− 638.10	− 509.89	− 394.75	− 298.22	− 221.03	− 161.20	− 116.08	− 82.68
$Q_{0\cdot0}(n-1.-n+1)-\delta$	− 412.18	− 865.89	− 765.43	− 638.10	− 509.89	− 394.75	− 298.22	− 221.03	− 161.20	− 116.08	− 82.68
$Q_{2\cdot0}(n+1.-n)$	−1477										
$Q_{2\cdot0}(n-1.-n)$	−1477				+ 3919						
$Q_{2\cdot0}(n-3.-n)$					−11748				−17938		
$Q_{2\cdot1}(n.-n+1)$		[+ 6916]									
$Q_{2\cdot1}(n-2.-n+1)$		+ 4691				− 8364					
$Q_{2\cdot1}(n.-n-1)$				− 4841							
$Q_{2\cdot1}(n-2.-n-1)$				+40197				+78052			
$Q_{1\cdot2}(n-1.-n+2)$			− 3793								
$Q_{1\cdot2}(n+1.-n)$	−5525										
$Q_{1\cdot2}(n-1.-n)$	−5525				+17415						
$Q_{1\cdot2}(n+1.-n-2)$			− 1102								
$Q_{1\cdot2}(n-1.-n-2)$			−43556				−112546				
$Q_{0\cdot2}(n.-n+1)$		+ 6023									
$Q_{0\cdot2}(n.-n-1)$				− 7678							
$Q_{0\cdot2}(n.-n-3)$		+11251				+53719					
$Q_{1\cdot0}(n.-n+1)+\sigma$		− 7257									
$Q_{1\cdot0}(n.-n-1)-\sigma$				− 2252							
$Q_{1\cdot0}(n-2.-n-1)-\sigma$				− 7356				− 4612			

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$Q_{1,0}(n.-n-1)+\delta$												
$Q_{1,0}(n.-n+1)-\delta$		++ 5525										
$Q_{1,0}(n-2.-n+1)-\delta$		++ 5525										
$Q_{0,1}(n+1.-n)+\sigma$	+2345											
$Q_{0,1}(n-1.-n)-\sigma$	+2345			+10615	+ 8633							
$Q_{0,1}(n-1.-n-2)-\sigma$	−2345											
$Q_{0,1}(n+1.-n)+\delta$				−10615								
$Q_{0,1}(n+1.-n-2)+\delta$				− 4492								
$Q_{0,1}(n-1.-n+2)-\delta$	−2345					− 363	− 7795	+ 5965	+ 7443			
$Q_{0,1}(n-1.-n)-\delta$												

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TABLE VIw. Unit=1″.

n	0	1	2	3	4	5	6	7	8	9	10
$Q_{0\cdot0}(n.-n)$	− 127.95	− 210.93	− 316.24	− 286.03	− 241.82	− 195.08	− 152.08	− 115.52	− 85.94	− 62.90	− 45.41
$Q_{1\cdot0}(n+1.-n)$	+ 577.26	+ 782.77	+ 357.68	+ 74.16	− 113.43	− 218.86	− 263.20	− 267.07	− 247.01	− 215.35	− 180.02
$Q_{1\cdot0}(n-1.-n)$	+ 577.26	+ 1415.55	+ 2255.10	+ 2648.42	+ 2788.44	+ 2707.42	+ 2474.2	+ 2158.8	+ 1815.4	+ 1482.8	+1182.3
$Q_{0\cdot1}(n.-n+1)$	− 705.22	− 888.23	− 357.68	+ 68.85	+ 355.25	+ 511.49	+ 567.4	+ 556.2	+ 504.8	+ 435.5	+ 361.7
$Q_{0\cdot1}(n.-n-1)$	− 705.22	− 1731.94	− 2887.57	− 3363.50	− 3513.91	− 3390.22	− 3082.5	− 2678.6	− 2245.1	− 1828.8	−1454.8
$Q_{2\cdot0}(n+2.-n)$	− 1553.0	− 1821.2	− 1090.6	− 711.5	− 552.9	− 506.4	− 502.3	− 502.8	− 490.7	− 462.9	
$Q_{2\cdot0}(n.-n)$	− 3106.1	− 5779.9	− 3694.8	− 1232.6	+ 1222.5	+ 3245.6	+ 4645.2	+ 5411.6	+ 5633.9	+ 5454.9	
$Q_{2\cdot0}(n-2.-n)$	− 1553.0	− 4907.9	− 8296.6	− 12105	− 15636	− 18197	− 19489	− 19558	− 18624	−17008	
$Q_{1\cdot1}(n+1.-n+1)$	+ 4613.3	+ 5366.2	+ 3033.8	+ 1854.9	+ 1411.8	+ 1354.2	+ 1440.5	+ 1533.4	+ 1566.8	+ 1529.5	
$Q_{1\cdot1}(n-1.-n+1)$	+ 3908.1	+ 7142.7	+ 4822.2	+ 1304.0	− 2495.9	− 5806.9	− 8204.8	− 9585.8	− 10044	− 9792.9	
$Q_{1\cdot1}(n+1.-n-1)$	+ 3908.1	+ 7187.2	+ 4272.7	+ 1097.5	− 1982.2	− 4462.6	− 6133.7	− 7006.8	− 7207.3	− 6919.9	
$Q_{1\cdot1}(n-1.-n-1)$	+ 4613.3	+14115	+ 24486	+ 34732	+ 43698	+ 49782	+ 52434	+ 51923	+ 48920	+44286	
$Q_{0\cdot2}(n.-n+2)$	− 3540.7	− 4121.0	− 2280.3	− 1287.6	− 923.1	− 909.1	− 1034.6	− 1170.6	− 1253.3	− 1265.5	
$Q_{0\cdot2}(n.-n)$	− 4260.6	− 7609.0	− 4093.8	+ 551.5	+ 5318.5	+ 9292.2	+ 12016	+ 13425	+ 13691	+13103	
$Q_{0\cdot2}(n.-n-2)$	− 3540.7	−10416	− 18424	− 25346	− 31030	− 34574	− 35777	− 34932	− 32536	−29175	
$Q_{0\cdot0}(n+1.-n+1)+\sigma$	− 2113.2	− 4260.5	− 4015.4	− 3628.0	− 3157.4	− 2660.8	− 2181.9	− 1748.6	− 1373.5	− 1060.8	− 807.4
$Q_{0\cdot0}(n-1.-n-1)-\sigma$	− 2113.2	− 4260.5	− 4015.4	− 3628.0	− 3157.4	− 2660.8	− 2181.9	− 1748.6	− 1373.5	− 1060.8	− 807.4
$Q_{0\cdot0}(n+1.-n-1)+\delta$	+ 2113.2	+ 4260.5	+ 4015.4	+ 3628.0	+ 3157.4	+ 2660.8	+ 2181.9	+ 1748.6	+ 1373.5	+ 1060.8	+ 807.4
$Q_{0\cdot0}(n-1.-n+1)-\delta$	+ 2113.2	+ 4260.5	+ 4015.4	+ 3628.0	+ 3157.4	+ 2660.8	+ 2181.9	+ 1748.6	+ 1373.5	+ 1060.8	+ 807.4
$Q_{2\cdot0}(n+1.-n)$	+10758										
$Q_{2\cdot0}(n-1.-n)$	+10758				− 2344						
$Q_{2\cdot0}(n-3.-n)$					+ 58043				+124030		
$Q_{2\cdot1}(n.-n+1)$		[−48681]									
$Q_{2\cdot1}(n-2.-n+1)$		−31039				+ 28058					
$Q_{2\cdot1}(n.-n-1)$				− 24814							
$Q_{2\cdot1}(n-2.-n-1)$				−181030				−490100			
$Q_{1\cdot2}(n-1.-n+2)$			+ 26574								
$Q_{1\cdot2}(n+1.-n)$	+35095										
$Q_{1\cdot2}(n-1.-n)$	+35095				− 36497						
$Q_{1\cdot2}(n+1.-n-2)$			+ 34613								
$Q_{1\cdot2}(n-1.-n-2)$			+182000				+636060				
$Q_{0\cdot3}(n.-n+1)$		−38182									
$Q_{0\cdot3}(n.-n-1)$				+ 395							
$Q_{0\cdot3}(n.-n-3)$		−54985				−270330					
$Q_{1\cdot0}(n.-n+1)+\sigma$		+43616									
$Q_{1\cdot0}(n.-n-1)-\sigma$				+ 17970							
$Q_{1\cdot0}(n-2.-n-1)-\sigma$				+ 46993				+ 37989			

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Table VI.

Unit=1″

$Q_{1\cdot0}(n.-n-1)+\delta$				−54250									
$Q_{1\cdot0}(n.-n+1)-\delta$		−35095					− 43097						
$Q_{1\cdot0}(n-2.-n+1)-\delta$		−35095				− 1666							
								− 56539					
$Q_{0\cdot1}(n+1.-n)+\sigma$	−15520												
$Q_{0\cdot1}(n-1.-n)-\sigma$	−15520		− 62277										
$Q_{0\cdot1}(n-1.-n-2)-\sigma$													
$Q_{0\cdot1}(n+1.-n)+\delta$	+15520												
$Q_{0\cdot1}(n+1.-n-2)+\delta$			+ 62277										
$Q_{0\cdot1}(n-1.-n+2)-\delta$			++ 30150										
$Q_{0\cdot1}(n-1.-n)-\delta$	+15520					+ 52184							

$Q_{0\cdot0}(n.-n)$	+ 171.1	+ 331.2	+ 382.8	+ 406.2	+ 405.5	+ 382.2	+ 343.2	+ 298.0	+ 247.1	+ 200.8	+ 159.5
$Q_{1\cdot0}(n+1.-n)$	− 1132	− 1771	− 1250	− 741	− 284	+ 83	+ 346	+ 509	+ 537	+ 602	
$Q_{1\cdot0}(n-1.-n)$	− 1132	− 2765	− 3546	− 4396	− 5150	− 5650	− 5830	− 5707	− 5335	− 4818	
$Q_{0\cdot1}(n.-n+1)$	+ 1303	+ 1937	+ 1250	+ 538	− 121	− 657	− 1033	− 1249	− 1329	− 1305	
$Q_{0\cdot1}(n.-n-1)$	+ 1308	+ 3262	+ 4312	+ 5411	+ 6368	+ 6988	+ 7203	+ 7040	+ 6578	+ 5923	
$Q_{2\cdot0}(n.-n)$			+ 14502								
$Q_{2\cdot0}(n-2.-n)$			+ 17506			+ 41211	[+48544]				
$Q_{1\cdot1}(n+1.-n-1)$		− 20144									
$Q_{1\cdot1}(n-1.-n+1)$				− 10618							
$Q_{1\cdot1}(n-1.-n-1)$		− 33161			−83436	−110380					
$Q_{0\cdot2}(n.-n+2)$	+ 8100		+ 16618								
$Q_{0\cdot2}(n.-n)$				+46088	+60916						
$Q_{0\cdot2}(n.-n-2)$	+ 8100										
$Q_{0\cdot0}(n+1.-n+1)+\sigma$		+ 10988				+ 8601					
$Q_{0\cdot0}(n-1.-n-1)-\sigma$		++ 10988									
$Q_{0\cdot0}(n+1.-n-1)+\delta$		− 10988									
$Q_{0\cdot0}(n-1.-n+1)-\delta$		[− 10988]		−10223							
$Q_{1\cdot0}(n.-n+1)+\sigma$		−138800									
$Q_{1\cdot0}(n.-n+1)-\delta$		+118820									
$Q_{1\cdot0}(n-2.-n+1)-\delta$		+118820									
$Q_{0\cdot1}(n+1.-n)+\sigma$	+53159										
$Q_{0\cdot1}(n-1.-n)-\sigma$	+53159										
$Q_{0\cdot1}(n+1.-n)+\delta$	−53159										
$Q_{0\cdot1}(n-1.-n+2)-\delta$			−104980								
$Q_{0\cdot1}(n-1.-n)-\delta$	[−53159]										

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TABLE VII. Unit=1″.

	n	0	1	2	3	4	5	6
	$R_{0\cdot0}(n.-n+1)+\pi'$	− 79.10	− 191.93	− 142.48	− 101.43	− 70.45	− 48.14	− 32.52
	$R_{0\cdot0}(n.-n-1)-\pi'$	+ 79.10	+ 191.93	+ 142.48	+ 101.43	+ 70.45	+ 48.14	+ 32.52
	$R_{1\cdot0}(n+1.-n+1)+\pi'$	+ 372.6	+ 482.0	+ 266.7	+ 130.9	+ 52.0	+ 9.6	− 10.7
	$R_{1\cdot0}(n-1.-n+1)+\pi'$	+ 293.5	+ 865.9	+ 979.2	+ 942.4	+ 826.9	+ 683.6	+542.1
	$R_{1\cdot0}(n+1.-n-1)-\pi'$	− 293.5	− 290.1	− 124.2	− 29.5	+ 18.5	+ 38.5	+ 43.2
	$R_{1\cdot0}(n-1.-n-1)-\pi'$	− 372.6	− 1057.8	− 1121.6	− 1043.8	− 897.4	− 731.7	−574.6
	$R_{0\cdot1}(n.-n+2)+\pi'$	− 649.5	− 1057.8	− 622.9	− 333.8	− 157.6	− 57.8	− 5.6
	$R_{0\cdot1}(n.-n)+\pi'$	− 333.1	− 1057.8	− 1192.9	− 1145.3	− 1003.0	− 828.0	−655.9
	$R_{0\cdot1}(n.-n)-\pi'$	+ 333.1	+ 290.1	+ 53.0	− 71.9	− 124.1	− 134.8	−124.5
	$R_{0\cdot1}(n.-n-2)-\pi'$	+ 649.5	+ 1825.5	+ 1762.8	+ 1551.0	+ 1284.8	+ 1020.6	+786.0
	$R_{2\cdot0}(n.-n+1)+\pi'$		− 3119					
	$R_{2\cdot0}(n-2.-n+1)+\pi'$		− 2330					
	$R_{1\cdot1}(n-1.-n+2)+\pi'$			+ 5118				
	$R_{1\cdot1}(n+1.-n)+\pi'$	+ 2012						
	$R_{1\cdot1}(n-1.-n)+\pi'$	+ 2012						
	$R_{1\cdot1}(n-1.-n)-\pi'$	− 2012						
	$R_{1\cdot1}(n-1.-n)-\pi'$	− 2012						
	$R_{0\cdot2}(n.-n+1)+\pi'$		− 6583					
	$R_{0\cdot2}(n.-n+1)-\pi'$		+ 1656					
	$R_{0\cdot0}(n-1.-n)-\sigma+\pi'$	− 533						
	$R_{0\cdot0}(n+1.-n)+\delta+\pi'$	+ 533						
	$R_{0\cdot0}(n-1.-n+2)-\delta+\pi'$			+ 873				
	$R_{0\cdot0}(n+1.-n)+\sigma-\pi'$	+ 533						
	$R_{0\cdot0}(n-1.-n)-\delta-\pi'$	− 533						
Factor w	$R_{0\cdot0}(n.-n+1)+\pi'$	+ 327.5	+ 705.2	+ 605.3	+ 493.0	+ 386.9	+ 295.2	
	$R_{0\cdot0}(n.-n-1)-\pi'$	− 327.5	− 705.2	− 605.3	− 493.0	− 386.9	− 295.2	
	$R_{1\cdot0}(n+1.-n+1)+\pi'$	− 1950	− 2850	− 1897	− 1163	− 643	− 299	
	$R_{1\cdot0}(n-1.-n+1)+\pi'$	− 1622	− 4260	− 4923	− 5107	− 4898	− 4432	
	$R_{1\cdot0}(n+1.-n-1)-\pi'$	+ 1622	+ 2145	+ 1292	+ 670	+ 256	+ 4	
	$R_{1\cdot0}(n-1.-n-1)-\pi'$	+ 1950	+ 4966	+ 5529	+ 5600	+ 5285	+ 4727	
	$R_{0\cdot1}(n.-n+2)+\pi'$	+ 3096	+ 4966	+ 3410	+ 2149	+ 1223	+ 594	
	$R_{0\cdot1}(n.-n)+\pi'$	+ 1786	+ 4966	+ 5831	+ 6093	+ 5866	+ 5318	
	$R_{0\cdot1}(n.-n)-\pi'$	− 1786	− 2145	− 989	− 177	+ 325	+ 587	
	$R_{0\cdot1}(n.-n-2)-\pi'$	− 3096	− 7786	− 8252	− 8065	− 7413	− 6499	
	$R_{2\cdot0}(n.-n+1)+\pi'$		+23018					
	$R_{2\cdot0}(n-2.-n+1)+\pi'$		+15418					
	$R_{1\cdot1}(n-1.-n+2)+\pi'$			−33562				
	$R_{1\cdot1}(n+1.-n)+\pi'$	−13734						
	$R_{1\cdot1}(n-1.-n)+\pi'$	−13734						
	$R_{1\cdot1}(n+1.-n)-\pi'$	+13734						
	$R_{1\cdot1}(n-1.-n)-\pi'$	+13734						
	$R_{0\cdot2}(n.-n+1)+\pi'$		+40061					
	$R_{0\cdot2}(n.-n+1)-\pi'$		−11862					
	$R_{0\cdot0}(n-1.-n)-\sigma+\pi'$	+ 3280						
	$R_{0\cdot0}(n+1.-n)+\delta+\pi'$	− 3280						
	$R_{0\cdot0}(n-1.-n+2)-\delta+\pi'$			− 5854				
	$R_{0\cdot0}(n+1.-n)+\sigma-\pi'$	− 3280						
	$R_{0\cdot0}(n-1.-n)-\delta-\pi'$	+ 3280						
Factor w^2	$R_{0\cdot0}(n.-n+1)+\pi'$		− 1303		− 1127			
	$R_{0\cdot0}(n.-n-1)-\pi'$		+ 1303		+ 1127		+ 838	
	$R_{1\cdot0}(n+1.-n+1)+\pi'$							
	$R_{1\cdot0}(n-1.-n+1)+\pi'$				+13600			
	$R_{1\cdot0}(n+1.-n-1)-\pi'$		− 7080				−14465	
	$R_{1\cdot0}(n-1.-n-1)-\pi'$		−12290					
	$R_{0\cdot1}(n.-n+2)+\pi'$	− 7475						
	$R_{0\cdot1}(n.-n)+\pi'$			−14430				
	$R_{0\cdot1}(n.-n)-\pi'$			+ 4532				
	$R_{0\cdot1}(n.-n-2)-\pi'$	+ 7475				+20370		

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With these tables we compute terms of the first order in the mass in Hansen's differential equations for the function W and the perturbation in the third coordinate. See Z 7, eq. (33) and Z 8, eq. (39). The first order parts of the equations are expressed in Z 41, eqs. (82), (83), in the form of trigonometric series, in which the coefficients are computed from the formulæ given in B 67. These coefficients comprise Tables VIII–XIVw^2 (cf. Z, 42–48).

Table XV (cf. Z 50, eq. (88)) is an auxiliary table of the same type of construction, which is employed in the computation of terms of the second order in the mass in the differential equation for W (cf. Z 53).

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Logarithmic TABLE VIII.[1] Unit=1".

	n	0	1	2	3	4	5	6	7	8	9	10
	$F_{0\cdot0}(n.-n)$	———	1. 673285	2. 433759	2. 334671	2. 203773	2. 056133	1. 898177	1. 733342	1. 56326	1. 38947	1. 21271
	$F_{1\cdot0}(n+1.-n)$	1. 935920$_n$	2. 318698$_n$	0. 79226$_n$	2. 031587	2. 206222	2. 234945	2. 201667	2. 13346	2. 04226	1. 93508	1. 81622
	$F_{1\cdot0}(n-1.-n)$	1. 935920	2. 056618$_n$	3. 071724$_n$	3. 175139$_n$	3. 180222$_n$	3. 135409$_n$	3. 060270$_n$	2. 964820$_n$	2. 85447$_n$	2. 73315$_n$	2. 60315$_n$
	$F_{0\cdot1}(n.-n+1)$	———	2. 112925	2. 117596$_n$	2. 496822$_n$	2. 589989$_n$	2. 590169$_n$	2. 542637$_n$	2. 465958$_n$	2. 36921$_n$	2. 25822$_n$	2. 13634$_n$
	$F_{0\cdot1}(n.-n-1)$	———	2. 502714	3. 309811	3. 357816	3. 336243	3. 275716	3. 190207	3. 087397	2. 97156	2. 84596	2. 71256
	$F_{2\cdot0}(n+2.-n)$	2. 324886	2. 512335	2. 336833	2. 278726	2. 297482	2. 326457	2. 336014	2. 321047	2. 28339	2. 22709	2. 15527
	$F_{2\cdot0}(n.-n)$	———	2. 878904	2. 107541	2. 945378$_n$	3. 251084$_n$	3. 372758$_n$	3. 414280$_n$	3. 409205$_n$	3. 37282$_n$	3. 31422$_n$	3. 23878$_n$
	$F_{2\cdot0}(n-2.-n)$	2. 324886$_n$	1. 51151$_n$	3. 351267	3. 677441	3. 827111	3. 890251	3. 901422	3. 877911	3. 82926	3. 76195	3. 67998
	$F_{1\cdot1}(n+1.-n+1)$	2. 584171$_n$	2. 913604$_n$	2. 717323$_n$	2. 660799$_n$	2. 727273$_n$	2. 800935$_n$	2. 841198$_n$	2. 846629$_n$	2. 82297$_n$	2. 77663$_n$	2. 71219$_n$
	$F_{1\cdot1}(n-1.-n+1)$	2. 584171	2. 454799$_n$	2. 320098	3. 236866	3. 511858	3. 632828	3. 677530	3. 675961	3. 64277	3. 58693	3. 51384
	$F_{1\cdot1}(n+1.-n-1)$	2. 584171$_n$	3. 154992$_n$	2. 583997	3. 258957	3. 466673	3. 550454	3. 571125	3. 552680	3. 50693	3. 44139	3. 36058
	$F_{1\cdot1}(n-1.-n-1)$	2. 584171	2. 984357$_n$	4. 007854$_n$	4. 235981$_n$	4. 339837$_n$	4. 376075$_n$	4. 369477$_n$	4. 333339$_n$	4. 27524$_n$	4. 20058$_n$	4. 11275$_n$
	$F_{0\cdot2}(n.-n+2)$	———	2. 654194	2. 515602	2. 429622	2. 548932	2. 673834	2. 746793	2. 772863	2. 76294	2. 72623	2. 66888
	$F_{0\cdot2}(n.-n)$	———	2. 881187	3. 175443$_n$	3. 593121$_n$	3. 769305$_n$	3. 844500$_n$	3. 862392$_n$	3. 843126$_n$	3. 79735$_n$	3. 73211$_n$	3. 65170$_n$
	$F_{0\cdot2}(n.-n-2)$	———	3. 199336	4. 028642	4. 184785	4. 250188	4. 262271	4. 239041	4. 190732	4. 12334	4. 04134	3. 94757
	$F_{0\cdot0}(n+1.-n+1)+\sigma$	2. 375282	3. 061292	3. 108012	3. 085352	3. 023966	2. 93777	2. 83432	2. 71823	2. 59217	2. 45846	2. 31847
	$F_{0\cdot0}(n-1.-n-1)-\sigma$	2. 375282$_n$	———	2. 630891	2. 784322	2. 802117	2. 76168	2. 68819	2. 59329	2. 48303	2. 36155	2. 23132
	$F_{0\cdot0}(n+1.-n-1)+\delta$	2. 375282$_n$	3. 061292$_n$	3. 108012$_n$	3. 085352$_n$	3. 023966$_n$	2. 93777$_n$	2. 83432$_n$	2. 71823$_n$	2. 59217$_n$	2. 45846$_n$	2. 31847$_n$
	$F_{0\cdot0}(n-1.-n+1)-\delta$	2. 375282	———	2. 630891$_n$	2. 784322$_n$	2. 802117$_n$	2. 76168$_n$	2. 68819$_n$	2. 59329$_n$	2. 48303$_n$	2. 36155$_n$	2. 23132$_n$
Factor w	$F_{0\cdot0}(n.-n)$	———	2. 17407$_n$	2. 80385$_n$	2. 81634$_n$	2. 77327$_n$	2. 69826$_n$	2. 60231$_n$	2. 49160$_n$	2. 36959$_n$	2. 23903$_n$	2. 10154$_n$
	$F_{1\cdot0}(n+1.-n)$	2. 40808	2. 89260	2. 61060	1. 30651$_n$	2. 57056$_n$	2. 77368$_n$	2. 84137$_n$	2. 84714$_n$	2. 81568$_n$	2. 75940$_n$	2. 68487$_n$
	$F_{1\cdot0}(n-1.-n)$	2. 40808$_n$	2. 58559	3. 47166	3. 67505	3. 76194	3. 78630	3. 77099	3. 72820	3. 66492	3. 58609	3. 49478
	$F_{0\cdot1}(n.-n+1)$	———	2. 68438$_n$	1. 09094	2. 84603	3. 09732	3. 19490	3. 22147	3. 20585	3. 16154	3. 09673	3. 01623
	$F_{0\cdot1}(n.-n-1)$	———	3. 03370$_n$	3. 70589$_n$	3. 85493$_n$	3. 91601$_n$	3. 92517$_n$	3. 89984$_n$	3. 84993$_n$	3. 78132$_n$	3. 69833$_n$	3. 60374$_n$
	$F_{2\cdot0}(n+2.-n)$	2. 95358$_n$	3. 23873$_n$	3. 11084$_n$	3. 00363$_n$	2. 96993$_n$	2. 99436$_n$	3. 03311$_n$	3. 06053$_n$	3. 06783$_n$	3. 05500$_n$	3. 02391$_n$
	$F_{2\cdot0}(n.-n)$	———	3. 51482$_n$	3. 50053$_n$	2. 44665$_n$	3. 59612	3. 90886	4. 05465	4. 12446	4. 14800	4. 14015	4. 10902
	$F_{2\cdot0}(n-2.-n)$	2. 95358	2. 34335	3. 79017$_n$	4. 20261$_n$	4. 42532$_n$	4. 55258$_n$	4. 62051$_n$	4. 64765$_n$	4. 64471$_n$	4. 61891$_n$	4. 57492$_n$
	$F_{1\cdot1}(n+1.-n+1)$	3. 14935	3. 57940	3. 46185	3. 35842	3. 36644	3. 44584	3. 52668	3. 58147	3. 60643	3. 60539	
	$F_{1\cdot1}(n-1.-n+1)$	3. 14935$_n$	3. 08298	3. 14736	[3. 48489$_n$]	4. 00073$_n$	4. 23280$_n$	4. 35556$_n$	4. 41647$_n$	4. 43623$_n$	4. 42674$_n$	
	$F_{1\cdot1}(n+1.-n-1)$	3. 14935	3. 78804	3. 55943	3. 07546$_n$	3. 85262$_n$	4. 09729$_n$	4. 21567$_n$	4. 26985$_n$	4. 28308$_n$	4. 26782$_n$	
	$F_{1\cdot1}(n-1.-n-1)$	3. 14935$_n$	3. 54815	4. 44151	4. 75558	4. 93430	5. 03585	5. 08675	5. 10174	5. 08965	5. 05672	
	$F_{0\cdot2}(n.-n+2)$	———	3. 27657$_n$	3. 23290$_n$	3. 13811$_n$	3. 19242$_n$	3. 32448$_n$	3. 44020$_n$	3. 51600$_n$	3. 55420$_n$	3. 56205$_n$	
	$F_{0\cdot2}(n.-n)$	———	3. 52284$_n$	2. 65282	3. 94249	4. 27919	4. 45222	4. 54409	4. 58551	4. 59187	4. 57252	
	$F_{0\cdot2}(n.-n-2)$	———	3. 76082$_n$	4. 45251$_n$	4. 69857$_n$	4. 84101$_n$	4. 91959$_n$	4. 95455$_n$	4. 95782$_n$	4. 93673$_n$	4. 89677$_n$	

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Factor w^2												
$F_{0,0}(n+1.-n+1)+\sigma$	2.99236[a]	3.62646[a]	3.73619[a]	3.77204[a]	3.76368[a]	3.72546[a]	3.66561[a]	3.58938[a]	3.50001[a]	3.40022[a]	3.29177[a]
$F_{0,0}(n-1.-n-1)-\sigma$	2.99236	———	3.25907[a]	3.47101[a]	3.54183[a]	3.54937[a]	3.51948[a]	3.46444[a]	3.39087[a]	3.30331[a]	3.20462[a]
$F_{0,0}(n+1.-n-1)+\delta$	2.99236	3.62646	3.73619	3.77204	3.76368	3.72546	3.66561	3.58938	3.50001	3.40022	3.29177
$F_{0,0}(n-1.-n+1)-\delta$	2.99236[a]	———	3.25907	3.47101	3.54183	3.54937	3.51948	3.46444	3.39087	3.30331	3.20462
$F_{0,0}(n.-n)$	———	2.2697	2.7214	2.8744	2.9388	2.9505	2.9272	2.8790	2.8119	2.7301	2.6366
$F_{1,0}(n+1.-n)$	2.5342[a]	3.1189[a]	3.1164[a]	2.9280[a]	2.2494[a]	2.6910	3.0131	3.1434	3.1968	3.2065	
$F_{1,0}(n-1.-n)$	2.5342	2.7389[a]	3.4676[a]	3.7755[a]	3.9531[a]	4.0555[a]	4.1079[a]	4.1245[a]	4.1140[a]	4.0825[a]	
$F_{0,1}(n.-n+1)$	———	2.9072	2.8557	1.9557[a]	3.0636[a]	3.3610[a]	3.4950[a]	3.5591[a]	3.5794[a]	3.5695[a]	
$F_{0,1}(n.-n-1)$	———	3.1908	3.6927	3.9492	4.1032	4.1917	4.2348	4.2448	4.2292	4.1938	
$F_{2,0}(n.-n)$	———		4.0329								
$F_{2,0}(n-2.-n)$			3.8756		4.6503		4.9726				
$F_{1,1}(n-1.-n+1$		3.3936[a]		3.5361[a]							
$F_{1,1}(n+1.-n-1)$		4.1034[a]									
$F_{1,1}(n-1.-n-1)$		[3.7670]		4.8998[a]		5.3223[a]					
$F_{0,2}(n.-n+2)$			3.9171								
$F_{0,2}(n.-n)$	———		4.5068		5.0514						
$F_{0,2}(n.-n-2)$	———										
$F_{0,0}(n-1.-n-1)-\sigma$	———			3.8301		4.0023					
$F_{0,0}(n+1.-n-1)+\delta$	3.8928[a]										
$F_{0,0}(n-1.-n+1)-\delta$	———			3.8301[a]							

[a] It is convenient to have this table in seconds of arc also.

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Logarithmic. TABLE IX.[1] Unit=1″.

	n	0	1	2	3	4	5	6	7	8	9	10
	$G_{0\cdot0}(n.-n)$	1.634891	1.439686	1.826715_n	1.770059_n	1.660318_n	1.525471_n	1.376121_n	1.217493_n	1.05211_n	0.88196_n	0.70814_n
	$G_{1\cdot0}(n+1.-n)$	2.023857_n	1.740753_n	1.917856_n	1.927654_n	1.914277_n	1.86965_n	1.79959_n	1.71025_n	1.60585_n	1.48983_n	1.36459_n
	$G_{1\cdot0}(n-1.-n)$	2.283141_n	2.350404_n	2.436203	2.622307	2.654871	2.623379	2.555976	2.465550	2.35871	2.23993	2.11185
	$G_{0\cdot1}(n.-n+1)$	2.283141	2.049382	2.025986	2.104641	2.131548	2.109818	2.053762	1.973612	1.87535	1.76428	1.64259
	$G_{0\cdot1}(n.-n-1)$	2.283141	2.346611	2.634099_n	2.763025_n	2.775609_n	2.733913_n	2.660317_n	2.565735_n	2.45592_n	2.33487_n	2.20508_n
	$G_{2\cdot0}(n+2.-n)$	2.251669	2.054473	0.542701	1.673240_n	1.872876_n	1.941516_n	1.953253_n	1.930618_n	1.88358_n	1.81832_n	1.73875_n
	$G_{2\cdot0}(n.-n)$	2.754592	2.710784	2.728285	2.864698	2.965479	3.013074	3.018683	2.993200	2.94396	2.87668	2.79502
	$G_{2\cdot0}(n-2.-n)$	2.590848	2.894131	2.597213_n	3.136437_n	3.325120_n	3.401835_n	3.419045_n	3.398628_n	3.35168_n	3.28538_n	3.20413_n
	$G_{1\cdot1}(n+1.-n+1)$	2.886452_n	2.671647_n	2.059927_n	1.782544	2.223601	2.361811	2.411416	2.413615	2.38430	2.33246	2.26325
	$G_{1\cdot1}(n-1.-n+1)$	2.983112_n	3.001797_n	2.886806_n	3.001111_n	3.124901_n	3.196128_n	3.220094_n	3.208491_n	3.16994_n	3.11102_n	3.03610_n
	$G_{1\cdot1}(n+1.-n-1)$	2.761928_n	2.718191_n	2.972738_n	3.072386_n	3.138813_n	3.163746_n	3.154462_n	3.118772_n	3.06222_n	2.98945_n	2.90362_n
	$G_{1\cdot1}(n-1.-n-1)$	3.062127_n	3.315456_n	3.197526	3.628655	3.787099	3.847462	3.854044	3.826068	3.77341	3.70266	3.61772
	$G_{0\cdot2}(n.-n+2)$	2.912119	2.693921	2.188198	1.29898	1.84986_n	2.13017_n	2.23936_n	2.27652_n	2.27057_n	2.23551_n	2.17899_n
	$G_{0\cdot2}(n.-n)$	2.937465	2.957065	3.055278	3.200797	3.307904	3.360489	3.370043	3.347712	3.30104	3.23589	3.15597
	$G_{0\cdot2}(n.-n-2)$	2.912119	3.114004	3.258778_n	3.553495_n	3.669745_n	3.707950_n	3.700505_n	3.662731_n	3.60283_n	3.52641_n	3.43700_n
	$G_{0\cdot0}(n+1.-n+1)+\sigma$	2.404818	1.992001	1.951744_n	2.238919_n	2.289146_n	2.26227_n	2.19592_n	2.10522_n	1.99764_n	1.87801_n	1.74907_n
	$G_{0\cdot0}(n-1.-n-1)-\sigma$	2.756164	2.937464	2.681654	2.366191	1.94049	0.9836	1.4304_n	1.60533_n	1.61843_n	1.57046_n	1.48964_n
	$G_{0\cdot0}(n+1.-n-1)+\delta$	2.404818_n	1.992001_n	1.951744	2.238919	2.289146	2.26227	2.19592	2.10522	1.99764	1.87801	1.74907
	$G_{0\cdot0}(n-1.-n+1)-\delta$	2.756164_n	2.937464_n	2.681654_n	2.366191_n	1.94049_n	0.9836_n	1.4304	1.60533	1.61843	1.57046	1.48964
Factor w	$G_{0\cdot0}(n.-n)$	2.10705_n	2.04685_n	2.03404	2.17823	2.18669	2.13895	2.05974	1.96028	1.84633	1.72180	1.58899
	$G_{1\cdot0}(n+1.-n)$	2.65255	2.54290	2.41806	2.40791	2.44888	2.47354	2.46930	2.43899	2.38700	2.31799	2.23511
	$G_{1\cdot0}(n-1.-n)$	2.84832	3.00240	2.25278_n	2.98478_n	3.16887_n	3.23317_n	3.23892_n	3.20883_n	3.15392_n	3.08094_n	2.99390_n
	$G_{0\cdot1}(n.-n+1)$	2.84832_n	2.75275_n	2.56336_n	2.60078_n	2.68023_n	2.72655_n	2.73464_n	2.71161_n	2.66487_n	2.59934_n	2.51902_n
	$G_{0\cdot1}(n.-n-1)$	2.84832_n	3.00496_n	2.69838	3.14924	3.29676	3.34661	3.34462	3.30967	3.25141	3.17597	3.08714
	$G_{2\cdot0}(n+2.-n)$	3.04285_n	2.96317_n	2.49592_n	8.9445	2.28955	2.51168	2.60979	2.65226	2.65962	2.64211	
	$G_{2\cdot0}(n.-n)$	3.49220_n	3.56162_n	3.39880_n	3.41162_n	3.52203_n	3.62520_n	3.69230_n	3.72425_n	3.72689_n	3.70623_n	
	$G_{2\cdot0}(n-2.-n)$	3.30153_n	3.62263_n	3.17426_n	3.36313	3.81210	4.00369	4.09959	4.14189	4.14774	4.12750	
	$G_{1\cdot1}(n+1.-n+1)$	3.59196	3.50310	3.08664	2.36611	2.59736_n	2.93011_n	3.07018_n	3.13789_n	3.16324_n	3.15887_n	
	$G_{1\cdot1}(n-1.-n+1)$	3.66401	3.76125	3.57678	3.57435	3.69858	3.82142	3.90454	3.94867	3.96061	3.94722	
	$G_{1\cdot1}(n+1.-n-1)$	3.50554	3.57892	3.52922	3.58063	3.68601	3.77403	3.82803	3.85016	3.84548	3.81929	
	$G_{1\cdot1}(n-1.-n-1)$	3.72579	4.02457	3.35713	3.92118_n	4.28687_n	4.45373_n	4.53669_n	4.57015_n	4.56981_n	4.54488_n	
	$G_{0\cdot2}(n.-n+2)$	3.54909_n	3.45647_n	3.05713_n	2.56977_n	2.06145	2.69745	2.90437	3.00723	3.05504	3.06688	
	$G_{0\cdot2}(n.-n)$	3.62947_n	3.73135_n	3.64281_n	3.72335_n	3.86693_n	3.98197_n	4.05380_n	4.08795_n	4.09194_n	4.07225_n	
	$G_{0\cdot2}(n.-n-2)$	3.54909_n	3.81772_n	2.67578	3.90075	4.18183	4.31858	4.38499	4.40764	4.39955	4.36878	

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$G_{0,0}(n+1.-n+1)+\sigma$		3.16381$_n$	3.15827$_n$	2.58410$_n$	2.49979	2.85224	2.94559	2.95660	2.92500	2.86611	2.78862	2.69707
$G_{0,0}(n-1.-n-1)-\sigma$		3.44221$_n$	3.62942$_n$	3.44791$_n$	3.21904$_n$	2.92221$_n$	2.47531$_n$	1.36173	2.28744	2.42556	2.44648	2.41578
$G_{0,0}(n+1.-n-1)+\delta$		3.16381	3.15827	2.58410	2.49979$_n$	2.85224$_n$	2.94559$_n$	2.95660$_n$	2.92500$_n$	2.86611$_n$	2.78862$_n$	2.69707$_n$
$G_{0,0}(n-1.-n+1)-\delta$		3.44221	3.62942	3.44791	3.21904	2.92221	2.47531	1.36173$_n$	2.28744$_n$	2.42556$_n$	2.44648$_n$	2.41578$_n$
Factor w^2	$G_{0,0}(n.-n)$	2.2332	2.3164	1.5024	1.9685$_n$	2.2393$_n$	2.3276$_n$	2.3436$_n$	2.3193$_n$	2.2674$_n$	2.1964$_n$	2.1116$_n$
	$G_{1,0}(n+1.-n)$	2.9825$_n$	3.0110$_n$	2.7515$_n$	2.5962$_n$	2.5991$_n$	2.6724$_n$	2.7396$_n$	2.7789$_n$	2.7896$_n$	2.7761$_n$	
	$G_{1,0}(n-1.-n)$	3.1143$_n$	3.3396$_n$	3.0680$_n$	2.1166	3.1618	3.4063	3.5186	3.5672	3.5769	3.5573	
	$G_{0,1}(n.-n+1)$	3.1148	3.1438	2.8873	2.7768	2.8368	2.9416	3.0219	3.0667	3.0800	3.0680	
	$G_{0,1}(n.-n-1)$	3.1148	3.3478	3.0112	2.7163$_n$	3.3194$_n$	3.5286$_n$	3.6278$_n$	3.6697$_n$	3.6743$_n$	3.6527$_n$	
	$G_{2,0}(n-2.-n)$			3.9709								
	$G_{1,1}(n-1.-n-1)$		4.5912									
	$G_{0,2}(n.-n-2)$	3.9035										
	$G_{0,0}(n-1.-n-1)-\sigma$		4.0409									

[1] It is convenient to have this table in seconds of arc also.

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Logarithmic. TABLE X.[1] Unit=1′.

	n	0	1	2	3	4	5	6	7	8	9	10
	$H_{0\cdot0}(n.-n)$	1. 634891$_n$	1. 955978$_n$	2. 469664$_n$	2. 360312$_n$	2. 223808$_n$	2. 072607$_n$	1. 912179$_n$	1. 745521$_n$	1. 57402$_n$	1. 39915$_n$	1. 22149$_n$
	$H_{1\cdot0}(n+1.-n)$	2. 283141	2. 497539	2. 132783	1. 289495$_n$	2. 007687$_n$	2. 122538$_n$	2. 124651$_n$	2. 075490$_n$	1. 99603$_n$	1. 89695$_n$	1. 78371$_n$
	$H_{1\cdot0}(n-1.-n)$	2. 023857	2. 639706	3. 175028	3. 237886	3. 225052	3. 170212	3. 088688	2. 988817	2. 87524	2. 75144	2. 61949
	$H_{0\cdot1}(n.-n+1)$	2. 283141$_n$	2. 454799$_n$	1. 836524	2. 464367	2. 583580	2. 591242	2. 546486	2. 470917	2. 37459	2. 26368	2. 14175
	$H_{0\cdot1}(n.-n-1)$	2. 283141$_n$	2. 810509$_n$	3. 359932$_n$	3. 390883$_n$	3. 360857$_n$	3. 295300$_n$	3. 206462$_n$	3. 101284$_n$	2. 98368$_n$	2. 85672$_n$	2. 72223$_n$
	$H_{2\cdot0}(n+2.-n)$	2. 590848$_n$	2. 707952$_n$	2. 376382$_n$	2. 175386$_n$	2. 130812$_n$	2. 164244$_n$	2. 196499$_n$	2. 203577$_n$	2. 18377$_n$	2. 14150$_n$	2. 08066$_n$
	$H_{2\cdot0}(n.-n)$	2. 754592$_n$	3. 199457$_n$	2. 990444$_n$	2. 248049	3. 086389	3. 284047	3. 355296	3. 365683	3. 33861	3. 28624	3. 21513
	$H_{2\cdot0}(n-2.-n)$	2. 251669$_n$	3. 013336$_n$	3. 555780$_n$	3. 787322$_n$	3. 900910$_n$	3. 945468$_n$	3. 945418$_n$	3. 914428$_n$	3. 86046$_n$	3. 78915$_n$	3. 70406$_n$
	$H_{1\cdot1}(n+1.-n+1)$	3. 062127	3. 172492	2. 868916	2. 667577	2. 657025	2. 721975	2. 772070	2. 788666	2. 77427	2. 73517	2. 67639
	$H_{1\cdot1}(n-1.-n+1)$	2. 761928	3. 197028	2. 649500	3. 173785$_n$	3. 512606$_n$	3. 644848$_n$	3. 691986$_n$	3. 690517$_n$	3. 65672$_n$	3. 60008$_n$	3. 52617$_n$
	$H_{1\cdot1}(n+1.-n-1)$	2. 983112	3 357877	2. 854083	2. 930833$_n$	3. 328509$_n$	3. 464872$_n$	3. 509819$_n$	3. 505195$_n$	3. 46829$_n$	3. 40891$_n$	3. 33258$_n$
	$H_{1\cdot1}(n-1.-n-1)$	2. 886452	3. 578026	4. 131226	4. 308446	4. 390309	4. 414545	4. 400460	4. 359230	4. 29747	4. 22004	4. 13004
	$H_{0\cdot2}(n.-n+2)$	2. 912119$_n$	3. 039643$_n$	2. 771808$_n$	2. 578026$_n$	2. 603308$_n$	2. 693928$_n$	2. 756447$_n$	2. 779086$_n$	2. 76791$_n$	2. 73067$_n$	2. 67316$_n$
	$H_{0\cdot2}(n.-n)$	2. 937465$_n$	3. 283318$_n$	2. 935144	3. 560728	3. 763936	3. 846773	3. 867352	3. 849057	3. 80356	3. 73829	3. 65773
	$H_{0\cdot2}(n.-n-2)$	2. 912119$_n$	3. 532772$_n$	4. 094393$_n$	4. 226022$_n$	4. 279805$_n$	4. 285230$_n$	4. 257724$_n$	4. 206449$_n$	4. 13689$_n$	4. 05324$_n$	3. 95818$_n$
	$H_{0\cdot0}(n+1.-n+1)+\sigma$	2. 756164$_n$	3. 213148$_n$	3. 209606$_n$	3. 161233$_n$	3. 084354$_n$	2. 98785$_n$	2. 87705$_n$	2. 75548$_n$	2. 62518$_n$	2. 48808$_n$	2. 34533$_n$
	$H_{0\cdot0}(n-1.-n-1)-\sigma$	2. 404818$_n$	2. 937464$_n$	3. 021354$_n$	3. 018627$_n$	2. 969687$_n$	2. 89202$_n$	2. 79476$_n$	2. 68338$_n$	2. 56103$_n$	2. 43031$_n$	2. 29280$_n$
	$H_{0\cdot0}(n+1.-n-1)+\delta$	2. 756164	3. 213148	3. 209606	3. 161233	3. 084354	2. 98785	2. 87705	2. 75548	2. 62518	2. 48808	2. 34533
	$H_{0\cdot0}(n-1.-n+1)-\delta$	2. 404818	2. 937464	3. 021354	3. 018627	2. 969687	2. 89202	2. 79476	2. 68338	2. 56103	2. 43031	2. 29280
Factor w	$H_{0\cdot0}(n.-n)$	2. 10705	2. 49201	2. 86960	2. 85902	2. 80438	2. 72253	2. 62211	2. 50826	2. 38394	2. 25162	2. 11274
	$H_{1\cdot0}(n+1.-n)$	2. 84832$_n$	3. 11926$_n$	2. 94344$_n$	2. 51758$_n$	2. 05090	2. 60473	2. 74368	2. 77970	2. 76467	2. 71844	2. 65095
	$H_{1\cdot0}(n-1.-n)$	2. 65255$_n$	3. 23704$_n$	3. 62990$_n$	3. 76534$_n$	3. 82325$_n$	3. 83204$_n$	3. 80720$_n$	3. 75803$_n$	3. 69020$_n$	3. 60798$_n$	3. 51407$_n$
	$H_{0\cdot1}(n\ -n+1)$	2. 84832	3. 08298	2. 54339	2. 72959$_n$	3. 07532$_n$	3. 19194$_n$	3. 22466$_n$	3. 21114$_n$	3. 16786$_n$	3. 10328$_n$	3. 02273$_n$
	$H_{0\cdot1}(n.-n-1)$	2. 84832	3. 38959	3. 79758	3. 91047	3. 95464	3. 95433	3. 92304	3. 86908	3. 79756	3. 71242	3. 61614
	$H_{2\cdot0}(n+2.-n)$	3. 30153	3. 47658	3. 26944	3. 06962	2. 93550	2. 89904	2. 92186	2. 95450	2. 97290	2. 97130	
	$H_{2\cdot0}(n.-n)$	3. 49221	3. 91454	3. 87105	3. 58162	3. 01092$_n$	3. 75726$_n$	3. 97365$_n$	4. 07117$_n$	4. 10899$_n$	4. 10968$_n$	
	$H_{2\cdot0}(n-2.-n)$	3. 04285	3. 70335	4. 09904	4. 35862	4. 52456	4. 62359	4. 67529	4. 69189	4. 68170	4. 65065	
	$H_{1\cdot1}(n+1.-n+1)$	3. 72579$_n$	3. 89596$_n$	3. 68472$_n$	3. 47854$_n$	3. 37752$_n$	3. 40062$_n$	3. 46926$_n$	3. 52757$_n$	3. 55905$_n$	3. 56441$_n$	
	$H_{1\cdot1}(n-1.-n+1)$	3. 50554$_n$	3. 91340$_n$	3. 76928$_n$	2. 83064	3. 96157	4. 23549	4. 36784	4. 43113	4. 45108	4. 44105	
	$H_{1\cdot1}(n+1.-n-1)$	3. 66401$_n$	4. 05317$_n$	3. 93197$_n$	3. 49986$_n$	[3. 52156]	3. 96647	4. 13685	4. 21423	4. 24034	4. 23321	
	$H_{1\cdot1}(n-1.-n-1)$	3. 59196$_n$	4. 22680$_n$	4. 63658$_n$	4. 86351$_n$	5. 00543$_n$	5. 08771$_n$	5. 12705$_n$	5. 13448$_n$	5. 11710$_n$	5. 08030$_n$	
	$H_{0\cdot2}(n.-n+2)$	3. 54909	3. 73089	3. 53403	3. 34318	3. 29267	3. 36481	3. 45811	3. 52608	3. 56130	3. 56791	
	$H_{0\cdot2}(n.-n)$	3. 62947	3. 99260	3. 57911	3. 80556$_n$	4. 25522$_n$	4. 44991$_n$	4. 54840$_n$	4. 59212$_n$	4. 59923$_n$	4. 57993$_n$	
	$H_{0\cdot2}(n.-n-2)$	3. 54909	4. 15411	4. 57196	4. 76826	4. 88795	4. 95411	4. 98146	4. 97967	4. 95504	4. 91236	

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Factor w^s													
$H_{0,0}(n+1.-n+1)+\sigma$		3 44221	3. 85011	3. 88349	3. 87921	3. 84673	3. 79264	3. 72171	3. 63735	3. 54179	3. 43716	3. 32484
$H_{0,0}(n-1.-n-1)-\sigma$		3. 16381	3. 62946	3. 71816	3. 74819	3. 73868	3. 70094	3. 64215	3. 56714	3. 47906	3. 38042	3. 27308
$H_{0,0}(n+1.-n-1)+\delta$		3. 44221$_n$	3. 85011$_n$	3. 88349$_n$	3. 87921$_n$	3. 84673$_n$	3. 79264$_n$	3. 72171$_n$	3. 63735$_n$	3. 54179$_n$	3. 43716$_n$	3. 32484$_n$
$H_{0,0}(n-1.-n+1)-\delta$		3. 16381$_n$	3. 62946$_n$	3. 71816$_n$	3. 74819$_n$	3. 73868$_n$	3. 70094$_n$	3. 64215$_n$	3. 56714$_n$	3. 47906$_n$	3. 38042$_n$	3. 27308$_n$
$H_{0,0}(n.-n)$		2. 2332$_n$	2. 6582$_n$	2. 8656$_n$	2. 9668$_n$	2. 9832$_n$	2. 9899$_n$	2. 9576$_n$	2. 9034$_n$	2. 8321$_n$	2$_n$.7473$_n$	2. 6516$_n$
$H_{1,0}(n+1.-n)$		3. 1148	3. 4217	3. 3589	3. 2004	2. 8749	1. 6284$_n$	2. 8458$_n$	3. 0473$_n$	3. 1331$_n$	3. 1592$_n$	
$H_{1,0}(n-1.-n)$		2. 9825	3. 5079	3. 7460	3 9255	4. 0482	4. 1224	4. 1583	4. 1644	4. 1465	4. 1102	
$H_{0,1}(n.-n+1)$		3. 1148$_n$	3. 3936$_n$	3. 2375$_n$	2. 6792$_n$	2. 9683	3. 3400	3. 4938	3. 5640	3. 5866	3. 5773	
$H_{0,1}(n.-n-1)$		3. 1148$_n$	3. 6331$_n$	3. 8807$_n$	4. 0647$_n$	4. 1709$_n$	4. 2394$_n$	4. 2707$_n$	4. 2731$_n$	4. 2524$_n$	4. 2133$_n$	
$H_{2,0}(n.-n)$				4. 3637$_n$				[5. 0467$_n$]				
$H_{2,0}(n-2.-n)$					4. 1173		5. 3997					
$H_{1,1}(n-1.-n+1)$			4. 4525									
$H_{1,1}(n+1.-n-1)$												
$H_{1,1}(n-1.-n-1)$		3. 9085$_n$		4. 3449$_n$		5. 1334$_n$						
$H_{0,2}(n.-n+2)$												
$H_{0,2}(n.-n)$							4. 1848$_n$					
$H_{0,2}(n.-n-2)$			4. 2095		4. 1683							
$H_{0,0}(n-1.-n-1)-\sigma$												
$H_{0,0}(n+1.-n-1)+\delta$												
$H_{0,0}(n-1.-n+1)-\delta$												

It is convenient to have this table in seconds of arc also

TABLE XI. Unit=1″

n	0	1	2	3	4	5	6	7	8
$_{3\cdot0}(n+1.-n)$	$H=+1656$								
$_{3\cdot0}(n-1.-n)$	$G=-1656$		$F=-1955$		$H=-5097$				
$_{3\cdot0}(n-3.-n)$					$G=+6177$		$F=-35276$		$H=+45199$
$_{2\cdot1}(n.-n+1)$		$H=[-9270]$							
$_{2\cdot1}(n-2.-n+1)$		$G=+4207$		$F=-3730$		$H=+22898$			
$_{2\cdot1}(n.-n-1)$		$F=+6733$		$H=+5323$					
$_{2\cdot1}(n-2.-n-1)$				$G=-13730$		$F=+142854$		$H=-200024$	
$_{1\cdot2}(n-1.-n+2)$	$F=+1634$		$H=+5239$						
$_{1\cdot2}(n+1.-n)$	$H=+6391$								
$_{1\cdot2}(n-1.-n)$	$G=-6391$		$F=+3445$		$H=-52183$				
$_{1\cdot2}(n+1.-n-2)$	$F=-1634$		$H=+2712$						
$_{1\cdot2}(n-1.-n-2)$			$G=+4409$		$F=-190467$		$H=+294332$		
$_{0\cdot3}(n.-n+1)$		$H=-7839$							
$_{0\cdot3}(n.-n-1)$		$F=+3479$		$H=+27512$					
$_{0\cdot3}(n.-n-3)$		$G=+6634$		$F=+83314$		$H=-144023$			
$_{1\cdot0}(n.-n+1)+\sigma$		$H=+11006$							
$_{1\cdot0}(n.-n-1)-\sigma$		$F=-1732$		$H=+4022$					
$_{1\cdot0}(n-2.-n-1)-\sigma$				$G=-3616$		$F=-7798$		$H=+9526$	
$_{1\cdot0}(n.-n-1)+\delta$		$F=+6534$		$H=-17562$					
$_{1\cdot0}(n.-n+1)-\delta$		$H=-6391$							
$_{1\cdot0}(n-2.-n+1)-\delta$		$G=+6391$		$F=+4175$		$H=-10846$			
$_{0\cdot1}(n+1.-n)+\sigma$	$H=-3012$								
$_{0\cdot1}(n-1.-n)-\sigma$	$G=+3012$		$F=+159$		$H=+1108$				
$_{0\cdot1}(n-1.-n-2)-\sigma$			$G=+7090$		$F=+11564$		$H=-15308$		
$_{0\cdot1}(n+1.-n)+\delta$	$H=+3012$								
$_{0\cdot1}(n+1.-n-2)+\delta$	$F=-1948$		$H=+21192$						
$_{0\cdot1}(n-1.-n+2)-\delta$	$F=+1948$		$H=+5738$						
$_{0\cdot1}(n-1.-n)-\delta$	$G=-3012$		$F=-3579$		$H=+13813$				
$_{3\cdot0}(n+1.-n)$	$H=-11862$								
$_{3\cdot0}(n-1.-n)$	$G=+11862$		$F=+14862$		$H=-4328$				
$_{3\cdot0}(n-3.-n)$					$G=-16310$		$F=+189348$		$H=-304998$
$_{2\cdot1}(n.-n+1)$		$H=[+61512]$							
$_{2\cdot1}(n-2.-n+1)$		$G=-28940$		$F=[+1693]$		$H=-84425$			
$_{2\cdot1}(n.-n-1)$		$F=-33547$		$H=+55061$					
$_{2\cdot1}(n-2.-n-1)$				$G=+9080$		$F=-673242$		$H=+1218446$	
$_{1\cdot2}(n-1.-n+2)$	$F=-7082$		$H=-35125$						
$_{1\cdot2}(n+1.-n)$	$H=-39356$								
$_{1\cdot2}(n-1.-n)$	$G=+39356$		$F=+15177$		$H=[+143461]$				
$_{1\cdot2}(n+1.-n-2)$	$F=+7082$		$H=-60586$						
$_{1\cdot2}(n-1.-n-2)$			$G=+41698$		$F=+772598$		$H=-1600036$		

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Factor w										
$0^{\cdot}5\langle n.-n+1\rangle$		$H=+47423$								
$0^{\cdot}5\langle n.-n-1\rangle$		$F=+17683$								
$0^{\cdot}5\langle n.-n-3\rangle$		$G=-36904$								
$1^{\cdot}0\langle n.-n+1\rangle+\sigma$		$H=-60629$								
$1^{\cdot}0\langle n.-n-1\rangle-\sigma$		$F=+\ 8521$								
$1^{\cdot}0\langle n-2.-n-1\rangle-\sigma$										
$1^{\cdot}0\langle n.-n-1\rangle+\delta$		$F=-28325$								
$1^{\cdot}0\langle n.-n+1\rangle-\delta$		$H=+39356$								
$1^{\cdot}0\langle n-2.-n+1\rangle-\delta$		$G=-39356$								
	$H=+19091$									
$0^{\cdot}1\langle n+1.-n\rangle+\sigma$	$G=-19091$			$F=-\ 2967$						
$0^{\cdot}1\langle n-1.-n\rangle-\sigma$				$G=-45771$						
$0^{\cdot}1\langle n-1.-n-2\rangle-\sigma$										
$0^{\cdot}1\langle n+1.-n\rangle+\delta$	$H=-19091$			$H=-111790$						
$0^{\cdot}1\langle n+1.-n-2\rangle+\delta$	$F=+\ 9287$			$H=-36971$						
$0^{\cdot}1\langle n-1.-n+2\rangle-\delta$	$F=-\ 9287$			$F=+17494$						
$0^{\cdot}1\langle n-1.-n\rangle-\delta$	$G=+19091$									
Factor w^2										
$1^{\cdot}0\langle n.-n+1\rangle+\sigma$		$H=+179581$								
$1^{\cdot}0\langle n.-n-1\rangle+\delta$		$F=+64794$								
$1^{\cdot}0\langle n.-n+1\rangle-\delta$		$H=[-127808]$								
$1^{\cdot}0\langle n-2.-n+1\rangle-\delta$		$G=[+127808]$								
				$F=+58926$						
	$H=-62910$									
$0^{\cdot}1\langle n+1.-n\rangle+\sigma$	$G=[+62910]$									
$0^{\cdot}1\langle n-1.-n\rangle-\sigma$	$H=+62910$									
$0^{\cdot}1\langle n+1.-n+2\rangle+\delta$				$H=+123344$						
$0^{\cdot}1\langle n-1.-n+2\rangle-\delta$	$G=-62910$									
$0^{\cdot}1\langle n-1.-n\rangle-\delta$										

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TABLE XII.

Unit=1″.

	n	0	1	2	3	4	5
	$F_{1\cdot0}(n+1.-n+1)+\pi'$	− 79.1	− 191.9	− 142.5	− 101.4	− 70.4	− 48.1
	$F_{1\cdot0}(n-1.-n+1)+\pi'$	+ 79.1	+ 191.9	+ 142.5	+ 101.4	+ 70.4	+ 48.1
	$F_{1\cdot0}(n+1.-n-1)-\pi'$	+ 79.1	+ 191.9	+ 142.5	+ 101.4	+ 70.4	+ 48.1
	$F_{1\cdot0}(n-1.-n-1)-\pi'$	− 79.1	− 191.9	− 142.5	− 101.4	− 70.4	− 48.1
Factor w	$F_{1\cdot0}(n+1.-n+1)+\pi'$	+ 327	+ 705	+ 605	+ 493	+ 387	+ 295
	$F_{1\cdot0}(n-1.-n+1)+\pi'$	− 327	− 705	− 605	− 493	− 387	− 295
	$F_{1\cdot0}(n+1.-n-1)-\pi'$	− 327	− 705	− 605	− 493	− 387	− 295
	$F_{1\cdot0}(n-1.-n-1)-\pi'$	+ 327	+ 705	+ 605	+ 493	+ 387	+ 295
Factor w^2	$F_{1\cdot0}(n-1.-n+1)+\pi'$		+1303				
	$F_{1\cdot0}(n-1.-n-1)-\pi'$				−1127		

TABLE XIII.

Unit=1″.

	$2G_{0\cdot0}(n.-n+1)+\pi'$	+ 79.10	+ 191.93	+ 142.48	+ 101.43	+ 70.45	+ 48.14
	$2G_{0\cdot0}(n.-n-1)-\pi'$	− 79.10	− 191.93	− 142.48	− 101.43	− 70.45	− 48.14
	$2G_{1\cdot0}(n+1.-n+1)+\pi'$	− 451.7	− 674.0	− 409.2	− 232.4	− 122.4	− 57.8
	$2G_{1\cdot0}(n-1.-n+1)+\pi'$	− 530.8	− 1441.7	−1406.6	−1246.7	−1038.3	− 828.0
	$2G_{1\cdot0}(n+1.-n-1)-\pi'$	+ 372.6	+ 482.0	+ 266.7	+ 130.9	+ 52.0	+ 9.6
	$2G_{1\cdot0}(n-1.-n-1)-\pi'$	+ 609.9	+ 1633.6	+1549.1	+1348.1	+1108.7	+ 876.1
	$2G_{0\cdot1}(n.-n+2)+\pi'$	+ 649.5	+ 1057.8	+ 622.9	+ 333.8	+ 157.6	+ 57.8
	$2G_{0\cdot1}(n.-n)+\pi'$	+ 333.1	+ 1057.8	+1192.9	+1145.3	+1003.0	+ 828.0
	$2G_{0\cdot1}(n.-n)-\pi'$	− 333.1	− 290.1	− 53.0	+ 71.9	+ 124.2	+ 134.8
	$2G_{0\cdot1}(n.-n-2)-\pi'$	− 649.5	− 1825.5	−1762.8	−1551.0	−1284.8	−1020.6
	$G_{2\cdot0}(n.-n+1)+\pi'$						
	$G_{2\cdot0}(n-2.-n+1)+\pi'$		+ 2656				
	$G_{1\cdot1}(n-1.-n+2)+\pi'$						
	$G_{1\cdot1}(n+1.-n)+\pi'$						
	$G_{1\cdot1}(n-1.-n)+\pi'$	−1506					
	$G_{1\cdot1}(n+1.-n)-\pi'$						
	$G_{1\cdot1}(n-1.-n)-\pi'$	+1506					
	$G_{0\cdot2}(n.-n+1)+\pi'$						
	$G_{0\cdot2}(n.-n+1)-\pi'$						
	$G_{0\cdot0}(n-1.-n)-\delta+\pi'$	+ 266					
	$G_{0\cdot0}(n+1.-n)+\delta+\pi'$						

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$G_{0,0}(n-1.-n+2)-\delta+\pi'$							
$G_{0,0}(n+1.-n)+\sigma-\pi'$		+ 266					
$G_{0,0}(n-1.-n)-\delta-\pi'$							
Factor w	$2G_{0,0}(n.-n+1)+\pi'$	− 327.5	− 705.2	− 606.3	− 493.0	− 386.9	− 295.2
	$2G_{0,0}(n.-n-1)-\pi'$	+ 327.5	+ 705.2	+ 606.3	+ 493.0	+ 386.9	+ 295.2
	$2G_{1,0}(n+1.-n+1)+\pi'$	+2277	+3555	+2501	+1656	+1030	+ 594
	$2G_{1,0}(n-1.-n+1)+\pi'$	+2604	+6376	+6739	+6586	+6059	+5318
	$2G_{1,0}(n+1.-n-1)-\pi'$	−1949	− 2850	−1897	−1163	− 643	− 299
	$2G_{1,0}(n-1.-n-1)-\pi'$	−2932	− 7081	− 7344	− 7079	−6446	−5613
	$2G_{0,1}(n.-n+2)+\pi'$	−3096	− 4966	−3410	−2149	−1223	− 594
	$2G_{0,1}(n.-n)+\pi'$	−1786	− 4966	−5831	−6093	−5866	−5318
	$2G_{0,1}(n.-n)-\pi'$	+1786	+ 2145	+ 989	+ 177	− 325	− 587
	$2G_{0,1}(n.-n-2)-\pi'$	+3096	+ 7786	+8252	+8065	+7413	+6499
	$G_{2,0}(n.-n+1)+\pi'$						
	$G_{2,0}(n-2.-n+1)+\pi'$		−14804				
	$G_{1,1}(n-1.-n+2)+\pi'$						
	$G_{1,1}(n+1.-n)+\pi'$						
	$G_{1,1}(n-1.-n)+\pi'$	+9645					
	$G_{1,1}(n+1.-n)-\pi'$						
	$G_{1,1}(n-1.-n)-\pi'$	−9645					
	$G_{0,2}(n.-n+1)+\pi'$						
	$G_{0,2}(n.-n+1)-\pi'$						
	$G_{0,0}(n-1.-n)-\sigma+\pi'$						
	$G_{0,0}(n+1.-n)+\delta+\pi'$	−1640					
	$G_{0,0}(n-1.-n+2)-\delta+\pi'$						
	$G_{0,0}(n+1.-n)+\sigma-\pi'$						
	$G_{0,0}(n-1.-n)-\delta-\pi'$	−1640					
Factor w^2	$2G_{1,0}(n-1.-n-1)-\pi'$		+12292				
	$2G_{0,1}(n.-n-2)-\pi'$	−7474					

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TABLE XIV. Unit=1″.

	n	0	1	2	3	4	5
	$2H_{0\cdot0}(n.-n+1)+\pi'$	− 79.10	− 191.93	− 142.48	− 101.43	− 70.45	− 48.14
	$2H_{0\cdot0}(n.-n-1)-\pi'$	+ 79.10	+ 191.93	+ 142.48	+ 101.43	+ 70.45	+ 48.14
	$2H_{1\cdot0}(n+1.-n+1)+\pi'$	+ 609.9	+ 1057.8	+ 694.2	+ 435.2	+ 263.3	+ 154.0
	$2H_{1\cdot0}(n-1.-n+1)+\pi'$	+ 372.6	+ 1057.8	+ 1121.6	+ 1043.8	+ 897.4	+ 731.7
	$2H_{1\cdot0}(n+1.-n-1)-\pi'$	− 530.8	− 865.9	− 551.7	− 333.8	− 192.9	− 105.9
	$2H_{1\cdot0}(n-1.-n-1)-\pi'$	− 451.7	− 1249.8	− 1264.1	− 1145.2	− 967.8	− 779.8
	$2H_{0\cdot1}(n.-n+2)+\pi'$	− 649.4	− 1057.8	− 623.0	− 338.2	− 157.7	− 57.8
	$2H_{0\cdot1}(n.-n)+\pi'$	− 333.0	− 1057.8	− 1192.9	− 1145.2	− 1003.0	− 828.0
	$2H_{0\cdot1}(n.-n)-\pi'$	+ 333.0	+ 290.2	+ 53.0	− 71.9	− 124.2	− 134.8
	$2H_{0\cdot1}(n.-n-2)-\pi'$	+ 649.4	+ 1825.6	+ 1762.8	+ 1551.0	+ 1284.8	+ 1020.6
	$H_{2\cdot0}(n.-n+1)+\pi'$		− 3292				
	$H_{2\cdot0}(n-2.-n+1)+\pi'$						
	$H_{1\cdot1}(n-1.-n+2)+\pi'$			+ 2871			
	$H_{1\cdot1}(n+1.-n)+\pi'$	+1506					
	$H_{1\cdot1}(n-1.-n)+\pi'$						
	$H_{1\cdot1}(n+1.-n)-\pi'$	−1506					
	$H_{1\cdot1}(n-1.-n)-\pi'$						
	$H_{0\cdot2}(n.-n+1)+\pi'$		− 3292				
	$H_{0\cdot2}(n.-n+1)-\pi'$		+ 828				
	$H_{0\cdot0}(n-1.-n)-\sigma+\pi'$						
	$H_{0\cdot0}(n+1.-n)+\delta+\pi'$	+ 266					
	$H_{0\cdot0}(n-1.-n+2)-\delta+\pi'$			+ 437			
	$H_{0\cdot0}(n+1.-n)+\sigma-\pi'$	+ 266					
	$H_{0\cdot0}(n-1.-n)-\delta-\pi'$						
	$2H_{0\cdot0}(n.-n+1)+\pi'$	+ 327.5	+ 705.2	+ 605.3	+ 493.0	+ 386.9	+ 295.2
	$2H_{0\cdot0}(n.-n-1)-\pi'$	− 327.5	− 705.2	− 605.3	− 493.0	− 386.9	− 295.2
	$2H_{1\cdot0}(n+1.-n+1)+\pi'$	−2932	− 4966	− 3713	− 2642	− 1803	− 1185
	$2H_{1\cdot0}(n-1.-n+1)+\pi'$	−1949	− 4966	− 5529	− 5600	− 5285	− 4728
	$2H_{1\cdot0}(n+1.-n-1)-\pi'$	+2604	+ 4261	+ 3108	+ 2149	+ 1416	+ 889
	$2H_{1\cdot0}(n-1.-n-1)-\pi'$	+2277	+ 5671	+ 6134	+ 6093	+ 5672	+ 5023
	$2H_{0\cdot1}(n.-n+2)+\pi'$	+3096	+ 4966	+ 3410	+ 2149	+ 1223	+ 594
	$2H_{0\cdot1}(n.-n)+\pi'$	+1786	+ 4966	+ 5831	+ 6093	+ 5866	+ 5318
	$2H_{0\cdot1}(n.-n)-\pi'$	−1786	− 2145	− 989	− 177	+ 325	+ 587
	$2H_{0\cdot1}(n.-n-2)-\pi'$	−3096	− 7786	− 8252	− 8065	− 7413	− 6499
Factor w	$H_{2\cdot0}(n.-n+1)+\pi'$		+20030				
	$H_{2\cdot0}(n-2.-n+1)+\pi'$						
	$H_{1\cdot1}(n-1.-n+2)+\pi'$			−18486			
	$H_{1\cdot1}(n+1.-n)+\pi'$	−9545					

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$H_{1,1}(n-1.-n)+\pi'$	+9545					
$H_{1,1}(n+1.-n)-\pi'$						
$H_{1,1}(n-1.-n)-\pi'$						
		+20039				
$H_{0,2}(n.-n+1)+\pi'$		− 5931				
$H_{0,2}(n.-n+1)-\pi'$						
			− 2927			
$H_{0,0}\langle(n-1.-n)-\sigma+\pi'$	−1640					
$H_{0,0}\langle(n+1.-n)+\delta+\pi'$						
$H_{0,0}\langle(n-1.-n+2)-\delta+\pi'$						
$H_{0,0}\langle(n+1.-n)+\sigma-\pi'$	−1640					
$H_{0,0}\langle(n-1.-n)-\delta-\pi'$						
				+ 1128		
$2H_{0,0}\langle(n.-n+1)+\pi'$		− 1302				
$2H_{0,0}\langle(n.-n-1)-\pi'$						
				+14730		
$2H_{1,0}\langle(n+1.-n+1)+\pi'$						
$2H_{1,0}\langle(n-1.-n+1)+\pi'$		−10988				
$2H_{1,0}\langle(n+1.-n-1)-\pi'$						
$2H_{1,0}\langle(n-1.-n-1)-\pi'$						−15302
	−7474					
			−14432			
$2H_{0,1}\langle(n.-n+2)+\pi'$			+ 4532			
$2H_{0,1}\langle(n.-n)+\pi'$						
$2H_{0,1}\langle(n.-n)-\pi'$					+20360	
$2H_{0,1}\langle(n.-n-2)-\pi'$						
Factor w^2						

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TABLE XV.

Unit=1″

Factor u	n	0	1	2	3	4	5	6	7
	$S_{0\cdot0}(n.-n)$	——	+ 15.71	+ 90.50	+ 72.04	+ 53.29	+ 37.93	+ 26.37	+ 18.04
	$S_{1\cdot0}(n+1.-n)$	——	− 19.7	+ 134.2	+ 140.7	+ 129.7	+ 110.8	+ 89.9	
	$S_{1\cdot0}(n-1.-n)$	——	− 66.8	− 406.6	− 507.7	+ − 509.8	− 458.2	− 384.7	− 308.4
	$S_{0\cdot1}(n.-n+1)$	——	+ 43.2	− 43.7	− 104.6	− 129.7	− 129.7	− 116.3	
	$S_{0\cdot1}(n.-n-1)$	——	+ 106.1	+ 680.3	+ 759.8	+ 723.0	+ 628.9	+ 516.5	
	$S_{2\cdot0}(n+2.-n)$			− 777				+ 2678	
	$S_{2\cdot0}(n.-n)$			+ 838					
	$S_{2\cdot0}(n-2.-n)$								
	$S_{1\cdot1}(n+1.-n+1)$				+ 560				
	$S_{1\cdot1}(n-1.-n+1)$		− 116				− 7982		
	$S_{1\cdot1}(n+1.-n-1)$		− 540						
	$S_{1\cdot1}(n-1.-n-1)$								
	$S_{0\cdot2}(n.-n+2)$	——							
	$S_{0\cdot2}(n.-n)$	——		− 499		+ 5930			
	$S_{0\cdot2}(n.-n-2)$	——							
	$S_{0\cdot0}(n+1.-n+1+\sigma$								
	$S_{0\cdot0}(n-1.-n-1)-\sigma$		———				+ 192.6		
	$S_{0\cdot0}(n+1.-n-1)+\delta$		− 383.9		− 202.9				
	$S_{0\cdot0}(n-1.-n+1)-\delta$								
	$S_{0\cdot0}(n.-n)$	——	− 49.8	− 212.2	− 218.4	− 197.8	− 166.4	− 133.4	− 103.4
	$S_{1\cdot0}(n+1.-n)$	——	+ 87	− 216	− 343	− 417	− 439	− 422	
	$S_{1\cdot0}(n-1.-n)$	——	+ 236	+1057	+1622	+ 1956	+ 2057	+ 1980	+1791
	$S_{0\cdot1}(n.-n+1)$	——	− 161	+ 4	+ 234	+ 417	+ 522	+ 555	
	$S_{0\cdot1}(n.-n-1)$	——	− 360	−1693	−2387	− 2747	− 2906	− 2647	
	$S_{2\cdot0}(n+2.-n)$			+1194					
	$S_{2\cdot0}(n.-n)$			−2573				−14096	
	$S_{2\cdot0}(n-2.-n)$								
	$S_{1\cdot1}(n+1.-n+1)$				− 906				
	$S_{1\cdot1}(n-1.-n+1)$								
	$S_{1\cdot1}(n+1.-n-1)$		+ 651				+36591		
	$S_{1\cdot1}(n-1.-n-1)$		+2092						
	$S_{0\cdot2}(n.-n+2)$	——							
	$S_{0\cdot2}(n.-n)$	——		+ 150		−23115			
	$S_{0\cdot2}(n.-n-2)$	——							

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	Factor w^3									
$S_{0,0}(n+1.-n+1)+\sigma$			+1410					− 1181		
$S_{0,0}(n-1.-n-1)-\sigma$										
$S_{0,0}(n+1.-n-1)+\delta$					+ 986					
$S_{0,0}(n-1.-n+1)-\delta$										
	$S_{0,0}(n.-n)$	———	+ 62	+ 175	+ 250	+ 289	+ 297	+ 282		
		———	− 176	− 63	+ 155	+ 405	+ 617	+ 760		
	$S_{1,0}(n+1.-n)$	———	− 362	−1117	−2092	− 3070	− 3844	− 4314		
	$S_{1,0}(n-1.-n)$	———								
		———	+ 269	+ 239	− 30	− 404	− 765	− 1042		
	$S_{0,1}(n.-n+1)$	———	+ 517	+1643	+2966	+ 4227	+ 5183	+ 5724		
	$S_{0,1}(n.-n-1)$	———								

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INTEGRATION OF THE DIFFERENTIAL EQUATION FOR W.

With the exception of Tables LVI and LVII all the following tables are concerned with the integration of functions whose coefficients can be derived, more or less directly, from the preceding tables. The terms of first order in the mass, before and after integration, are of the type

$$\Sigma C_{p\cdot q}(n+r.-n+s)\eta^p\eta'^q j^{2t}\begin{Bmatrix}\sin\\ \cos\end{Bmatrix}\begin{Bmatrix}A\\ A+\varepsilon-\psi\\ A-\varepsilon+\psi\end{Bmatrix}$$

where $$C_{p\cdot q}=C_{0\cdot p\cdot q}+C_{1\cdot p\cdot q}\cdot w+C_{2\cdot p\cdot q}\cdot w^2+\cdots \text{(see } Z \text{ 25)}$$

and $$A=[n+r-\tfrac{1}{2}(n-s)]\varepsilon+(n-s)\theta+i\,\Pi+i'\,\Pi'$$

In the argument A the factor n is always a positive integer; the factors r, s, i, and i' are positive and negative integers. Evidently, the factor of ε is $\pm\frac{k}{2}$ where k is any positive integer, and the arguments in a series are $\Sigma_n\Sigma_r\Sigma_s A$. Within the extent of Bohlin's tables all of the coefficients can be written in symbolic form from B 188, XVII, XVIII. In the notation for the coefficients the particular values of r and s are given, and there remains to be found only the positive value of n, if there is one, for each multiple of $\frac{\varepsilon}{2}$.

The following tables present, in skeleton form, any series of the given type. There are properly two tables, one for perturbations in the plane of the orbit, and the other for perturbations perpendicular to the same. The headings Δ and Σ are defined by

$$\Delta=\Pi-\Pi'$$
$$\Sigma=\Pi+\Pi'$$

Considering first the tables referring to the plane of the orbit, omitting for the moment the arguments bearing the subscripts $\pm\delta$ or $\pm\sigma$, the argument A for any term is read from a main heading $\pm\frac{k\varepsilon}{2}$ and the first two columns under this heading. The tabulated numbers are the respective factors of θ, Δ, and Σ. The degree of the factors in the eccentricities is indicated in the subscripts $p\cdot q$ in the symbol for the coefficient. Further, when $j^{2t}=1$

$$i\,\Pi+i'\Pi'=n(\Pi-\Pi')=n\Delta.$$

Hence the coefficient of Δ is also the number n in the proper table of the numerical values of the coefficients. For instance, in the function T_2 (Z 41, eq. 82) we have for one term

$$F_{1\cdot 0}(n-1.-n)\eta\sin(\varepsilon+4\theta+4\Delta)$$

where F, taken from Table VIII, is numerically

$$F_{1\cdot 0}(n-1.-n)_{n=4}=-1514''+5780''w-8976''w^2.$$

Adding $\varepsilon-\psi$ to the argument and taking the coefficients from Table IX, we have also in the function T,

$$G_{1\cdot 0}(n-1.-n)_{n=4}\eta\sin(2\varepsilon-\psi+4\theta+4\Delta)$$

where $$G_{1\cdot 0}(n-1.-n)_{n=4}=+452''-1475''w+1451''w^2.$$

In this manner the series is built up.

The coefficients having subscripts $\pm\delta$ and $\pm\sigma$ belong to terms depending upon the mutual inclination of the orbit planes. They differ from the preceding type of terms in three ways. In the first place the subscript signifies the addition of $\pm\Delta$ and $\pm\Sigma$ to the argument, respectively. Evidently, if $\pm\Delta$ is added to the argument, the factor of Δ is not n but $n\pm1$, from which we determine n. Lastly, these terms contain the factor j^2, i. e., within the extent of our tables the exponent t is not greater than unity.

For the tables referring to functions which concern the perturbations in the third coordinate the same explanations hold, with the exception that the additional subscript $\pm\pi'$ signifies the addition of $\pm\Pi'$ to the argument.

These tables, in connection with the proper tables of numerical coefficients, enable the computer to write a complete series by inspection or segregate any term of given degree and given argument.

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TABLE XVa.

$$\Sigma C_{p,q}(n+r.-n+s)\eta^p\gamma'^q\eta^{2k}\begin{matrix}\sin\\\cos\end{matrix}\begin{cases}A+\epsilon-\phi\\A-\epsilon+\phi\end{cases}$$

In the Plane of the Orbit.

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TABLE XV*b*.

$$\Sigma C_{p,q}(n+r.-n+s)\eta^p\rho_7{}^r q_j{}^{2t}\begin{matrix}\sin\\ \cos\end{matrix}\begin{cases}A\\ A+\epsilon-\psi\\ A-\epsilon+\psi\end{cases}$$

Perpendicular to the Plane of the Orbit.

				$+\epsilon$			$-\epsilon$			$+2\epsilon$			-2ϵ			$+\frac{1}{2}\epsilon$			$-\frac{1}{2}\epsilon$			$+\frac{3}{2}\epsilon$			$-\frac{3}{2}\epsilon$		
$C_{p,q}$	θ	A	Σ	θ	A	Σ	θ	A	Σ	θ	A	Σ	θ	A	Σ	θ	A	Σ	θ	A	Σ	θ	A	Σ	θ	A	Σ
$0,0(n.-n+1)+\pi'$				0	1					2	3					−1	0					1	2				
$0,0(n.-n-1)-\pi'$	2	1		4	3					6	5					3	2		1	0		5	4				
$1,0(n+1.-n+1)+\pi'$										0	1											−1	0				
$1,0(n-1.-n+1)+\pi'$	0	1		2	3					4	5					1	2		−1	0		3	4				
$1,0(n+1.-n-1)-\pi'$				2	1					4	3					1	0					3	2				
$1,0(n-1.-n-1)-\pi'$	4	3		6	5		2	1		8	7					5	4		3	2		7	6		1	0	
$0,1(n.-n+2)+\pi'$				−2	0					0	2											−1	1				
$0,1(n.-n)+\pi'$	0	0		2	2					4	4					1	1					3	3				
$0,1(n.-n)-\pi'$	0	0		2	2					4	4					1	1					3	3				
$0,1(n.-n-2)-\pi'$	4	2		6	4		2	0		8	6					5	3		3	1		7	5				
$2,0(n+2.-n+1)+\pi'$																											
$2,0(n.-n+1)+\pi'$				0	1					2	3					−1	0					1	2				
$2,0(n-2.-n+1)+\pi'$	2	3		4	5		0	1		6	7					3	4		1	2		5	6		−1	0	
$2,0(n+2.-n-1)-\pi'$										2	1											1	0				
$2,0(n.-n-1)-\pi'$	2	1		4	3					6	5					3	2		1	0		5	4				
$2,0(n-2.-n-1)-\pi'$	6	5		8	7		4	3		10	9		2	1		7	6		5	4		9	8		3	2	
$1,1(n+1.-n+2)+\pi'$										−2	0																
$1,1(n-1.-n+2)+\pi'$	−2	0		0	2					2	4					−1	1					1	3				
$1,1(n+1.-n)+\pi'$				0	0					2	2											1	1				
$1,1(n-1.-n)+\pi'$	2	2		4	4		0	0		6	6					3	3		1	1		5	5				
$1,1(n+1.-n)-\pi'$				0	0					2	2											1	1				
$1,1(n-1.-n)-\pi'$	2	2		4	4		0	0		6	6					3	3		1	1		5	5				
$1,1(n+1.-n-2)-\pi'$	2	0		4	2					6	4					3	1					5	3				
$1,1(n-1.-n-2)-\pi'$	6	4		8	6		4	2		10	8		2	0		7	5		5	3		9	7		3	1	
$0,2(n.-n+3)+\pi'$										−2	1											−3	0				
$0,2(n.-n+1)+\pi'$				0	1					2	3					−1	0					1	2				
$0,2(n.-n-1)+\pi'$	2	1		4	3					6	5					3	2		1	0		5	4				
$0,2(n.-n+1)-\pi'$				0	1					2	3					−1	0					1	2				
$0,2(n.-n-1)-\pi'$	2	1		4	3					6	5					3	2		1	0		5	4				
$0,2(n.-n-3)-\pi'$	6	3		8	5		4	1		10	7					7	4		5	2		9	6		3	0	
$0,0(n+1.-n+2)+\sigma+\pi'$										−2	0	1															
$0,0(n-1.-n)-\sigma+\pi'$	2	2	−1	4	4	−1	0	0	−1	6	6	−1				3	3	−1	1	1	−1	5	5	−1			
$0,0(n+1.-n)+\delta+\pi'$				0	1					2	3											1	2				
$0,0(n-1.-n+2)-\delta+\pi'$	−2	−1		0	1					2	3					−1	0					1	2				
$0,0(n+1.-n)+\sigma-\pi'$				0	0	1				2	2	1										1	1	1			
$0,0(n-1.-n-2)-\sigma-\pi'$	6	4	−1	8	6	−1	4	2	−1	10	8	−1	2	0	−1	7	5	−1	5	3	−1	9	7	−1	3	1	−1
$0,0(n+1.-n-2)+\delta-\pi'$	2	1		4	3					6	5					3	2					5	4				
$0,0(n-1.-n)-\delta-\pi'$	2	1		4	3		0	−1		6	5					3	2		1	0		5	4				

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Our problem is now the integration of the partial differential equations Z 7, eq. (33), Z 8, eqs. (37) and (39), and Z 9, eq. (47[1]).

In the trigonometric series to be integrated the argument is a function of θ, ε, ψ, Δ, Σ. The last two are constants. According to the principles of Hansen, ψ occurs outside the operation. Numerically, however, it is equal to ε. The argument θ contains ε implicitly. See Z 9, eq. (43). Hence we must, in general, write

$$F(\varepsilon, \theta)$$

and

$$\frac{dF}{d\varepsilon} = \frac{\partial F}{\partial \varepsilon} + \frac{\partial F}{\partial \theta} \cdot \frac{d\theta}{d\varepsilon}$$

In order to set up the partial differential equations from the total derivative, the following notation is introduced:

$$F(\varepsilon, \theta) = [F(\varepsilon, \theta)] + F(\varepsilon, \theta) - [F(\varepsilon, \theta)]$$

where $[F(\varepsilon, \theta)]$ signifies that part of the function which is independent of ε. Again, since ε has the period of the planet, there can be no secular terms in ε (with the exception of the function θ), i. e.,

$$\left[\frac{\partial F}{\partial \varepsilon}\right] = 0$$

On the other hand, the argument θ varies much more slowly, and there may be secular terms in θ. Hence

$$\left[\frac{\partial F}{\partial \theta}\right] \neq 0$$

and θ may occur outside the sign of integration.

Owing to the presence of the required function in the differential equation, the integrations must be performed rank by rank where rank is defined as follows:

In the course of the developments there arise negative powers of w. Since w is a small quantity, these factors increase the numerical value of the terms, or, in other words, they lower the order. Therefore, it is better to define order in terms of both the disturbing mass m' and w. For this purpose v. Zeipel makes the assumption that both w and $\sqrt{m'}$ are quantities of the first order. Order so defined is called "rank," and the word "order" is reserved as usual for the powers of m'. The factors $\frac{m'^{\alpha}}{w^{\beta}}$ are arranged according to rank in Z 53.

Any function is then written in the form

$$F(\varepsilon, \theta) = F_1(\varepsilon, \theta) + F_2(\varepsilon, \theta) + F_3(\varepsilon, \theta) + \cdots\cdots$$

where the subscript denotes the term of *lowest* rank, for F_i (ε, θ) contains terms of more than one rank since each coefficient is itself a Taylor's series in w. In assigning rank it is to be noted that the coefficients in all the preceding tables contain the factor m' implicitly. The implicit mass factor is indicated at the foot of each table which follows.

On the basis of the foregoing principles, the differential equation for W,

$$\frac{dW}{d\varepsilon} = \frac{\partial W}{\partial \varepsilon} + \frac{\partial W}{\partial \theta} \cdot \frac{d\theta}{d\varepsilon} = T$$

expressed in Z 52, eq. (91), is broken up into four equations, Z 53, eqs. (95_1 — 95_4), according to rank, and before integration they are again subdivided according to parts which contain ε and parts which are independent of ε. The total derivative is then in the form of eight equivalent equations, and the integration can be performed in the following order:

$$W_1;\ W_2 - [W_2];\ [W_2];\ W_3 - [W_3];\ \text{etc.}$$

It is possible to avoid the computation of T_3, as v. Zeipel did, by the introduction of some auxiliary functions, but we found it preferable to tabulate them.

Employing Table XV*a*, and by inspection of Tables VIII, IX, X, XI, T_2 is written directly. (T has no terms of first rank.)

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TABLE XVc.

T_2 Unit=1″

	Sin	w^0	w	w^2
	$\epsilon-\phi$	+ 43.1	− 128.0	+ 171
	$-\epsilon+\phi$	− 43.1	+ 128.0	− 171
	$\epsilon+2\theta+2\Delta$	+ 271.5	− 636.6	+ 526
	$2\epsilon-\phi+2\theta+2\Delta$	− 67.1	+ 106.2	+ 32
	$\phi+2\theta+2\Delta$	− 294.9	+ 740.6	− 734
	$2\epsilon+4\theta+4\Delta$	+ 159.9	− 593.3	+ 869
	$3\epsilon-\phi+4\theta+4\Delta$	− 45.7	+ 122.1	− 174
	$\epsilon+\phi+4\theta+4\Delta$	− 167.4	+ 637.4	− 984
	$2\epsilon+\phi+6\theta+6\Delta$	− 81.7	+ 418.9	− 907
η	$2\theta+2\Delta$	−1180	+2962	− 2935
	$\epsilon-\phi+2\theta+2\Delta$	+ 273	− 179	− 1170
	$-\epsilon+\phi+2\theta+2\Delta$	+1496	−4265	+ 5572
η	ϵ	− 173	+ 512	− 684
	$2\epsilon-\phi$	− 211	+ 899	− 1921
	ϕ	+ 384	−1410	+ 2605
η	$\epsilon+4\theta+4\Delta$	−1514	+5780	− 8976
	$2\epsilon-\phi+4\theta+4\Delta$	+ 452	−1475	+ 1451
	$\phi+4\theta+4\Delta$	+1679	−6656	+11172
η	$2\epsilon+2\theta+2\Delta$	− 6	+ 408	− 1307
	$3\epsilon-\phi+2\theta+2\Delta$	− 83	+ 262	− 564
	$\epsilon+\phi+2\theta+2\Delta$	+ 136	− 878	+ 2285
η	$2\epsilon+6\theta+6\Delta$	−1149	+5902	−12820
	$3\epsilon-\phi+6\theta+6\Delta$	+ 360	−1734	+ 3301
	$\epsilon+\phi+6\theta+6\Delta$	+1227	−6415	+14400
η	$2\epsilon+\phi+4\theta+4\Delta$	− 102	+ 112	
η	$2\epsilon+\phi+8\theta+8\Delta$	+ 750	−4900	
η'	$2\theta+\Delta$	+ 318	−1081	+ 1552
	$\epsilon-\phi+2\theta+\Delta$	+ 222	−1012	+ 2227
	$-\epsilon+\phi+2\theta+\Delta$	− 646	+2452	− 4296
η'	$\epsilon+\Delta$	+ 130	− 484	+ 808
	$2\epsilon-\phi+\Delta$	+ 112	− 565	+ 1393
	$\phi+\Delta$	− 285	+1211	− 2475
η'	$\epsilon+4\theta+3\Delta$	+2279	−7160	+ 8896
	$2\epsilon-\phi+4\theta+3\Delta$	− 580	+1410	− 520
	$\phi+4\theta+3\Delta$	−2460	+8138	−11342
η'	$2\epsilon+2\theta+3\Delta$	− 314	+ 702	− 90
	$3\epsilon-\phi+2\theta+3\Delta$	+ 127	− 399	+ 598
	$\epsilon+\phi+2\theta+3\Delta$	+ 291	− 537	− 478
η'	$2\epsilon+6\theta+5\Delta$	+1887	−8417	+15550
	$3\epsilon-\phi+6\theta+5\Delta$	− 542	+2221	− 3377
	$\epsilon+\phi+6\theta+5\Delta$	−1974	+9002	−17350
η'	$2\epsilon+\phi+4\theta+5\Delta$	+ 390	−1556	
η'	$2\epsilon+\phi+8\theta+7\Delta$	−1263	+7397	
η^2	$\epsilon-\phi$	+ 568	− 3106	
	$-\epsilon+\phi$	− 568	+ 3106	
η^2	$4\theta+4\Delta$	+6716	− 26627	+ 44700
	$\epsilon-\phi+4\theta+4\Delta$	−2114	+ 6488	
	$-\epsilon+\phi+4\theta+4\Delta$	−7960	+ 33462	
η^2	$\epsilon+2\theta+2\Delta$	+ 128	− 3166	
	$2\epsilon-\phi+2\theta+2\Delta$	+ 535	− 2505	
	$\phi+2\theta+2\Delta$	− 978	+ 7431	− 23105
			m'	

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TABLE XVc—Continued.

T_2 Unit=1″

	Sin	w^0	w	w^2
η^2	$\epsilon+6\theta+6\Delta$	+ 7969	− 41736	
	$2\epsilon-\phi+6\theta+6\Delta$	− 2624	+ 12577	
	$\phi+6\theta+6\Delta$	− 8819	+ 47347	−111337
η^2	$-\epsilon+2\theta+2\Delta$	+ 2245	− 6168	
	$-\phi+2\theta+2\Delta$	− 396	− 1494	+ 9351
	$-2\epsilon+\phi+2\theta+2\Delta$	− 3596	+ 12561	
η^2	2ϵ	+ 423	− 1797	
	$3\epsilon-\phi$	+ 357	− 2207	
	$\epsilon+\phi$	− 780	+ 4005	
η^2	$2\epsilon+4\theta+4\Delta$	− 1783	+ 3946	
	$3\epsilon-\phi+4\theta+4\Delta$	+ 924	− 3327	
	$\epsilon+\phi+4\theta+4\Delta$	+ 1220	− 1026	
η^2	$2\epsilon+8\theta+8\Delta$	+ 6749	− 44127	
	$3\epsilon-\phi+8\theta+8\Delta$	− 2247	+ 14052	
	$\epsilon+\phi+8\theta+8\Delta$	− 7252	+ 48051	
$\eta\,\eta'$	Δ	− 285	+ 1210	− 2475
	$\epsilon-\phi+\Delta$	− 1004	+ 5771	
	$-\epsilon+\phi+\Delta$	+ 1574	− 8192	
$\eta\,\eta'$	$4\theta+3\Delta$	−17218	+ 56961	− 79400
	$\epsilon-\phi+4\theta+3\Delta$	+ 4253	− 8340	
	$-\epsilon+\phi+4\theta+3\Delta$	+20345	− 73031	
$\eta\,\eta'$	$\epsilon+2\theta+\Delta$	− 1429	+ 6138	
	$2\epsilon-\phi+2\theta+\Delta$	− 523	+ 3792	
	$\phi+2\theta+\Delta$	+ 2280	− 11302	+ 28347
$\eta\,\eta'$	$\epsilon+2\theta+3\Delta$	+ 1725	− 3054	
	$2\epsilon-\phi+2\theta+3\Delta$	− 1003	+ 3753	
	$\phi+2\theta+3\Delta$	− 1492	+ 677	+ 13097
$\eta\,\eta'$	$\epsilon+6\theta+5\Delta$	−23773	+108605	
	$2\epsilon-\phi+6\theta+5\Delta$	+ 7038	− 28427	
	$\phi+6\theta+5\Delta$	+25974	−122380	+251019
$\eta\,\eta$	$-\epsilon+2\theta+\Delta$	− 965	+ 3533	
	$-\phi+2\theta+\Delta$	− 2068	+ 10582	+ 39011
	$-2\epsilon+\phi+2\theta+\Delta$	+ 3785	− 16928	
$\eta\,\eta'$	$2\epsilon+\Delta$	− 820	+ 3797	
	$3\epsilon-\phi+\Delta$	− 470	+ 3185	
	$\epsilon+\phi+\Delta$	+ 1488	− 7870	
$\eta\,\eta'$	$2\epsilon+4\theta+3\Delta$	+ 1815	− 1190	
	$3\epsilon-\phi+4\theta+3\Delta$	− 1181	+ 3807	
	$\epsilon+\phi+4\theta+3\Delta$	− 853	− 3161	
$\eta\,\eta'$	$2\epsilon+4\theta+5\Delta$	+ 4294	− 17092	
	$3\epsilon-\phi+4\theta+5\Delta$	− 1571	+ 6629	
	$\epsilon+\phi+4\theta+5\Delta$	− 4414	+ 17198	
$\eta\,\eta'$	$2\epsilon+8\theta+7\Delta$	−21544	+126397	
	$3\epsilon-\phi+8\theta+7\Delta$	+ 6700	− 37167	
	$\epsilon+\phi+8\theta+7\Delta$	+22868	−136294	
η'^2	$\epsilon-\phi$	+ 866	− 4261	
	$-\epsilon+\phi$	− 866	+ 4261	
η'^2	$4\theta+2\Delta$	+10682	− 28347	+ 32120
	$\epsilon-\phi+4\theta+2\Delta$	− 1815	+ 474	
	$-\epsilon+\phi+4\theta+2\Delta$	−12428	+ 37322	
η'^2	$\epsilon+2\theta+2\Delta$	− 1498	+ 450	
	$2\epsilon-\phi+2\theta+2\Delta$	+ 1136	− 4394	
	$\phi+2\theta+2\Delta$	+ 861	+ 3794	− 22127
		m'		

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XVc—Continued.

T_2 Unit=1″

	Sin	w^0	w	w^2
η'^2	$\varepsilon+ 6\theta+4\Delta$	+17790	− 69344	
	$2\varepsilon-\psi+6\theta+4\Delta$	− 4675	+ 15200	
	$\psi+6\theta+4\Delta$	−19046	+ 77260	−135954
η'^2	$-\psi+2\theta$	+ 1634	− 7081	+ 16199
	$-2\varepsilon+\psi+2\theta$	− 1634	+ 7081	
η'^2	$2\varepsilon+ 2\Delta$	+ 328	− 1710	
	$3\varepsilon-\psi+ 2\Delta$	+ 154	− 1141	
	$\varepsilon+\psi+ 2\Delta$	− 591	+ 3420	
η'^2	$2\varepsilon+ 4\theta+4\Delta$	− 5879	+ 19019	
	$3\varepsilon-\psi+4\theta+4\Delta$	+ 2032	− 7361	
	$\varepsilon+\psi+4\theta+4\Delta$	+ 5807	− 17998	
η'^2	$2\varepsilon+ 8\theta+6\Delta$	+17340	− 90064	
	$3\varepsilon-\psi+8\theta+6\Delta$	− 5018	+ 24266	
	$\varepsilon+\psi+8\theta+6\Delta$	−18102	+ 95820	
j^2	$\varepsilon-\psi$	− 866	+ 4260	
	$-\varepsilon+\psi$	+ 866	− 4260	
j^2	$4\theta+3\Delta-\Sigma$	+ 609	− 2958	+ 6763
	$\varepsilon-\psi+4\theta+3\Delta-\Sigma$	+ 232	− 1656	
	$-\varepsilon+\psi+4\theta+3\Delta-\Sigma$	− 1044	+ 5600	
j^2	$\varepsilon+ 2\theta+2\Delta$	− 1760	+ 7189	
	$2\varepsilon-\psi+2\theta+2\Delta$	− 331	+ 3096	
	$\psi+2\theta+2\Delta$	+ 2677	− 12681	+ 30930
j^2	$\varepsilon+ 6\theta+5\Delta-\Sigma$	+ 578	− 3543	
	$2\varepsilon-\psi+6\theta+5\Delta-\Sigma$	+ 10	− 299	
	$\psi+6\theta+5\Delta-\Sigma$	− 780	+ 5023	− 15302
j^2	$-\psi+2\theta+\Delta-\Sigma$	+ 866	− 4260	+ 10988
	$-2\varepsilon+\psi+2\theta+\Delta-\Sigma$	− 866	+ 4260	
j^2	$2\varepsilon+ \Delta+\Sigma$	+ 1152	− 4231	
	$3\varepsilon-\psi+ \Delta+\Sigma$	+ 98	− 1440	
	$\varepsilon+\psi+ \Delta+\Sigma$	− 1634	+ 7081	
j^2	$2\varepsilon+ 4\theta+4\Delta$	− 1795	+ 9459	
	$3\varepsilon-\psi+4\theta+4\Delta$	+ 164	− 17	
	$\varepsilon+\psi+4\theta+4\Delta$	+ 2229	− 12595	
j^2	$2\varepsilon+ 8\theta+7\Delta-\Sigma$	+ 392	− 2914	
	$3\varepsilon-\psi+8\theta+7\Delta-\Sigma$	− 40	+ 194	
	$\varepsilon+\psi+8\theta+7\Delta-\Sigma$	− 482	+ 3691	
	$\frac{1}{2}\varepsilon+ \theta+ \Delta$	+ 47.1	− 149.3	+ 186
	$\frac{1}{2}\varepsilon-\psi+ \theta+ \Delta$	+ 27.5	− 111.4	+ 207
	$-\frac{1}{2}\varepsilon+\psi+ \theta+ \Delta$	− 90.4	+ 310.5	− 455
	$\frac{1}{2}\varepsilon+ 3\theta+3\Delta$	+ 216.1	− 655.2	+ 749
	$\frac{1}{2}\varepsilon-\psi+3\theta+3\Delta$	− 58.9	+ 150.7	− 93
	$\frac{1}{2}\varepsilon+\psi+3\theta+3\Delta$	− 229.3	+ 722.8	− 905
	$\frac{1}{2}\varepsilon+ 5\theta+5\Delta$	+ 113.8	− 499.2	+ 892
	$\frac{1}{2}\varepsilon-\psi+5\theta+5\Delta$	− 33.5	+ 137.7	− 213
	$\frac{1}{2}\varepsilon+\psi+5\theta+5\Delta$	− 118.2	+ 527.9	− 977
	$\frac{1}{2}\varepsilon+ 7\theta+7\Delta$	+ 54.1	− 310.2	+ 757
	$\frac{1}{2}\varepsilon-\psi+7\theta+7\Delta$	− 16.5	+ 91.3	− 209
	$\frac{1}{2}\varepsilon+\psi+7\theta+7\Delta$	− 55.7	+ 322.3	− 801
	$\frac{1}{2}\varepsilon+ 9\theta+9\Delta$	+ 24.5	− 173.5	+ 537
	$1\frac{1}{2}\varepsilon-\psi+9\theta+9\Delta$	− 7.6	+ 52.7	− 157
	$\frac{1}{2}\varepsilon+\psi+9\theta+9\Delta$	− 25.1	+ 178.5	− 559
		m'		

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XVc—Continued.

T_2 Unit—1″

	Sin	w^0	w	w^2
η	$\frac{1}{2}\varepsilon+ 3\theta+3\Delta$	−1497	+ 4732	− 5963
	$\frac{1}{2}\varepsilon-\phi+3\theta+3\Delta$	+ 419	− 966	+ 131
	$-\frac{1}{2}\varepsilon+\phi+3\theta+3\Delta$	+1729	− 5826	+ 8424
η	$-\frac{1}{2}\varepsilon+ \theta+ \Delta$	− 114	+ 385	− 548
	$\frac{1}{2}\varepsilon-\phi+ \theta+ \Delta$	− 224	+ 1006	− 2186
	$-\frac{1}{2}\varepsilon+\phi+ \theta+ \Delta$	+ 436	− 1726	+ 3220
η	$\frac{1}{2}\varepsilon+ \theta+ \Delta$	− 208	+ 781	− 1315
	$\frac{1}{2}\varepsilon-\phi+ \theta+ \Delta$	− 55	+ 349	− 1026
	$\frac{1}{2}\varepsilon+\phi+ \theta+ \Delta$	+ 314	− 1316	+ 2641
η	$\frac{1}{2}\varepsilon+ 5\theta+5\Delta$	−1366	+ 6114	−11363
	$\frac{1}{2}\varepsilon-\phi+5\theta+5\Delta$	+ 420	− 1711	+ 2548
	$\frac{1}{2}\varepsilon+\phi+5\theta+5\Delta$	+1480	− 6793	+13254
η	$\frac{1}{2}\varepsilon+ 3\theta+3\Delta$	+ 108	− 20	− 847
	$\frac{1}{2}\varepsilon-\phi+3\theta+3\Delta$	− 85	+ 256	− 395
	$\frac{1}{2}\varepsilon+\phi+3\theta+3\Delta$	− 19	− 329	+ 1586
η	$\frac{1}{2}\varepsilon+ 7\theta+7\Delta$	− 922	+ 5348	−13320
	$\frac{1}{2}\varepsilon-\phi+7\theta+7\Delta$	+ 292	− 1618	+ 3692
	$\frac{1}{2}\varepsilon+\phi+7\theta+7\Delta$	+ 975	− 5728	+14602
η	$\frac{1}{2}\varepsilon+ 5\theta+5\Delta$	+ 172	− 594	+ 491
	$\frac{1}{2}\varepsilon-\phi+5\theta+5\Delta$	− 74	+ 298	− 470
	$\frac{1}{2}\varepsilon+\phi+5\theta+5\Delta$	− 133	+ 402	− 42
η	$\frac{1}{2}\varepsilon+ 9\theta+9\Delta$	− 541	+ 3856	−12092
	$\frac{1}{2}\varepsilon-\phi+9\theta+9\Delta$	+ 174	− 1205	+ 3608
	$\frac{1}{2}\varepsilon+\phi+9\theta+9\Delta$	+ 564	− 4055	+12887
η'	$\frac{1}{2}\varepsilon+3\theta+2\Delta$	+2041	− 5080	+ 4928
	$\frac{1}{2}\varepsilon-\phi+3\theta+2\Delta$	− 431	+ 499	− 1026
	$-\frac{1}{2}\varepsilon+\phi+3\theta+2\Delta$	−2290	+ 6274	− 7597
η'	$\frac{1}{2}\varepsilon-\phi+ \theta$	+ 384	− 1410	+ 2605
	$-\frac{1}{2}\varepsilon+\phi+ \theta$	− 384	+ 1410	− 2605
η'	$\frac{1}{2}\varepsilon+ \theta+2\Delta$	− 131	+ 12	+ 717
	$\frac{1}{2}\varepsilon-\phi+ \theta+2\Delta$	+ 106	− 366	+ 772
	$\frac{1}{2}\varepsilon+\phi+ \theta+2\Delta$	+ 69	+ 350	− 1728
η'	$\frac{1}{2}\varepsilon+5\theta+4\Delta$	+2169	− 8241	+12680
	$\frac{1}{2}\varepsilon-\phi+5\theta+4\Delta$	− 596	+ 1980	− 2086
	$\frac{1}{2}\varepsilon+\phi+5\theta+4\Delta$	−2295	+ 9008	−14823
η'	$\frac{1}{2}\varepsilon+3\theta+4\Delta$	− 389	+ 1251	− 1212
	$\frac{1}{2}\varepsilon-\phi+3\theta+4\Delta$	+ 135	− 479	+ 687
	$\frac{1}{2}\varepsilon+\phi+3\theta+4\Delta$	+ 383	− 1189	+ 930
η'	$\frac{1}{2}\varepsilon+7\theta+6\Delta$	+1550	− 7940	+17170
	$\frac{1}{2}\varepsilon-\phi+7\theta+6\Delta$	− 457	+ 2211	− 4245
	$\frac{1}{2}\varepsilon+\phi+7\theta+6\Delta$	−1609	+ 8376	−18650
η'	$\frac{1}{2}\varepsilon+5\theta+6\Delta$	− 349	+ 1665	− 3127
	$\frac{1}{2}\varepsilon-\phi+5\theta+6\Delta$	+ 113	− 543	+ 1052
	$\frac{1}{2}\varepsilon+\phi+5\theta+6\Delta$	+ 352	− 1678	+ 3117
η'	$\frac{1}{2}\varepsilon+9\theta+8\Delta$	+ 937	− 6044	+16950
	$\frac{1}{2}\varepsilon-\phi+9\theta+8\Delta$	− 286	+ 1784	− 4724
	$\frac{1}{2}\varepsilon+\phi+9\theta+8\Delta$	− 963	+ 6274	−17880
η^2	$\frac{1}{2}\varepsilon+ \theta+ \Delta$	+ 757	− 3272	
	$\frac{1}{2}\varepsilon-\phi+ \theta+ \Delta$	+ 514	− 3644	
	$-\frac{1}{2}\varepsilon+\phi+ \theta+ \Delta$	−1583	+ 8214	
η^2	$\frac{1}{2}\varepsilon+5\theta+5\Delta$	+7767	−35692	
	$\frac{1}{2}\varepsilon-\phi+5\theta+5\Delta$	−2522	+10085	
	$-\frac{1}{2}\varepsilon+\phi+5\theta+5\Delta$	−8820	+42033	
		m'		

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TABLE XVc—Continued.

T_2 Unit—1″

	Sin	w^0	w	w^2
η^2	$-\frac{1}{2}\varepsilon + 3\theta + 3\Delta$	+ 4758	− 15945	
	$\frac{1}{2}\varepsilon - \psi + 3\theta + 3\Delta$	− 1369	+ 2307	
	$-\frac{1}{2}\varepsilon + \psi + 3\theta + 3\Delta$	− 6128	+ 22836	
η^2	$\frac{1}{2}\varepsilon + 3\theta + 3\Delta$	− 882	− 280	
	$\frac{1}{2}\varepsilon - \psi + 3\theta + 3\Delta$	+ 732	− 2580	
	$\frac{1}{2}\varepsilon + \psi + 3\theta + 3\Delta$	+ 177	+ 3816	
η^2	$\frac{1}{2}\varepsilon + 7\theta + 7\Delta$	+ 7549	− 44427	
	$\frac{1}{2}\varepsilon - \psi + 7\theta + 7\Delta$	− 2504	+ 13864	
	$\frac{1}{2}\varepsilon + \psi + 7\theta + 7\Delta$	− 8212	+ 49192	
η^2	$-\frac{1}{2}\varepsilon + \theta + \Delta$	− 32	+ 220	
	$-\frac{1}{2}\varepsilon - \psi + \theta + \Delta$	+ 784	− 4194	
	$-\frac{1}{2}\varepsilon + \psi + \theta + \Delta$	− 1031	+ 5051	
η^2	$\frac{1}{2}\varepsilon + 9\theta + 9\Delta$	+ 5780	− 41583	
	$\frac{1}{2}\varepsilon - \psi + 9\theta + 9\Delta$	− 1929	+ 13412	
	$\frac{1}{2}\varepsilon + \psi + 9\theta + 9\Delta$	− 6154	+ 44735	
$\eta\,\eta'$	$\frac{1}{2}\varepsilon + \theta$	− 768	+ 2821	
	$\frac{1}{2}\varepsilon - \psi + \theta$	− 1156	+ 6406	
	$-\frac{1}{2}\varepsilon + \psi + \theta$	+ 1924	− 9227	
$\eta\,\eta'$	$\frac{1}{2}\varepsilon + \theta + 2\Delta$	+ 209	+ 1404	
	$\frac{1}{2}\varepsilon - \psi + \theta + 2\Delta$	− 771	+ 3774	
	$-\frac{1}{2}\varepsilon + \psi + \theta + 2\Delta$	+ 446	− 5879	
$\eta\,\eta'$	$\frac{1}{2}\varepsilon + 5\theta + 4\Delta$	−21869	+ 85960	
	$\frac{1}{2}\varepsilon - \psi + 5\theta + 4\Delta$	+ 6125	− 19358	
	$-\frac{1}{2}\varepsilon + \psi + 5\theta + 4\Delta$	+24564	−101260	
$\eta\,\eta'$	$-\frac{1}{2}\varepsilon + 3\theta + 2\Delta$	−10182	+ 27638	
	$\frac{1}{2}\varepsilon - \psi + 3\theta + 2\Delta$	+ 1576	+ 2276	
	$-\frac{1}{2}\varepsilon + \psi + 3\theta + 2\Delta$	+13528	− 43309	
$\eta\,\eta'$	$\frac{1}{2}\varepsilon + 3\theta + 2\Delta$	+ 384	+ 3626	
	$\frac{1}{2}\varepsilon - \psi + 3\theta + 2\Delta$	− 939	+ 3382	
	$\frac{1}{2}\varepsilon + \psi + 3\theta + 2\Delta$	+ 715	− 8550	
$\eta\,\eta'$	$\frac{1}{2}\varepsilon + 3\theta + 4\Delta$	+ 3250	− 10017	
	$\frac{1}{2}\varepsilon - \psi + 3\theta + 4\Delta$	− 1333	+ 4996	
	$\frac{1}{2}\varepsilon + \psi + 3\theta + 4\Delta$	− 3256	+ 9153	
$\eta\,\eta'$	$\frac{1}{2}\varepsilon + 7\theta + 6\Delta$	−23414	+122108	
	$\frac{1}{2}\varepsilon - \psi + 7\theta + 6\Delta$	+ 7146	− 34410	
	$\frac{1}{2}\varepsilon + \psi + 7\theta + 6\Delta$	+25145	−133985	
$\eta\,\eta'$	$-\frac{1}{2}\varepsilon + \theta$	+ 768	− 2821	
	$-\frac{1}{2}\varepsilon - \psi + \theta$	− 2308	+ 10637	
	$-\frac{1}{2}\varepsilon + \psi + \theta$	+ 1540	− 7816	
$\eta\,\eta'$	$\frac{1}{2}\varepsilon + 9\theta + 8\Delta$	−18847	+122928	
	$\frac{1}{2}\varepsilon - \psi + 9\theta + 8\Delta$	+ 5935	− 37138	
	$\frac{1}{2}\varepsilon + \psi + 9\theta + 8\Delta$	+19837	−130949	
η'^2	$\frac{1}{2}\varepsilon + \theta + \Delta$	+ 761	− 3333	
	$\frac{1}{2}\varepsilon - \psi + \theta + \Delta$	+ 906	− 5387	
	$-\frac{1}{2}\varepsilon + \psi + \theta + \Delta$	− 1920	+ 9831	
η'^2	$\frac{1}{2}\varepsilon + 5\theta + 3\Delta$	+15303	− 49954	
	$\frac{1}{2}\varepsilon - \psi + 5\theta + 3\Delta$	− 3577	+ 7957	
	$-\frac{1}{2}\varepsilon + \psi + 5\theta + 3\Delta$	−16828	+ 58649	
η'^2	$-\frac{1}{2}\varepsilon + 3\theta + \Delta$	+ 1582	− 5765	
	$\frac{1}{2}\varepsilon - \psi + 3\theta + \Delta$	+ 1300	− 6572	
	$-\frac{1}{2}\varepsilon + \psi + 3\theta + \Delta$	− 3410	+ 14260	
η'^2	$\frac{1}{2}\varepsilon - \theta + \Delta$	+ 451	− 1890	
	$\frac{1}{2}\varepsilon - \psi - \theta + \Delta$	+ 494	− 2861	
	$\frac{1}{2}\varepsilon + \psi - \theta + \Delta$	− 1096	+ 5381	
		m'		

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TABLE XVc—Continued.

T_2 Unit=1″

	Sin	w^0	w	w^2
η'^2	$\frac{1}{2}\epsilon$ $+3\theta+3\Delta$	− 3918	+ 8760	
	$\frac{1}{2}\epsilon-\phi+3\theta+3\Delta$	+ 1588	− 5289	
	$\frac{1}{2}\epsilon+\phi+3\theta+3\Delta$	+ 3637	− 6391	
η'^3	$\frac{1}{2}\epsilon$ $+7\theta+5\Delta$	+ 18292	− 83098	
	$\frac{1}{2}\epsilon-\phi+7\theta+5\Delta$	− 5104	+ 20825	
	$\frac{1}{2}\epsilon+\phi+7\theta+5\Delta$	− 19286	+ 89973	
j^2	$\frac{1}{2}\epsilon$ $+\theta+\Delta$	− 902	+ 3781	
	$\frac{1}{2}\epsilon-\phi+\theta+\Delta$	− 988	+ 5721	
	$-\frac{1}{2}\epsilon+\phi+\theta+\Delta$	+ 2191	− 10762	
j^2	$\frac{1}{2}\epsilon$ $+5\theta+4\Delta-\Sigma$	+ 634	− 3482	
	$\frac{1}{2}\epsilon-\phi+5\theta+4\Delta-\Sigma$	+ 87	− 836	
	$-\frac{1}{2}\epsilon+\phi+5\theta+4\Delta-\Sigma$	− 933	+ 5479	
j^2	$-\frac{1}{2}\epsilon$ $+3\theta+2\Delta-\Sigma$	+ 428	− 1816	
	$\frac{1}{2}\epsilon-\phi+3\theta+2\Delta-\Sigma$	+ 480	− 2805	
	$-\frac{1}{2}\epsilon+\phi+3\theta+2\Delta-\Sigma$	− 1050	+ 5226	
j^2	$\frac{1}{2}\epsilon$ $+3\theta+3\Delta$	− 1916	+ 8929	
	$\frac{1}{2}\epsilon-\phi+3\theta+3\Delta$	+ 2	+ 1220	
	$\frac{1}{2}\epsilon+\phi+3\theta+3\Delta$	+ 2553	− 13126	
j^2	$\frac{1}{2}\epsilon$ $+7\theta+6\Delta-\Sigma$	+ 488	− 3307	
	$\frac{1}{2}\epsilon-\phi+7\theta+6\Delta-\Sigma$	− 27	+ 23	
	$\frac{1}{2}\epsilon+\phi+7\theta+6\Delta-\Sigma$	− 623	+ 4387	
j^2	$-\frac{1}{2}\epsilon$ $+\theta\quad-\Sigma$	− 475	+ 1965	
	$-\frac{1}{2}\epsilon-\phi+\theta\quad-\Sigma$	+ 1141	− 5536	
	$-\frac{1}{2}\epsilon+\phi+\theta\quad-\Sigma$	− 508	+ 2916	
j^2	$\frac{1}{2}\epsilon$ $+\theta+2\Delta+\Sigma$	+ 1282	− 5447	
	$\frac{1}{2}\epsilon-\phi+\theta+2\Delta+\Sigma$	− 90	− 384	
	$\frac{1}{2}\epsilon+\phi+\theta+2\Delta+\Sigma$	− 1620	+ 7647	
j^2	$\frac{1}{2}\epsilon$ $+5\theta+5\Delta$	− 1544	+ 9111	
	$\frac{1}{2}\epsilon-\phi+5\theta+5\Delta$	+ 222	− 735	
	$\frac{1}{2}\epsilon+\phi+5\theta+5\Delta$	+ 1838	− 11413	
j^2	$\frac{1}{2}\epsilon$ $+9\theta+8\Delta-\Sigma$	+ 304	− 2460	
	$\frac{1}{2}\epsilon-\phi+9\theta+8\Delta-\Sigma$	− 42	+ 266	
	$\frac{1}{2}\epsilon+\phi+9\theta+8\Delta-\Sigma$	− 364	+ 3013	
η^3	$2\theta+2\Delta$	− 1955	+ 14862	
	$6\theta+6\Delta$	− 35276	+ 189348	
	ϕ	+ 3312	− 23724	
	$\phi+4\theta+4\Delta$	− 5097	− 4328	
	$-\phi+4\theta+4\Delta$	+ 6177	− 16310	
	$\phi+8\theta+8\Delta$	+ 45199	− 304998	
$\eta^2\eta'$	$2\theta+\Delta$	+ 6733	− 33547	
	$2\theta+3\Delta$	− 3730	+ 1693	
	$6\theta+5\Delta$	+142854	− 673242	
	$\phi\quad+\Delta$	− 9270	+ 61512	
	$-\phi\quad+\Delta$	+ 4207	− 28940	
	$\phi+4\theta+3\Delta$	+ 5323	+ 55061	
	$-\phi+4\theta+3\Delta$	− 13730	+ 9080	
	$\phi+4\theta+5\Delta$	+ 22898	− 84425	
	$\phi+8\theta+7\Delta$	−200024	+1218446	
$\eta\,\eta'^2$	2θ	− 3268	+ 14164	
	$2\theta+2\Delta$	+ 3445	+ 15177	
	$6\theta+4\Delta$	−190467	+ 772593	
	ϕ	+ 12782	− 78712	
	$\phi\quad+2\Delta$	+ 5239	− 35125	
	$\phi+4\theta+2\Delta$	+ 2712	− 60586	
	$-\phi+4\theta+2\Delta$	+ 4409	+ 41693	
	$\phi+4\theta+4\Delta$	− 52183	+ 143461	
	$\phi+8\theta+6\Delta$	+294332	−1600036	
			m'	

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TABLE XVc—Continued.

T_2 Unit=1″

	Sin	w^0	w	w^2
η'^3	$2\theta+\Delta$	+ 3479	− 17883	
	$6\theta+3\Delta$	+ 83314	− 283500	
	$\phi \quad +\Delta$	− 7839	+ 47423	
	$-\phi+4\theta+\Delta$	+ 6634	− 36904	
	$\phi+4\theta+3\Delta$	+ 27512	− 44330	
	$\phi+8\theta+5\Delta$	−144023	+ 688658	
$j^2\eta$	$2\theta+2\Delta$	+ 10709	− 50725	
	$2\theta+\Delta-\Sigma$	− 1732	+ 8521	
	$6\theta+5\Delta-\Sigma$	− 7799	+ 50227	
	ϕ	− 12782	+ 78712	
	$\phi \quad +\Delta+\Sigma$	+ 11006	− 60629	
	$\phi+4\theta+3\Delta-\Sigma$	+ 4022	− 29208	
	$-\phi+4\theta+3\Delta-\Sigma$	− 3616	+ 27235	
	$\phi+4\theta+4\Delta$	− 28408	+ 176052	
	$\phi+8\theta+7\Delta-\Sigma$	+ 9526	− 75678	
$j^2\ \eta'$	$2\theta+\Delta$	− 7475	+ 36068	
	$2\theta+2\Delta-\Sigma$	+ 159	− 2967	
	$6\theta+4\Delta-\Sigma$	+ 11564	− 66719	
	$\phi \quad +\Delta$	+ 11762	− 75153	
	$\phi \quad +\Sigma$	− 6024	+ 38182	
	$-\phi+4\theta+2\Delta-\Sigma$	+ 7090	− 45771	
	$\phi+4\theta+3\Delta$	+ 35006	− 199168	
	$\phi+4\theta+4\Delta-\Sigma$	+ 1108	− 281	
	$\phi+8\theta+6\Delta-\Sigma$	− 15308	+ 111481	
		m'		

An inspection of the preceding table, which is typical of all the trigonometric series under consideration, shows readily that any function of this type is of the form

$$\Sigma k' \sin K' + \Sigma k \sin (K \pm \phi) = \Sigma k' \sin K' + \Sigma k \sin K \cos \phi \pm \Sigma k \cos K \sin \phi$$

or

$$\Sigma k' \cos K' + \Sigma k \cos (K \pm \phi) = \Sigma k' \cos K' + \Sigma k \cos K \cos \phi \mp \Sigma k \sin K \sin \phi$$

or, more briefly,

$$a + b \cos \phi + c \sin \phi$$

where a, b, c are trigonometric series and can be written by inspection from the tabulated function.

Hence, in v. Zeipel's notation (Z 54, eq. 96),

$$T_i = X_i + Y_i \cos \phi + Z_i \sin \phi$$

and the integral may be written

$$W_i = x_i + y_i \cos \phi + z_i \sin \phi$$

The functions T and W are to be used in this form in solving equations (95).

Considering only first order in the mass in T

$$T_2 = X_2 + Y_2 \cos \phi + Z_2 \sin \phi$$

where

$$X_2 = \Sigma k' \sin K';\ Y_2 = \Sigma k \sin K;\ Z_2 = \pm \Sigma k \cos K$$

or, X_2 is the part of T_2 which is independent of ϕ, Y_2 is a trigonometric sine series having the same numerical coefficients as the part of T_2 which contains ϕ in the argument, but in which ϕ is omitted from the argument, and Z_2 is the corresponding cosine series.

Considering the first two of the eqs. (95), the first one states that W_1 is not a function of ϵ alone, or,

$$W_1 - [W_1] = 0;\ W_1 = [W_1].$$

Making use of this fact in the second, W_1 can be obtained from (95_2). (See Z 54.) Introducing the auxiliary functions ψ_1 and u_1, defined by (99) and (101), the differential equation for W_1 is replaced by the equivalent differential equations, (100) and (102), for ψ_1 and u_1.

The series

$$[X_2] - \eta [Y_2]$$

and

$$[Y_2] \cos \phi + [Z_2] \sin \phi$$

can be written by inspection from T_2, or, better, the integration itself can be performed in part at the same time.

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The function ϕ_1 is given by Z 59, eq. (103), or,

$$\phi_1 - w = \frac{2}{w}\int^\theta ([X_2]-\eta[Y_2])d\theta - \frac{2}{w^3}\left\{\int^\theta ([X_2]-\eta[Y_2])d\theta\right\}^2 + \frac{4}{w^5}\left\{\int^\theta ([X_2]-\eta[Y_2])d\theta\right\}^3 + \ldots.$$

From the table of T_2, page 82, it is not difficult to write immediately

$$\int^\theta ([X_2]-\eta[Y_2])d\theta$$

The terms of higher order must be obtained by the usual method for the mechanical multiplication of series. A logarithmic multiplication is the most direct.

In each term in the expression for ϕ_1 the terms of lowest rank must be of the first rank. Recalling the tabulation of factors in Z 53, w, $\frac{m'}{w}$, $\frac{m'^2}{w^3}$, $\frac{m'^3}{w^5}$, etc., are all of first rank. But the coefficient for a given argument consists of three terms in ascending powers of w. Hence $\phi_1 - w$, within the limits of the given tabulation for T_2, is of rank 1, 2, 3 for each order in the mass. Table XVI, giving $\phi_1 - w$, is tabulated with double headings. The three subheadings indicate the expansion of the coefficients in a Taylor's series and the main headings give the factors in the development of the radical in Z 59, eq. (103).

Having found ϕ_1, its reciprocal, ϕ_1^{-1}, inclusive of first order in the mass, is given by

$$\phi_1^{-1} = \frac{1}{w} - \frac{2}{w^3}\int^\theta ([X_2]-\eta[Y_2])d\theta$$

The second term is the negative of the first three columns of Table XVI multiplied by w^{-2}. The product of $2\phi_1^{-1}$ and that part of T_2 which contains ψ gives $\frac{du_1}{d\theta}$, and integration with respect to θ gives u_1, tabulated in XVIII. The function u_1 is of first and higher rank because the factor ϕ_1^{-1} is of rank minus one and T_2 is of second rank.

From Table XVIII y_1 can be read by inspection, and ηy_1 added to Table XVI gives x_1, tabulated in Table XVII. The function W_1 is the sum of Tables XVII and XVIII.

In the integration those terms whose arguments are independent of θ are of the nature of constants. In accordance with the condition that there may be secular terms in θ, the integral contains such terms as the following:

$$\theta \cdot k \sin(\psi + \Delta).$$

As the constant of integration

$$\theta_0 \cdot k \sin(\psi + \Delta)$$

is added. Hence the integral contains terms such as

$$(\theta - \theta_0)\, k \sin(\psi + \Delta,$$

where θ_0 is the value of θ for the time $t = 0$.

In passing, it should be noted that, in order that the expansion of Z 59, eq. (103), shall represent the function, we must have

$$\left|\frac{4}{w^2}\int^\theta ([X_2]-\eta[Y_2])\, d\theta\right| < 1$$

and this condition should be tested for a given planet before applying this method of determining the perturbations.

To the computer the extent of auxiliary tables, the arrangement of series in logarithms or natural numbers, in seconds of arc or radians, inclusive or exclusive of numerical factors, and foresight in combining operations—all these are of the greatest importance. But considerations of this kind would carry the reader into complicated details which are best left to the computer's own judgment.

On the other hand, general considerations about the extent of the published tables are of importance in the discussion of the accuracy of the final tables. Yet, for a given limit of accuracy, it is so difficult to determine, for each table, the highest powers of m', w, η, η', and j^2 that little or nothing is said about it in connection with individual tables, but the discussion is reserved until later.

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TABLE XVI.

$\phi_1 - w = x_1 - \eta y_1 = [(1 - e\cos\varepsilon)\overline{W}_1]$ Unit = 4th decimal of a radian.

	Cos	w^{-1}			w^{-3}			w^{-5}	
		w^0	w	w^2	w^0	w	w^2	w^0	w
η^2					−0.0460	+0.231	−0.52		
$\eta\,\eta'^2$					−0.0060	+0.040	−0.127		
$\eta\,\eta'$	Δ				+0.0331	−0.195	+0.53		
η	$2\theta+2\Delta$	+ 42.889	− 107.72	+ 106.7					
$\eta\,\eta'$	$2\theta+\Delta$	− 15.427	+ 52.39	− 75.2					
η^2	$4\theta+4\Delta$	− 122.10	+ 484.1	− 813	−0.0460	+0.231	−0.52		
$\eta\,\eta'$	$4\theta+3\Delta$	+ 357.75	− 1183.5	+1650	+0.0331	−0.195	+0.53		
$\eta\,\eta'^2$	$4\theta+2\Delta$	− 258.93	+ 687.2	− 779	−0.0060	+0.040	−0.127		
j^2	$4\theta+3\Delta-\Sigma$	− 14.75	+ 71.7	− 164					
η^2	$2\theta+2\Delta$	+ 28.2	− 433		+0.262	−1.70		+0.0003	−0.0022
η^2	$6\theta+6\Delta$	+ 428	− 2295		+0.262	−1.70		+0.0001	−0.0008
$\eta^2\eta'$	$2\theta+\Delta$	− 316.1	+ 1592		−0.767	+4.46		−0.00021	+0.0018
$\eta^2\eta'$	$2\theta+3\Delta$	+ 108.5	− 49		−0.094	+0.69		−0.00011	+0.0009
$\eta^2\eta'$	$6\theta+5\Delta$	−1889	+ 8902		−0.86	+5.2		−0.0001	+0.001
$\eta\,\eta'^2$	2θ	+ 237.6	− 1030		+0.555	−2.87		+0.00004	−0.0002
$\eta\,\eta'^2$	$2\theta+2\Delta$	− 125.3	− 552		+0.276	−1.85		+0.00008	−0.0009
$\eta\,\eta'^2$	$6\theta+4\Delta$	+2770	−11237		+0.83	−4.7			
η'^2	$2\theta+\Delta$	− 168.7	+ 867		−0.200	+1.21			
η'^2	$6\theta+3\Delta$	−1346	+ 4581		−0.200	+1.21			
$j^2\eta$	$2\theta+2\Delta$	− 389.4	+ 1846	−4498					
$j^2\eta$	$2\theta+\Delta-\Sigma$	+ 126.0	− 620		+0.032	−0.23			
$j^2\eta$	$6\theta+5\Delta-\Sigma$	+ 113	− 731		+0.032	−0.23			
$j^2\,\eta'$	$2\theta+\Delta$	+ 362.4	− 1749						
$j^2\,\eta'$	$2\theta+2\Delta-\Sigma$	− 7.7	+ 144		−0.011	+0.09			
$j^2\,\eta'$	$6\theta+4\Delta-\Sigma$	− 187	+ 1078		−0.011	+0.1			
		m'			m'^2			m'^3	

TABLE XVII.

x_1 Unit = 1″.

	Cos	w^{-1}			w^{-3}		
		w^0	w	w^2	w^0	w	w^2
η	$2\theta+2\Delta$	+ 1179.6	− 2963	+ 2935			
$\eta\,\eta'$	$2\theta+\Delta$	− 318.2	+ 1081	− 1552			
η^2					− 0.95	+ 4.8	
η^2	$4\theta+4\Delta$	− 3358	+ 13313	− 22356	− 1.27	+ 6.4	
$\eta\,\eta'$	Δ				+ 0.68	− 4.0	
$\eta\,\eta'$	$4\theta+3\Delta$	+ 8609	− 28481	+ 39702	+ 0.79	− 4.7	
η'^2					− 0.12	+ 0.8	
η'^2	$4\theta+2\Delta$	− 5341	+ 14175	− 16063	− 0.12	+ 0.8	
j^2	$4\theta+3\Delta-\Sigma$	− 304	+ 1479	− 3383			
η^2	$2\theta+2\Delta$	+ 1955	− 14861		+ 7.2	− 46.6	
η^2	$6\theta+6\Delta$	+11758	− 63112		+ 7.2	− 46.6	
$\eta^2\eta'$	$2\theta+\Delta$	− 6732	+ 33547		−15.2	+ 88.0	
$\eta^2\eta'$	$2\theta+3\Delta$	+ 3730	− 1691		− 3.8	+ 27.9	
$\eta^2\eta'$	$6\theta+5\Delta$	−47616	+224423		−21.7	+130.0	
$\eta\,\eta'^2$	2θ	+ 3267	− 14165		+ 7.4	− 37.8	
$\eta\,\eta'^2$	$2\theta+2\Delta$	− 3446	− 15176		+ 7.8	− 52.5	
$\eta\,\eta'^2$	$6\theta+4\Delta$	+63489	−257533		+19.0	−108.1	
$j^2\eta$	$2\theta+\Delta-\Sigma$	+ 1733	− 8522		+ 0.4	− 3.1	
$j^2\eta$	$6\theta+5\Delta-\Sigma$	+ 2599	− 16744		+ 0.7	− 5.2	
$j^2\eta$	$2\theta+2\Delta$	−10709	+ 50748	−123705			
η'^2	$2\theta+\Delta$	− 3479	+ 17880		− 4.1	+ 24.9	
η'^2	$6\theta+3\Delta$	−27772	+ 94500		− 4.1	+ 24.9	
$j^2\,\eta'$	$2\theta+2\Delta-\Sigma$	− 159	+ 2966		− 0.2	+ 1.9	
$j^2\,\eta'$	$6\theta+4\Delta-\Sigma$	− 3855	+ 22240		− 0.2	+ 1.9	
$j^2\,\eta'$	$2\theta+\Delta$	+ 7475	− 36070				
$\eta\,\eta'$	$(\theta-\theta_0)\sin\Delta$	− 570	+ 2421	− 4950	− 0.45	+ 2.7	−7.2
		m'			m'^3		

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TABLE XVIII.

$u_1 = y_1 \cos\phi + z_1 \sin\phi$ Unit=1″.

	Cos	10^{-1}			10^{-2}		
		10^0	10	10^2	10^0	10	10^2
	$\phi+2\theta+2\Delta$	+ 294.89	− 740.6	+ 734			
η	$\phi+4\theta+4\Delta$	− 839.5	+ 3328	− 5586	− 0.316	+ 1.59	− 3.6
η'	$\phi+4\theta+3\Delta$	+ 1229.8	− 4069	+ 5671	+ 0.114	− 0.67	+ 1.8
η^2	$-\phi+2\theta+2\Delta$	+ 396	+ 1494	− 9351	− 2.62	+ 16.8	
η^2	$\phi+2\theta+2\Delta$	+ 978	− 7431	+ 23105	+ 4.42	− 28.4	
η^2	$\phi+6\theta+6\Delta$	+ 2940	− 15782	[+ 37112]	+ 1.80	− 11.7	
$\eta\,\eta'$	$-\phi+2\theta+\Delta$	+ 2068	− 10582	− 39010	+ 6.18	− 36.9	
$\eta\,\eta'$	$\phi+2\theta+3\Delta$	+ 1492	− 677	− 13098	− 1.91	+ 13.6	
$\eta\,\eta'$	$\phi+2\theta+\Delta$	− 2280	+ 11302	− 28348	− 5.57	+ 32.8	
$\eta\,\eta'$	$\phi+6\theta+5\Delta$	− 8658	+ 40793	− 83730	− 3.95	+ 23.6	
η'^2	$-\phi+2\theta$	− 1634	+ 7081	− 16199	− 4.04	+ 21.4	
η'^2	$\phi+2\theta+2\Delta$	− 861	− 3794	+ 22127	+ 2.12	− 14.4	
η'^2	$\phi+6\theta+4\Delta$	+ 6349	− 25753	+ 45318	+ 1.90	− 10.8	
ζ^2	$-\phi+2\theta+\Delta-\Sigma$	− 866	+ 4260	− 10988	− 0.22	+ 1.6	
ζ^2	$\phi+6\theta+5\Delta-\Sigma$	+ 260	− 1674	+ 5101	+ 0.07	− 0.5	
ζ^2	$\phi+2\theta+2\Delta$	− 2677	+ 12681	− 30930			
η^3	$\phi+4\theta+4\Delta$	+ 2549	+ 2164		[−11.9]	[+ 89]	
η^3	$-\phi+4\theta+4\Delta$	− 3089	+ 8155		[+ 3.9]	[− 25]	
η^3	$\phi+8\theta+8\Delta$	−11300	+ 76250		− 8.9	[+ 70]	
$\eta^2\eta'$	$\phi+4\theta+5\Delta$	−11449	+ 42212		+ 1.9	− 23	
$\eta^2\eta'$	$\phi+4\theta+3\Delta$	− 2661	− 27530		[+36.4]	[−241]	
$\eta^2\eta'$	$-\phi+4\theta+3\Delta$	+ 6865	− 4540		−20.3	[+118]	
$\eta^2\eta'$	$\phi+8\theta+7\Delta$	+50005	−304611		[+33.8]	[−248]	
$\eta\,\eta'^2$	$\phi+4\theta+4\Delta$	+26091	− 71730		−10.1	+ 83	
$\eta\,\eta'^2$	$\phi+4\theta+2\Delta$	− 1356	+ 30293		[−25.5]	[+153]	
$\eta\,\eta'^2$	$-\phi+4\theta+2\Delta$	− 2204	− 20846		[+28.0]	[−153]	
$\eta\,\eta'^2$	$\phi+8\theta+6\Delta$	−73583	+400009		[−41.9]	[+284]	
η'^2	$\phi+4\theta+3\Delta$	−13756	+ 22165		+10.1	− 65	
η'^2	$-\phi+4\theta+\Delta$	− 3317	+ 18452		−12.4	+ 64	
η'^2	$\phi+8\theta+5\Delta$	+36006	−172164		+16.6	−104	
$\zeta^2\eta$	$\phi+4\theta+3\Delta-\Sigma$	− 2011	+ 14604		− 1.9	+ 14	
$\zeta^2\eta$	$-\phi+4\theta+3\Delta-\Sigma$	+ 1808	− 13617		[+ 1.9]	[− 14]	
$\zeta^2\eta$	$\phi+8\theta+7\Delta-\Sigma$	− 2381	+ 18919		− 1.1	+ 10	
$\zeta^2\eta$	$\phi+4\theta+4\Delta$	+14204	− 88026		+ 5.7	[− 42]	
$\zeta^2\,\eta'$	$\phi+4\theta+4\Delta-\Sigma$	− 554	+ 140		+ 0.5	− 4	
$\zeta^2\,\eta'$	$-\phi+4\theta+2\Delta-\Sigma$	− 3545	+ 22886		− 1.8	+ 14	
$\zeta^2\,\eta'$	$\phi+8\theta+6\Delta-\Sigma$	+ 3827	− 27870		+ 1.3	− 11	
$\zeta^2\,\eta'$	$\phi+4\theta+3\Delta$	−17503	+ 99584		[− 3.7]	[+ 28]	
	$(\theta-\theta_0)$ sin						
η	ϕ	+ 767.72	− 2820.9	+ 5210	+ 1.265	− 6.35	+14.3
η'	$\phi+\Delta$	− 569.95	+ 2421.1	− 4950	− 0.455	+ 2.69	− 7.2
η^3	ϕ	+ 6624	− 47448		+23.8	[−221.9]	
$\eta^2\eta'$	$\phi+\Delta$	[−18540]	[+123024]		−73.4	+572.4	
$\eta^2\eta'$	$-\phi+\Delta$	+ 8414	− 57880		+36.0	−282.2	
$\eta\,\eta'^2$	$\phi+2\Delta$	+10478	− 70250		+55.2	−374.8	
$\eta\,\eta'^2$	ϕ	+25564	−157424		+87.3	−652.8	
η'^3	$\phi+\Delta$	−15678	+ 94846		−69.9	+438.6	
$\zeta^2\eta$	$\phi+\Delta+\Sigma$	+22012	−121258	+359162	+ 9.9	− 77.0	
$\zeta^2\eta$	ϕ	−25564	+157424	[−511232]	−23.1	+165.0	
$\zeta^2\,\eta'$	$\phi+\Sigma$	−12048	+ 76364	−251640	− 5.2	+ 45.8	
$\zeta^2\,\eta'$	$\phi+\Delta$	+23524	−150306	+498328	+14.8	−112.0	
		m'			m'^2		

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After the determination of W_1, the function $W_2-[W_2]$ is obtained from the solution of Z 53, eq. (95_2). The integral may be written as in Z 63, eqs. (105), (106), or, quite as simply, as follows:

$$W_2-[W_2]=W_2'+\int(T_2-[T_2])d\varepsilon$$

$$W_2'=-\int\left\{\frac{1}{2}(1-e\cos\varepsilon)(w+\overline{W}_1)-\frac{1}{2}[(1-e\cos\varepsilon)(w+\overline{W}_1)]\right\}\frac{dW_1}{d\theta}\,d\varepsilon$$

The function W_2' is given in Table XIX.

Anticipating some later developments, for which we shall need

$$[(1-e\cos\varepsilon)\,\overline{W}]$$

the function

$$[(1-e\cos\varepsilon)\,\overline{W}_2']$$

is tabulated in Table XX.

The determination of $[W_2]$ may be accomplished according to Z 65, eq. (108) – Z 67, eq. (116), or in the manner outlined below, which we regard as preferable.

Repeating Z 65, eq. (107),

$$\phi_1\frac{d[W_2]}{d\theta}+[x_2-\eta y_2+w^2]\frac{dW_1}{d\theta}=\left\{w\phi_1+\frac{3}{4}\left[\left(\overline{W}_1-\frac{1}{3}\Xi_1\right)\left(\overline{W}_1+\frac{1}{9}\Xi_1\right)(1-e\cos\varepsilon)\right]\right\}\frac{dW_1}{d\theta}$$

$$-\left[(1-e\cos\varepsilon)\left\{(w+\overline{W}_1)\frac{\partial}{\partial\theta}\int(T_2-[T_2])d\varepsilon+\frac{dW_1}{d\theta}\int\overline{(T_2-[T_2])d\varepsilon}\right\}\right]+2[T_2]$$

in which all the known parts are contained on the right-hand side, the development of equivalent equations proceeds in a manner analogous to that for W_1.

Writing

$$T_2=X_2+Y_2\cos\psi+Z_2\sin\psi$$

and introducing

$$\phi_2=[x_2]-\eta[y_2]+w^2$$

and equating parts independent of ψ, coefficients of $\cos\psi$ and coefficients of $\sin\psi$, the three equivalent equations are:

$$\phi_1\frac{d[x_2]}{d\theta}+\phi_2\frac{dx_1}{d\theta}=w\phi_1\frac{dx_1}{d\theta}+\frac{3}{4}\left[(1-e\cos\varepsilon)\left(\overline{W}_1-\frac{1}{3}\Xi_1\right)\left(\overline{W}_1+\frac{1}{9}\Xi_1\right)\right]\frac{dx_1}{d\theta}$$

$$-\left[(1-e\cos\varepsilon)(w+\overline{W}_1)\frac{\partial}{\partial\theta}\int(X_2-[X_2])d\varepsilon\right]-\left[(1-e\cos\varepsilon)\overline{\int(T_2-[T_2])d\varepsilon}\right]\frac{dx_1}{d\theta}+2[X_2]$$

$$\phi_1\frac{d[y_2]}{d\theta}+\phi_2\frac{dy_1}{d\theta}=w\phi_1\frac{dy_1}{d\theta}+\frac{3}{4}\left[(1-e\cos\varepsilon)\left(\overline{W}_1-\frac{1}{3}\Xi_1\right)\left(\overline{W}_1+\frac{1}{9}\Xi_1\right)\right]\frac{dy_1}{d\theta}$$

$$-\left[(1-e\cos\varepsilon)(w+\overline{W}_1)\frac{\partial}{\partial\theta}\int(Y_2-[Y_2])d\varepsilon\right]-\left[(1-e\cos\varepsilon)\overline{\int(T_2-[T_2])d\varepsilon}\right]\frac{dy_1}{d\theta}+2[Y_2]$$

$$\phi_1\frac{d[z_2]}{d\theta}+\phi_2\frac{dz_1}{d\theta}=w\phi_1\frac{dz_1}{d\theta}+\frac{3}{4}\left[(1-e\cos\varepsilon)\left(\overline{W}_1-\frac{1}{3}\Xi_1\right)\left(\overline{W}_1+\frac{1}{9}\Xi_1\right)\right]\frac{dz_1}{d\theta}$$

$$-\left[(1-e\cos\varepsilon)(w+\overline{W}_1)\frac{\partial}{\partial\theta}\int(Z_2-[Z_2])d\varepsilon\right]-\left[(1-e\cos\varepsilon)\overline{\int(T_2-[T_2])d\varepsilon}\right]\frac{dz_1}{d\theta}+2[Z_2].$$

Multiplying the second of these by η and subtracting from the first:

$$\frac{d}{d\theta}\left\{\phi_1\left(\phi_2-\frac{w}{2}\phi_1\right)\right\}=\frac{3}{4}\left[(1-e\cos\varepsilon)\left(\overline{W}_1-\frac{1}{3}\Xi_1\right)\left(\overline{W}_1+\frac{1}{9}\Xi_1\right)\right]\frac{d\phi_1}{d\theta}$$

$$-\left[(1-e\cos\varepsilon)(w+\overline{W}_1)\frac{\partial}{\partial\theta}\int\{X_2-\eta Y_2-[X_2-\eta Y_2]\}d\varepsilon\right]$$

$$-\left[(1-e\cos\varepsilon)\overline{\int(T_2-[T_2])\,d\varepsilon}\right]\frac{d\phi_1}{d\theta}+2[X_2-\eta Y_2].$$

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Multiplying the second by $\cos\psi$, the third by $\sin\psi$ and adding:

$$\phi_1\frac{d}{d\theta}(u_2-wu_1)=-\phi_2+\frac{3}{4}\Big[(1-e\cos\varepsilon)\Big(\overline{W}_1-\frac{1}{3}\Xi_1\Big)\Big(\overline{W}_1+\frac{1}{9}\Xi_1\Big)\Big]\frac{du_1}{d\theta}+2([Y_2]\cos\psi+[Z_2]\sin\psi)$$
$$-\Big[(1-e\cos\varepsilon)(w+\overline{W}_1)\frac{\partial}{\partial\theta}\int\{Y_2\cos\psi+Z_2\sin\psi-[Y_2\cos\psi+Z_2\sin\psi]\}d\varepsilon\Big]$$
$$-\Big[(1-e\cos\varepsilon)\int\overline{(T_2-[T_2])d\varepsilon}\Big]\frac{du_1}{d\theta}$$

in which

$$u_2=[y_2]\cos\psi+[z_2]\sin\psi$$

and $[X_2]$, $[Y_2]$, $[Z_2]$ are read by inspection from T_2, which is to be determined as follows:

If Z 50, eqs. (89), (90), are written in the form

$$\frac{1}{\cos^2\varphi}\frac{4\rho r^{-2}}{a^3\cos^2\varphi}[1-\cos(f-\omega)]=4\Big\{1-2\eta\cos\varepsilon-\cos(\varepsilon-\psi)+\eta\cos(2\varepsilon-\psi)+\eta\cos\psi+\dots\dots\Big\}$$

$$\frac{1}{\cos^2\varphi}\Big\{\frac{r^{-3}}{a^3}+\frac{2\rho r^{-2}}{a^3\cos^2\varphi}[1-\cos(f-\omega)]\Big\}=3+14\eta^2-8\eta\cos\varepsilon+2\eta^2\cos 2\varepsilon-2\cos(\varepsilon-\psi)$$
$$-8\eta^2\cos(\varepsilon-\psi)+2\eta\cos(2\varepsilon-\psi)+2\eta\cos\psi+\dots\dots$$

then $T_{\overline{W}}$ and T'_r, given by Z 49, eqs. (84), (85), in connection with Z 50, eq. (87), are given by

$$T_{\overline{W}}=-T_2-4\{1-2\eta\cos\varepsilon-\cos(\varepsilon-\psi)+\eta\cos(2\varepsilon-\psi)+\eta\cos\psi+\dots\}$$
$$\Sigma S_{p,q}(n+r.-n+s)\eta^p\eta'^q j^{2s}\sin A$$

$$T'_r=\{3+14\eta^2-8\eta\cos\varepsilon+2\eta^2\cos 2\varepsilon-2\cos(\varepsilon-\psi)-8\eta^2\cos(\varepsilon-\psi)+2\eta\cos(2\varepsilon-\psi)+2\eta\cos\psi$$
$$+\dots\dots\}\Sigma S_{p,q}(n+r.-n+s)\eta^p\eta'^q j^{2s}\sin A-\frac{3}{2}(1-w)\frac{\partial T_2}{\partial w}$$

and T_2 (Table XVIIIa) is computed by Z 53, eq. (94), in which

$$\Xi=x+2\eta y,\qquad \Xi_i=x_i+2\eta y_i$$

The function

$$\phi_2-w^2=[x_2]-\eta[y_2]$$

is tabulated in Table XXI; the function

$$u_2=[y_2]\cos\psi+[z_2]\sin\psi$$

is tabulated in Table XXII.

From the latter $[y_2]$ can be read by inspection, and $\eta[y_2]$ added to the former gives $[x_2]$. Finally, (Table XXIIa),

$$[W_2]=[x_2]+[y_2]\cos\psi+[z_2]\sin\psi$$

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TABLE XVIIIc.

T_3. Unit=1″

	Sin	w^{-3}			w^{-1}		
		w^0	w	w^2	w^0	w	w^2
	$-\varepsilon+\phi$				+0.339	− 2.01	
	$\varepsilon \quad +2\theta+2\Delta$				−0.375	+ 2.403	
	$2\varepsilon-\phi+2\theta+2\Delta$				−0.137	+ 0.847	
	$\phi+2\theta+2\Delta$				+0.498	− 3.223	+ 7.72
	$2\varepsilon \quad +4\theta+4\Delta$				−0.438	+ 2.234	
	$\varepsilon+\phi+4\theta+4\Delta$				+0.429	− 2.338	
	$2\varepsilon+\phi+6\theta+6\Delta$				+0.361	− 2.372	
η	$2\theta+2\Delta$	−0.00047	+0.0036	−0.0123	+2.199	−14.58	+ 33.34
η	$\varepsilon-\phi+2\theta+2\Delta$				+0.286	− 3.85	
η	$-\varepsilon+\phi+2\theta+2\Delta$				−3.294	+23.82	
η	ε				−2.811	+12.20	
η	ϕ	−0.00038	+0.0035	−0.0136	−0.688	+ 1.67	− 16.51
η	$\varepsilon \quad +4\theta+4\Delta$				+0.482	− 2.58	
η	$\phi+4\theta+4\Delta$	−0.00015	+0.0013	−0.0048	−4.536	+35.80	− 95.79
η	$\varepsilon+\phi+2\theta+2\Delta$				+1.017	− 6.333	
η	$\varepsilon+\phi+6\theta+6\Delta$				−3.219	+22.43	
η'	$2\theta+\Delta$	+0.00017	−0.0014	+0.0055	−2.520	+14.78	− 31.95
η'	$\varepsilon-\phi+2\theta+\Delta$				−1.253	+10.20	
η'	$-\varepsilon+\phi+2\theta+\Delta$				+4.372	−28.56	
η'	$\varepsilon \quad +\Delta$				−0.404	+ 4.06	
η'	$\phi \quad +\Delta$	+0.00014	−0.0014	+0.0060	+1.188	−11.30	+ 34.07
η'	$\varepsilon \quad +4\theta+3\Delta$				−0.224	+ 1.53	
η'	$\phi+4\theta+3\Delta$	+0.00005	−0.0005	+0.0021	+6.480	−47.37	+120.37
η'	$\varepsilon+\phi+2\theta+3\Delta$				+0.214	− 1.66	
η'	$\varepsilon+\phi+6\theta+5\Delta$				+5.977	−36.82	
	$(\theta-\theta_0)$ cos						
η	$2\theta+2\Delta$	−0.00188	+0.0143	−0.0489	−1.141	+ 7.14	− 20.54
η	$\varepsilon-\phi+2\theta+2\Delta$				+0.235	− 1.12	
η	$-\varepsilon+\phi+2\theta+2\Delta$				+1.12	− 7.39	
η	ϕ	−0.00059	+0.0051	−0.0189	−0.357	+ 2.62	− 8.20
η	$\varepsilon \quad +4\theta+4\Delta$				−0.975	+ 7.39	
η	$\phi+4\theta+4\Delta$	+0.00155	−0.0141	+0.0540	+0.939	− 7.27	+ 23.43
η	$\varepsilon+\phi+2\theta+2\Delta$				−1.12	+ 7.39	
η'	$2\theta+\Delta$	+0.00068	−0.0058	+0.0222	+0.847	− 5.79	+ 18.12
η'	$\varepsilon-\phi+2\theta+\Delta$				−0.17	+ 0.93	
η'	$-\varepsilon+\phi+2\theta+\Delta$				−0.828	+ 5.96	
η'	$\phi \quad +\Delta$	+0.00021	−0.0020	+0.0085	+0.265	− 2.10	+ 7.15
η'	$\varepsilon \quad +4\theta+3\Delta$				+0.724	− 5.90	
η'	$\phi+4\theta+3\Delta$	−0.00056	+0.0056	−0.0239	−0.697	+ 5.80	− 20.35
η'	$\varepsilon+\phi+2\theta+3\Delta$				+0.828	− 5.96	
		m'^3			m'^2		

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TABLE XIX.

W_2' Unit=1″.

	Cos	w^0			w^{-2}			w^{-4}	
		w^0	w	w^2	w^0	w	w^2	w^0	w
	$-\phi+\varepsilon$				+0.2108	− 1.059	+2.379		
	$\phi+\varepsilon+4\theta+4\Delta$				−0.2108	+ 1.059	−2.379		
η	ε				+0.843	− 4.236			
η	$\varepsilon+4\theta+4\Delta$				−0.843	+ 4.236			
η	$-\phi+\varepsilon-2\theta-2\Delta$	− 294.9	+ 740.6	−733.9	−1.200	+ 7.772			
η	$-\phi+\varepsilon+2\theta+2\Delta$				−0.875	+ 5.583			
η	$\phi+\varepsilon+2\theta+2\Delta$	+ 294.9	− 740.6	+733.9	+0.274	− 1.697			
η	$\phi+\varepsilon+6\theta+6\Delta$				+1.800	−11.658			
η	$-\phi+2\varepsilon$				−0.105	+ 0.529			
η	$\phi+2\varepsilon+4\theta+4\Delta$				+0.105	− 0.529			
η'	$\varepsilon + \Delta$				−0.227	+ 1.344			
η'	$\varepsilon+4\theta+3\Delta$				+0.227	− 1.344			
η'	$-\phi+\varepsilon-2\theta-\Delta$				+1.758	−10.233			
η'	$-\phi+\varepsilon+2\theta+\Delta$				+1.083	− 6.493			
η'	$\phi+\varepsilon+2\theta+3\Delta$				−0.204	+ 1.377			
η'	$\phi+\varepsilon+6\theta+5\Delta$				−2.637	+15.350			
η^2	$-\phi+\varepsilon$	+ 384	−1410						
η^2	$-\phi+\varepsilon-4\theta-4\Delta$	+1679	−6656						
η^2	$\varepsilon+2\theta+2\Delta$	+1180	−2963						
η^2	$-\varepsilon+2\theta+2\Delta$	−1180	+2963						
η^2	$\phi+\varepsilon$	− 384	+1410						
η^2	$\phi+\varepsilon+4\theta+4\Delta$	−1679	+6656						
$\eta\,\eta'$	$-\phi+\varepsilon - \Delta$	− 285	+1210						
$\eta\,\eta'$	$-\phi+\varepsilon-4\theta-3\Delta$	−2460	+8138						
$\eta\,\eta'$	$\varepsilon+2\theta+\Delta$	− 318	+1081						
$\eta\,\eta'$	$\varepsilon-2\theta-\Delta$	+ 318	−1081						
$\eta\,\eta'$	$\phi+\varepsilon + \Delta$	+ 285	−1210						
$\eta\,\eta'$	$\phi+\varepsilon+4\theta+3\Delta$	+2460	−8138						
	$(\theta-\theta_0)$ sin								
η	$-\phi+\varepsilon-2\theta-2\Delta$				+0.549	− 3.40		+0.00090	−0.0068
η	$\phi+\varepsilon+2\theta+2\Delta$				−0.549	+ 3.40		−0.00090	+0.0068
η'	$-\phi+\varepsilon-2\theta-\Delta$				−0.407	+ 2.75		−0.00032	+0.0027
η'	$\phi+\varepsilon+2\theta+3\Delta$				+0.407	− 2.75		+0.00032	−0.0027
		m'			m'^2			m'^3	

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TABLE XX.

$[(1-e\cos\epsilon)\ \overline{W}'_2]$ Unit=4th decimal of a radian.

Cos		w^0			w^{-2}			w^{-4}	
		w^0	w	w^2	w^0	w	w^2	w^0	w
					+0.01022	−0.0513	+0.115		
η^2		+ 18.6	− 68		+0.187	−1.76		+0.00043	−0.0039
η'^2					+0.296	−2.46		+0.00020	−0.0020
j^2					−0.186	+1.34	−4.8		
$\eta\,\eta'$	Δ	− 13.8	+ 59		−0.529	+4.34		−0.00075	+0.0065
η	$2\theta+2\Delta$	− 14.29	+ 35.9	− 36	−0.1006	+0.647	−1.91	−0.000055	+0.00041
η'	$2\theta+\Delta$				+0.1377	−0.811	+2.19	+0.000020	−0.00017
η^2	$4\theta+4\Delta$	+ 81.4	− 323		+0.477	−3.64			
$\eta\,\eta'$	$4\theta+3\Delta$	− 119.2	+ 395		−1.295	+9.36			
η'^2	$4\theta+2\Delta$				+0.921	−6.09			
j^2	$4\theta+3\Delta-\Sigma$				+0.036	−0.32			
	$(\theta-\theta_0)$ sin								
η	$2\theta+2\Delta$				−0.0266	+0.165	−0.49	−0.000044	+0.00033
η'	$2\theta+\Delta$				+0.0198	−0.134	+0.43	+0.000016	−0.00013
η^2	$4\theta+4\Delta$				+0.151	−1.16		+0.00031	−0.0027
$\eta\,\eta'$	$4\theta+3\Delta$				−0.334	+2.47		−0.00052	+0.0045
η'^2	$4\theta+2\Delta$				+0.165	−1.24		+0.00015	−0.0014
		m'			m'^2			m'^3	

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TABLE XXI.

$\phi_2 - w^2 = [z_2] - \eta[y_2] = [(1 - e\cos\epsilon)\,[W_2]]$.

Unit—4th decimal of a radian.

Cos	w^0	w^1	w^2		w^3	w^4	w^5	w^6
$\eta^3\ \eta'^3$	+0.000378	−0.00243		[+0.0052]	[0.022]	+ 0.03		
$\eta\ \eta'$	+0.000204	−0.00188		[−0.0030]	[+ 0.020]	− 0.06		
$\eta\ \eta'$	−0.000702	+0.00609		[+0.0063]	[− 0.039]	+ 0.12		
			+0.00002	−0.1765	+ 1.155	− 3.001	− 13.162	[+30.862] − 25.5
$\eta^3\ \eta'^3$	[+0.000108]	[−0.00104]	−0.00001	+0.2640	− 1.547	+ 3.747	+ 65.300	−249.32 −431
$\eta\ \eta'^3$	[−0.000034]	[+0.00047]		[+0.5874]	[− 4.946]	+ 8.72	−110.51	+ 347.1 +388
$\eta^3\ \eta'$	[−0.000262]	[+0.00172]		[−1.9416]	[+14.295]	− 25.3		
$\eta\ \eta'$	[−0.000026]	[+0.00026]		[+1.4798]	[− 9.831]	+ 19.8		
f^2				+0.0164	− 0.222	+ 0.52		
	m'^2				m'^2			m'
$(\theta-\theta_0)\sin$								
$2\theta+2\phi$	−0.000132	+0.00099	−0.00338	[−0.0798]	+ 0.4937	− 1.421		
$2\theta+\phi$	+0.000047	−0.00040	+0.00154	[+0.0592]	− 0.4005	+ 1.252		
$4\theta+4\phi$	+0.000030	−0.00016		[+0.3406]	− 2.604	+ 8.04		
$4\theta+3\phi$	−0.001166	+0.01017		[−0.7523]	+ 5.564	− 17.283		
ϕ	[+0.000002]	[−0.00001]		[0.0000]	[− 0.39]	− 5.97		
$4\theta+2\phi$	+0.000930	−0.00308		+0.3707	− 2.80]	+ 9.024		
$(\theta-\theta_0)^2\cos$								
ϕ	[+0.000171]	[−0.00149]		+0.0520	− 0.382	+ 1.055		
$\eta^3\ \eta'^3$	[−0.000188]	[+0.00177]		−0.0771	+ 0.610	− 1.798		
$\eta\ \eta'^3$	[+0.000046]	[−0.00046]		+0.0286	− 0.243	+ 0.765		
	m'^2				m'^2			m'

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TABLE XXII

$u_2 = [y_2] \cos\phi + [z_2] \sin\phi$ Unit=1″.

	Cos	w^{-4}			w^{-3}			w^0		
		w^0	w	w^2	w^0	w	w^2	w	w	w^2
η	$\phi+2\theta+2\Delta$	+0.00022	−0.00170	+0.0059	−0.6139	+ 4.059	−10.30			
η'	$\phi+4\theta+4\Delta$	+0.00132	−0.0123	+0.046	+2.790	[−23.39]	+70.40	+167.4	−637	+985
	$\phi+4\theta+3\Delta$	−0.00092	+0.0083	−0.032	[−4.559]	[+33.80]				
	$(\theta-\theta_0)$ sin									
η	ϕ	−0.00671	[+0.0620]	−0.240	+1.015	[+ 8.43]				
η	$\phi+4\theta+4\Delta$	+0.00158	−0.0141	+0.056	+0.781	− 5.96	+20.02			
η'	$\phi \quad + \Delta$	+0.00668	[−0.0577]	+0.220	+3.249	[−30.12]				
η'	$\phi+4\theta+3\Delta$	−0.00068	+0.0066	−0.029	−0.580	+ 4.76				
	$(\theta-\theta_0)^2$ cos									
η	ϕ	−0.00118	+0.01023	−0.040	−0.3572	+ 2.624				
η'	$\phi \quad + \Delta$	+0.00064	−0.0061	+0.026	+0.2652	− 2.100				
		m'^2			m'^2			m'		

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TABLE XXIIa.

$[W_2]$. Unit=1″

	Cos	w^{-2}			w^0	
		w^0	w	w^2	w^0	w
	$\phi+2\theta+2\Delta$	− 0.614	+ 4.059	−10.3		
η	$2\theta+2\Delta$	− 4.255	+27.89		− 271.5	+ 636.6
η	$\phi+4\theta+4\Delta$	+ 2.791	−23.39		+ 167.4	− 637.4
η'	$2\theta+\Delta$	+ 5.444	−31.91			
η'	$\phi+4\theta+3\Delta$	− 4.558	+33.80			
η^2		+ 0.11				
η^2	$4\theta+4\Delta$	+14.90			+1514	−5780
η^2	$\phi+2\theta+2\Delta$				+1360	−3387
η^2	$\phi+6\theta+6\Delta$				−1227	+6415
η^2	$-\phi+2\theta+2\Delta$				− 273	+ 179
$\eta\,\eta'$	Δ	+ 0.13				
$\eta\,\eta'$	$4\theta+3\Delta$	−44.62			−2279	+7160
$\eta\,\eta'$	$\phi+2\theta+\Delta$				− 646	+2452
$\eta\,\eta'$	$\phi+2\theta+3\Delta$				− 291	+ 536
$\eta\,\eta'$	$\phi+6\theta+5\Delta$				+1974	−9002
$\eta\,\eta'$	$-\phi+2\theta+\Delta$				− 222	+1012
η'^2		− 0.06				
η'^2	$4\theta+2\Delta$	+30.53				
f^2	$4\theta+3\Delta-\Sigma$	+ 0.34				
	$(\theta-\theta_0)$ sin					
η	$2\theta+2\Delta$	− 1.64	+10.18			
η	ϕ	+ 1.014	+ 8.43			
η	$\phi+4\theta+4\Delta$	+ 0.782	− 5.96			
η'	$2\theta+\Delta$	+ 1.22	− 8.26			
η'	$\phi\quad+\Delta$	+ 3.249	−30.12			
η'	$\phi+4\theta+3\Delta$	− 0.579	+ 4.74			
η^2	$4\theta+4\Delta$	+ 7.81				
$\eta\,\eta'$	Δ	+ 3.25				
$\eta\,\eta'$	$4\theta+3\Delta$	−16.10				
η'^2	$4\theta+2\Delta$	+ 7.64				
	$(\theta-\theta_0)^2$ cos					
η	ϕ	− 0.356	+ 2.62			
η'	$\phi\quad+\Delta$	+ 0.266	− 2.10			
		m'^2			m'	

In the construction of Tables XXI and XXII it is necessary to compute

$$\int (T_2-[T_2])\,d\epsilon$$

as one factor of a product, but the more complete tabulation is best arranged as follows. This function gives all of the terms of the first order in the mass in $W_2-[W_2]$. Let

$$W_2''=\int (T_2-[T_2])\,d\epsilon$$

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and denote first order terms in $W_3-[W_3]$ and $W_4-[W_4]$ by W_3'' and W_4'', respectively. Then because of the similarity in the equations for these functions of successive ranks, the sum

$$W_2''+W_3''+W_4''$$

can be computed by Z 70, eqs. (117), (118), (119). The coefficients $\bar{F}$, $\bar{G}$, $\bar{H}$ are tabulated in Tables XXIII, XXIV, XXV. The mass factor m' is, of course, implicitly contained in the tables.

Eliminating the distinction between ϕ and ϵ, the function is

$$\overline{W}_2''+\overline{W}_3''+\overline{W}_4''$$

in which the coefficients $\overline{A}_{p,q}$, determined by Z 71, eq. (121), are tabulated in Table XXVI.

The coefficients $A_{p,q}$ in the function

$$(1-e\cos\epsilon)\,(\overline{W}_2''+\overline{W}_3''+\overline{W}_4'')$$

are computed by Z 71, eq. (123) and are tabulated in Table XXVII.

By means of Table XXVII we readily compute

$$[(1-e\cos\epsilon)\,(\overline{W}_2''+\overline{W}_3''+\overline{W}_4'')]$$

tabulated in Table XXVIII.

Proceeding now to the determination of

$$[(1-e\cos\epsilon)\,\overline{W}_3]$$

(from which we shall subtract $[(1-e\cos)\ \overline{W}_3'']$, already included in Table XXVIII), we have by Z 53, eq. (95)

$$\frac{\partial}{\partial\epsilon}(W_3-[W_3])=(T_3-[T_3])\,(1-e\cos\epsilon)(w+\overline{W}_1)\frac{\partial\overline{W}_3}{\partial\theta}-\frac{1}{2}\frac{d\overline{W}_1}{d\theta}\Big\{(1-e\cos\epsilon)(\overline{W}_3-w\overline{W}_2)$$
$$-\frac{3}{4}(1-e\cos\epsilon)(\overline{W}_1-\tfrac{1}{3}\Xi_1)\,(\overline{W}_1+\tfrac{1}{9}\Xi_1)-\Big[(1-e\cos\epsilon)(w+\overline{W}_1)\frac{\partial\overline{W}_3}{\partial\theta}\Big]$$
$$-[(1-e\cos\epsilon)(\overline{W}_3-w\overline{W}_2)]-\frac{3}{4}[(1-e\cos\epsilon)(\overline{W}_1-\tfrac{1}{3}\Xi_1)\,(\overline{W}_1+\tfrac{1}{9}\Xi_1)]\Big\}$$

in which all quantities are known. The integration gives $W_3-[W_3]$.

Having computed $W_3-[W_3]$, $[W_3]$ can be obtained from Z 53, eq. (95).

$$[(1-e\cos\epsilon)(w+\overline{W}_1)]\frac{\partial[\overline{W}_3]}{\partial\theta}+[(1-e\cos\epsilon)[\overline{W}_3]]\frac{d\overline{W}_1}{d\theta}=2[T_4]-\Big[(1-e\cos\epsilon)(\overline{W}_3-w\overline{W}_2)\frac{\partial\overline{W}_3}{\partial\theta}\Big]$$
$$+[1-e\cos\epsilon)w\overline{W}_2]\frac{d\overline{W}_1}{d\theta}-[(1-e\cos\epsilon)(w+\overline{W}_1)\frac{\partial}{\partial\theta}(\overline{W}_3-[\overline{W}_3])]-[(1-e\cos\epsilon)(\overline{W}_3-[\overline{W}_3])]\frac{d\overline{W}_1}{d\theta}$$

The function $[T_4]$, computed from Z 53, eq. (94), is tabulated in Table XXVIII*a*.

In a manner similar to the development of equations for W_1 and $[W_2]$, the right-hand side of this equation, when computed, can be segregated into portions independent of ϕ, terms multiplied by $\cos\phi$, and terms multiplied by $\sin\phi$. It is of the form

$$A+B\cos\phi+C\sin\phi$$

where A, B, C are too complicated to be written analytically, but can be written by inspection after the computation has been performed.

The equation can then be written in the three following equivalent equations:

$$\phi_1\frac{d[x_3]}{d\theta}+(\phi_3-w^3)\frac{dx_1}{d\theta}=A$$
$$\phi_1\frac{d[y_3]}{d\theta}+(\phi_3-w^3)\frac{dy_1}{d\theta}=B$$
$$\phi\frac{d[z_3}{d\theta}+(\phi_3-w^3)\frac{dz_1}{d\theta}=C$$

in which we define

$$\phi_3-w^3=[x_3]-\eta[y_3]\,.$$

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From the first two equations we compute

$$\phi_2 - w^2 = \phi_1^{-1} \int (A - \eta B) d\theta$$

Let

$$u_3 = [y_3] \cos \phi + [z_3] \sin \phi.$$

Then from the second and the third equations

$$u_3 = \int \phi_1^{-1} \left\{ B \cos \phi + C \sin \phi - (\phi_2 - w^2) \frac{du_3}{d\theta} \right\} d\theta$$

By inspection of u_3 the function $[y_3]$ can be written, and $\eta[y_3]$ added to $[x_3] - \eta[y_3]$ gives $[x_3]$. Finally,

$$[W_3] = [x_3] + [y_3] \cos \phi + [z_3] \sin \phi$$

and

$$[(1 - e \cos \epsilon) W_3]$$

is readily computed from W_3, which is tabulated in Table XXVIII*b*.

But this function contains $[(1 - e \cos \epsilon) W_3']$, already included in Table XXVIII. By *Z* 69

$$[(1 - e \cos \epsilon) W_3'] = -\frac{w}{2} \left[(1 - e \cos \epsilon) \int \left\{ (1 - e \cos \epsilon) \frac{\partial W_2}{\partial \theta} - [(1 - e \cos \epsilon) \frac{\partial W_2}{\partial \theta}] \right\} d\epsilon \right]$$

Subtracting Table XXVIII*c* from $[(1 - e \cos \epsilon) W_3]$ we have

$$[(1 - e \cos \epsilon) (W_3 - W_3')]$$

which is tabulated in Table XXIX.

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TABLE XXIII.[1] Unit=1″

	n	0	1	2	3	4	5	6	7	8	9	10
	$\tilde{F}_{0\cdot0}(n.-n)$		− 94.26	− 271.49	− 144.07	− 79.94	− 45.52	− 26.37	− 15.46	− 9.15	− 5.45	− 3.26
	$\tilde{F}_{1\cdot0}(n+1.-n)$	+ 86.3	+ 138.9	+ 3.1	− 43.0	− 53.6	− 49.1	− 39.8	− 30.2	− 22.0	− 15.7	− 10.9
	$\tilde{F}_{1\cdot0}(n-1.-n)$	+ 86.3	− 227.8		+ 2993.4	+ 1514.3	+ 910.6	+ 574.4	+ 368.9	+ 238.4	+ 154.6	+ 100.3
	$\tilde{F}_{0\cdot1}(n.-n+1)$	———	− 129.7	+ 87.4	+ 157.0	+ 155.6	+ 129.7	+ 99.7	+ 73 1	+ 52.0	+ 36.2	+ 24.9
	$\tilde{F}_{0\cdot1}(n.-n-1)$	———		− 4081.7	− 2279.4	− 1445.9	− 943.4	− 619.8	− 407.6	− 267.6	− 175.4	− 114.6
	$\tilde{F}_{2\cdot0}(n+2.-n)$	− 106	− 130	− 72	− 54	− 50	− 47	− 43	− 38	− 32	− 26	− 20
	$\tilde{F}_{2\cdot0}(n.-n)$		−1513	− 128	+ 588	+ 891	+ 944	+ 865	+ 733	+ 590	+ 458	+ 347
	$\tilde{F}_{2\cdot0}(n-2.-n)$	− 106	− 22	+ 2245	+ 9516		− 15534	− 7969	− 5033	− 3375	− 2312	−1595
	$\tilde{F}_{1\cdot1}(n+1.-n+1)$	+ 256	+ 410	+ 209	+ 153	+ 152	+ 158	+ 154	+ 140	+ 121	+ 100	+ 79
	$\tilde{F}_{1\cdot1}(n-1.-n+1)$	+ 768		− 418	− 1725	− 2167	− 2147	− 1904	− 1581	− 1255	− 966	− 725
	$\tilde{F}_{1\cdot1}(n+1.-n-1)$	+ 768	+1429	− 256	− 908	− 1171	− 1184	− 1064	− 893	− 714	− 553	− 417
	$\tilde{F}_{1\cdot1}(n-1.-n-1)$	+ 256	− 965	−20365		+ 43739	+ 23772	+ 15609	+10772	+ 7539	+ 5290	+3704
	$\tilde{F}_{0\cdot2}(n.-n+2)$	———	− 301	− 164	− 108	− 118	− 135	− 140	− 132	− 116	− 97	− 78
	$\tilde{F}_{0\cdot2}(n.-n)$		−1521	+ 1498	+ 2612	+ 2940	+ 2796	+ 2428	+ 1991	+ 1568	+ 1199	+ 897
	$\tilde{F}_{0\cdot2}(n.-n-2)$	———	+3165		− 30607	− 17790	− 12195	− 8670	− 6206	− 4428	− 3142	−2216
	$\tilde{F}_{0\cdot0}(n+1.-n+1)+\sigma$	− 158	− 576	− 513	− 406	− 302	− 217	− 152	− 105	− 71	− 48	− 32
	$\tilde{F}_{0\cdot0}(n-1.-n-1)-\sigma$	− 158	———	+ 855		− 1268	− 578	− 325	− 196	− 122	− 77	− 49
	$\tilde{F}_{0\cdot0}(n+1.-n-1)+\delta$	+ 475	+1152	+ 855	+ 609	+ 423	+ 289	+ 195	+ 131	+ 87	+ 57	+ 38
	$\tilde{F}_{0\cdot0}(n-1.-n+1)-\delta$	+ 475		+ 855	+ 609	+ 423	+ 289	+ 195	+ 131	+ 87	+ 57	+ 38
	$\tilde{F}_{0\cdot0}(n.-n)$		+ 392.9	+ 908.1	+ 580.8	+ 376.6	+ 245.2	+ 159.8	+ 104.1	+ 67.7	+ 44.0	+ 28.5
	$\tilde{F}_{1\cdot0}(n+1.-n)$	− 256	− 598	− 341	− 53	+ 106	+ 172	+ 184	+ 168	+ 141	+ 113	+ 87
	$\tilde{F}_{1\cdot0}(n-1.-n)$	− 256	+ 637		− 18877	− 8969	− 5669	− 3852	− 2677	− 1871	− 1307	− 911
	$\tilde{F}_{0\cdot1}(n.-n+1)$	———	+ 483	− 37	− 429	− 594	− 609	− 547	− 456	− 363	− 279	− 209
	$\tilde{F}_{0\cdot1}(n.-n-1)$	———		+22406	+ 11719	+ 7904	+ 5624	+ 4044	+ 2903	+ 2071	+ 1467	+1032
	$\tilde{F}_{2\cdot0}(n+2.-n)$	+ 449	+ 747	+ 455	+ 293	+ 231	+ 218	+ 218	+ 214	+ 201	+ 182	
	$\tilde{F}_{2\cdot0}(n.-n)$		[+7969]	(+ 1846)	+ 2549	− 1403	− 3325	− 4111	− 4200	− 3889	− 3388	
	$\tilde{F}_{2\cdot0}(n-2.-n)$	+ 449	+ 216	(− 4830)	[− 12320]		+153610	+ 67367	+42222	+29290	+21073	
	$\tilde{F}_{1\cdot1}(n+1.-n+1)$	− 855	−1898	− 1183	− 759	− 663	− 712	− 778	− 803	− 778	− 714	
	$\tilde{F}_{1\cdot1}(n-1.-n+1)$	−3589		− 2303	[+ 4936]	+ 9000	+ 10823	+ 11073	+10350	+ 9108	+ 7680	
	$\tilde{F}_{1\cdot1}(n+1.-n-1)$	−3589	(−7249)	− 6243	− 777	+ 2574	+ 4411	+ 5139	+ 5139	+ 4711	+ 4092	
	$\tilde{F}_{1\cdot1}(n-1.-n-1)$	− 855	(+2886)	+ 6428		−397850	−182750	−119275	−85560	−69223	−47093	
	$\tilde{F}_{0\cdot2}(n.-n+2)$	———	+1160	+ 855	+ 571	+ 558	+ 661	+ 759	+ 802	+ 786	+ 719	
	$\tilde{F}_{0\cdot2}(n.-n)$		+8188	− 1947	− 8452	− 12449	− 14128	− 14095	−12992	−11336	− 9504	
	$\tilde{F}_{0\cdot2}(n.-n-2)$	———	−2036		+252940	+122720	+ 83854	+ 62371	+47468	+36194	+27464	

Factor w

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	$\bar{F}_{0\cdot0}(n+1.-n+1)+\sigma$	+ 602	+2116	+ 2281	+ 2107	+ 1788	+ 1437	+ 1113	+ 840	+ 620	+ 451	
	$\bar{F}_{0\cdot0}(n-1.-n-1)-\sigma$	+ 602	———	− 1067		+ 13304	+ 5276	+ 2964	+ 1849	+ 1203	+ 809	
	$\bar{F}_{0\cdot0}(n+1.-n-1)+\delta$	−2440	−5383	− 4487	− 3566	− 2744	− 2060	− 1518	− 1102	− 790	− 560	
	$\bar{F}_{0\cdot0}(n-1.-n+1)-\delta$	−2440		− 4487	− 3566	− 2744	− 2060	− 1518	− 1102	− 790	− 560	
Factor w^2	$\bar{F}_{0\cdot0}(n.-n)$		− 765	− 1434	− 1080	− 811	− 602	− 442	− 320	− 230	− 163	− 115
	$\bar{F}_{1\cdot0}(n+1.-n)$	+ 342	+1207	+ 1261	+ 719	+ 239	− 88	− 276	− 359	− 373	− 349	
	$\bar{F}_{1\cdot0}(n-1.-n)$	+ 342	− 852		+ 70290	+ 27660	+ 17430	+ 12430	+ 9221	+ 6920	+ 5193	
	$\bar{F}_{0\cdot1}(n.-n+1)$	———	− 808	− 466	+ 260	+ 841	+ 1171	+ 1291	+ 1248	+ 1126	+ 966	
	$\bar{F}_{0\cdot1}(n.-n-1)$	———		−77090	− 32330	− 21630	− 16210	− 12530	− 9728	− 7506	− 5740	

TABLE XXIV.

Unit=1″

	$\bar{G}_{0\cdot0}(n.-n)$	− 43.141	− 18.35	+ 33.55	+ 23.56	+ 15.25	+ 9.58	+ 5.94	+ 3.67	+ 2.25	+ 1.39	+ 0.85
	$\bar{G}_{1\cdot0}(n+1.-n)$	+ 52.8	+ 22.0	+ 27.6	+ 24.2	+ 20.5	+ 16.5	+ 12.6	+ 9.3	+ 6.7	+ 4.8	+ 3.3
	$G_{1\cdot0}(n-1.-n)$		+ 448.2	− 273.0	− 279.4	− 225.9	− 168.1	− 119.9	− 83.5	− 57.1	− 38.6	− 25.9
	$\bar{G}_{0\cdot1}(n.-n+1)$	− 128.0	− 56.0	− 42.5	− 42.4	− 38.7	− 32.2	− 25.2	− 18.8	− 13.7	− 9.7	− 6.8
	$G_{0\cdot1}(n.-n-1)$	− 383.9	− 222.1	+ 287.1	+ 289.7	+ 238.6	+ 180.6	+ 130.7	+ 92.0	+ 63.5	+ 43.2	+ 29.2
	$\bar{G}_{2\cdot0}(n+2.-n)$	− 60	− 32	[− 1]	+ 10	+ 15	+ 16	+ 15	+ 13	+ 12	+ 9	+ 7
	$\bar{G}_{2\cdot0}(n.-n)$	− 568	− 343	− 267	− 293	− 308	− 294	− 261	− 219	− 176	− 137	−104
	$G_{2\cdot0}(n-2.-n)$	+ 390	+ 1567		+ 2738	+ 2114	+ 1682	+ 1312	+ 1002	+ 749	+ 551	+400
	$\bar{G}_{1\cdot1}(n+1.-n+1)$	+ 308	+ 157	+ 33	− 15	− 37	− 46	− 47	− 43	− 37	− 31	− 24
	$\bar{G}_{1\cdot1}(n-1.-n+1)$	+1924	+ 1004	+ 514	+ 501	+ 533	+ 524	+ 474	+ 404	+ 329	+ 258	+198
	$\bar{G}_{1\cdot1}(n+1.-n-1)$	+ 385	+ 261	+ 376	+ 394	+ 393	+ 364	+ 317	+ 263	+ 210	+ 163	+123
	$G_{1\cdot1}(n-1.-n-1)$	−2308		−3152	− 4253	− 4080	− 3519	− 2858	− 2233	− 1696	− 1261	−922
	$\bar{G}_{0\cdot2}(n.-n+2)$	− 408	− 198	− 51	− 6	+ 18	+ 30	+ 35	+ 34	+ 31	+ 26	+ 22
	$\bar{G}_{0\cdot2}(n.-n)$	− 866	− 604	− 568	− 635	− 677	− 655	− 586	− 495	− 400	− 313	−239
	$G_{0\cdot2}(n.-n-2)$		− 2600	+1815	+ 2385	+ 2337	+ 2042	+ 1673	+ 1314	+ 1002	+ 747	+547
	$\bar{G}_{0\cdot0}(n+1.-n+1)+\sigma$	− 102	− 33	+ 26	+ 43	+ 43	+ 37	+ 29	+ 21	+ 15	+ 11	+ 7
	$\bar{G}_{0\cdot0}(n-1.-n-1)-\sigma$	+1141		− 961	− 232	− 58	− 5	+ 11	+ 13	+ 12	+ 9	+ 7
	$\bar{G}_{0\cdot0}(n+1.-n-1)+\delta$	+ 169	+ 49	− 36	− 58	− 56	− 46	− 35	− 25	− 18	− 13	− 9
	$\bar{G}_{0\cdot0}(n-1.-n+1)-\delta$	+1141	+ 866	+ 320	+ 116	+ 35	+ 3	− 8	− 10	− 9	− 7	− 6

[1] The terms inclosed by ( ) contain quantities which are functions of $W_1+[W_2]$. See Z 69.

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TABLE XXIV—Continued. Unit=1″

	n	0	1	2	3	4	5	6	7	8	9	10
Factor w	$\tilde{G}_{0\cdot0}(n.-n)$	+ 127.95	+ 80.4	− 70.9	− 74.4	− 61.4	− 46.2	− 33.1	− 23.1	− 15.8	− 10.7	− 7.2
	$\tilde{G}_{1\cdot0}(n+1.-n)$	− 225	− 148	− 85	− 73	− 73	− 70	− 63	− 54	− 44	− 34	− 26
	$G_{1\cdot0}(n-1.-n)$		− 2478	+ 485.5	+ 947	+ 979	+ 862	+ 704	+ 549	+ 416	+ 308	+224
	$\tilde{G}_{0\cdot1}(n.-n+1)$	+ 427	+ 283	+ 155	+ 147	+ 153	+ 149	+ 135	+ 114	+ 93	+ 73	+ 56
	$G_{0\cdot1}(n.-n-1)$	+1794	+ 1234	− 620	− 995	− 1031	− 921	− 762	− 602	− 460	− 343	−251
	$\tilde{G}_{2\cdot0}(n+2.-n)$	+ 368	+ 270	+ 85	+ 5	− 37	− 59	− 69	− 71	− 68	− 61	
	$\tilde{G}_{2\cdot0}(n.-n)$	+3106	+ 2700	+1264	+ 1055	+ 1160	+ 1307	+ 1346	+ 1290	+ 1167	+ 1009	
	$G_{2\cdot0}(n-2.-n)$	−2002	− 7289		−13668	−11168	− 9806	− 8437	− 7065	− 5759	− 4590	
	$\tilde{G}_{1\cdot1}(n+1.-n+1)$	−1476	− 1062	− 360	− 65	+ 87	+ 176	+ 224	+ 241	+ 237	+ 218	
	$\tilde{G}_{1\cdot1}(n-1.-n+1)$	−7175	− 5771	[−2701]	− 2148	− 2341	− 2580	− 2650	− 2538	− 2296	− 1985	
	$\tilde{G}_{1\cdot1}(n+1.-n-1)$	−2392	− 2138	−1406	− 1339	− 1497	− 1624	− 1641	− 1553	− 1394	− 1201	
	$G_{1\cdot1}(n-1.-n-1)$	+8713		+5765	+17425	+20109	[+19763]	+17949	+15489	+12872	+10396	
	$\tilde{G}_{0\cdot2}(n.-n+2)$	+1566	+ 1105	+ 380	+ 107	− 33	− 121	− 174	− 200	− 205	− 194	
	$\tilde{G}_{0\cdot2}(n.-n)$	+4261	+ 3793	+2481	+ 2497	+ 2905	+ 3209	+ 3269	+ 3106	+ 2792	+ 2403	
	$G_{0\cdot2}(n.-n-2)$		+20946	−4103	− 9279	−11106	−11195	−10319	− 8994	− 7525	− 6108	
	$\tilde{G}_{0\cdot0}(n+1.-n+1)+\sigma$	+ 563	+ 480	+ 106	− 90	− 173	− 191	− 178	− 151	− 121	− 94	
	$\tilde{G}_{0\cdot0}(n-1.-n-1)-\sigma$	−4396		+8493	+ 2121	+ 654	+ 157	− 24	− 83	− 91	− 82	
	$\tilde{G}_{0\cdot0}(n+1.-n-1)+\delta$	−1029	− 744	− 132	+ 144	+ 243	+ 255	+ 228	+ 189	+ 148	+ 113	
	$G_{0\cdot0}(n-1.-n+1)-\delta$	−4396	− 4260	−1977	− 886	− 355	− 102	+ 12	+ 56	+ 66	+ 62	
Factor w^2	$\tilde{G}_{0\cdot0}(n.-n)$	− 171	− 165	+ 20	+ 82	+ 99	+ 94	+ 80	+ 64	+ 50	+ 37	+ 28
	$\tilde{G}_{1\cdot0}(n+1.-n)$	+ 480	+ 456	+ 193	+ 112	+ 105	+ 118	+ 128	+ 129	+ 121	+ 108	
	$G_{1\cdot0}(n-1.-n)$		+ 6990	+ 613	− 1108	− 1766	− 1928	− 1837	− 1627	− 1375	− 1120	
	$\tilde{G}_{0\cdot1}(n.-n+1)$	− 726	− 696	− 340	− 248	− 262	− 293	− 309	− 302	− 278	− 243	
	$G_{0\cdot1}(n.-n-1)$	−4399	− 3461	[− 64]	+ 1255	+ 1866	+ 2047	+ 1975	+ 1771	+ 1510	+ 1242	

TABLE XXV.[1] Unit=1″

	$\tilde{H}_{0\cdot0}(n.-n)$	−43.141	−180.72		+ 458.50	+ 167.42	+ 78.80	+ 40.85	+ 22.26	+ 12.5	+ 7.2	+ 4.2
	$\tilde{H}_{1\cdot0}(n+1.-n)$		− 628.9	− 135.8	+ 13.0	+ 50.9	+ 53.0	+ 44.4	+ 34.0	+ 24.8	+ 17.5	+ 12.2
	$H_{1\cdot0}(n-1.-n)$	+ 52.8	+ 290.8	+ 1496.3	+ 3458.7		− 2959.7	− 1226.6	− 649.7	− 375.2	− 225.7	− 138.8
	$\tilde{H}_{0\cdot1}(n.-n+1)$	− 383.9		− 137.3	− 291.3	− 255.6	− 195.1	− 140.8	− 98.6	− 67.7	− 45.9	− 30.8
	$H_{0\cdot1}(n.-n-1)$	− 128.7	− 646.4	− 4581.0		+ 4590.8	+ 1973.8	+ 1072.4	+ 631.3	+ 385.3	+ 239.7	+ 150.7

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$H^{2\cdot0}_{2\cdot0}(n+2.-n)$	+ 390	+ 340	+ 119	+ 60	+ 45	+ 42	+ 39	+ 36	+ 31	+ 25	+ 20							
$H^{2\cdot0}_{2\cdot0}(n.-n)$	− 568	− 3166		− 354	− 1220	− 1282	− 1133	− 928	− 727	− 552	− 410							
$H^{2\cdot0}_{2\cdot0}(n-2.-n)$	− 60	− 412	− 1798	− 4085	− 7960	− 17640		− 16423	+ 7252	+ 4103	+2530							
$H^{1\cdot1}_{1\cdot1}\langle(n+1.-n+1)$	−2308	[− 1488]	− 493	− 233	− 182	− 176	− 169	− 154	− 132	− 109	− 86							
$H^{1\cdot1}_{1\cdot1}\langle(n-1.-n+1)$	+ 385	+ 1574	+ 892		− 6511	+ 4414	+ 3290	+ 2452	+ 1815	+ 1327	+ 960							
$H^{1\cdot1}_{1\cdot1}\langle(n+1.-n-1)$	+1924		− 1429	+ 853	+ 1420	+ 1458	+ 1294	+ 1067	+ 840	+ 641	+ 478							
$H^{1\cdot1}_{1\cdot1}\langle(n-1.-n-1)$	+ 308	+ 1892	+ 9019	+20344	+49129		− 50291	− 22868	− 13225	− 8299	−5396							
$H^{0\cdot2}_{0\cdot2}(n.-n+2)$		+ 2191	+ 591	+ 252	+ 201	+ 198	+ 190	+ 172	+ 147	+ 120	+ 94							
$H^{0\cdot2}_{0\cdot2}(n.-n)$	− 866	− 3840		− 7274	− 5807	− 4685	− 3684	− 2826	− 2121	− 1564	−1137							
$H^{0\cdot2}_{0\cdot2}(n.-n-2)$	− 408	− 2273	−12428	−33665		+ 38571	+ 18102	+ 10724	+ 6853	+ 4522	+3027							
$H^{0\cdot0}_{0\cdot0}\langle(n+1.-n+1)+\sigma$	+1141	+ 1634	+ 1060	+ 725	+ 486	+ 324	+ 215	+ 142	+ 94	+ 62	+ 40							
$H^{0\cdot0}_{0\cdot0}\langle(n-1.-n-1)-\sigma$	− 102	− 433	− 700	− 1044	− 1865		+ 1247	+ 482	+ 243	+ 135	+ 78							
$H^{0\cdot0}_{0\cdot0}\langle(n+1.-n-1)+\delta$	+1141		− 3241	− 1450	− 810	− 486	− 301	− 190	− 121	− 77	− 49							
$H^{0\cdot0}_{0\cdot0}\langle(n-1.-n+1)-\delta$	+ 169	+ 866	+ 2101		− 1865	− 780	− 416	− 241	− 146	− 90	− 56							
$H_{0\cdot0}(n.-n)$	+ 127. 95	+ 440. 2		− 2821. 1	− 972. 2	− 483. 2	− 270. 7	− 160. 1	− 97. 4	− 60. 2	− 37. 6							
$H^{1\cdot0}_{1\cdot0}\langle(n+1.-n)$		+ 3080	(+ 719)	+ 665	+ 60	− 135	− 188	− 184	− 158	− 127	− 98							
$H^{1\cdot0}_{1\cdot0}\langle(n-1.-n)$	− 225	− 983	(− 3063. 4)	− 2651		+ 28777	+ 10217	+ 5383	+ 3225	+ 2041	+1327							
$H^{0\cdot1}_{0\cdot1}\langle(n.-n+1)$	+1794		− 562	+ 828	+ 1048	+ 973	+ 812	+ 641	+ 488	+ 363	+ 265							
$H^{0\cdot1}_{0\cdot1}\langle(n.-n-1)$	+ 427	+ 1806. 0	− 1194		+ 40971	− 14923	− 8086	− 4961	− 3203	− 2119	−1417							
$H^{2\cdot0}_{2\cdot0}(n+2.-n)$	−2002	− 2321	(− 910)	− 498	− 283	− 218	− 205	− 201	− 193	− 176								
$H^{2\cdot0}_{2\cdot0}\langle n.-n)$	+3106	+13600		+ 3845	(+ 7260. 3)	+ 1105	+ 4632	+ 5150	+ 4786	+ 4120								
$H^{2\cdot0}_{2\cdot0}\langle n-2.-n)$	+ 368	+ 1880	(+ 4781)	+ 7680	(+ 20565)	+ 10663		[−217895]	− 78562	−42808								
$H^{1\cdot1}_{1\cdot1}\langle(n+1.-n+1)$	+8713	+ 7870	+ 3343	+ 1476	+ 910	+ 826	+ 862	+ 884	+ 855	+ 784								
$H^{1\cdot1}_{1\cdot1}\langle(n-1.-n+1)$	−2392	− 8192	−10728		+ 38605	− 26417	− 21252	− 17312	− 13937	−11034								
$H^{1\cdot1}_{1\cdot1}\langle(n+1.-n-1)$	−7175		+ 7645	(− 3464)	[+ 3068]	− 3855	− 5792	− 6040	− 5553	− 4779								
$H^{1\cdot1}_{1\cdot1}\langle(n-1.-n-1)$	−1476	− 7195	−15273	(−37263)	+ 20179		+627529	+230294	+128129	+81500								
$H^{0\cdot2}_{0\cdot2}(n.-n+2)$		− 8572	− 3420	− 1553	− 1081	− 1045	− 1084	− 1082	− 1020	− 915								
$H^{0\cdot2}_{0\cdot2}\langle n.-n)$	+4261	+15822		+34602	+ 29611	+ 26593	+ 23202	+ 19594	+ 16074	+12871								
$H^{0\cdot2}_{0\cdot2}(n.-n-2)$	+1566	++ 7232	+12466	−50979		−449040	−168230	− 95788	− 62214	−42638								
$H^{0\cdot0}_{0\cdot0}\langle(n+1.-n+1)+\sigma$	−4396	− 7081	− 5458	− 4148	− 3102	− 2284	− 1659	− 1191	− 847	− 596								
$H^{0\cdot0}_{0\cdot0}\langle(n-1.-n-1)-\sigma$	+ 563	+ 1914	+ 2784	+ 3512	+ 1632		+ 17501	+ 5620	+ 2737	+ 1537								
$H^{0\cdot0}_{0\cdot0}\langle(n+1.-n-1)+\delta$	−4396		+25016	+10471	+ 6033	+ 3831	+ 2529	+ 1699	+ 1150	+ 780								
$H^{0\cdot0}_{0\cdot0}\langle(n-1.-n+1)-\delta$	−1029	− 4261	− 8351		+ 16563	+ 6582	+ 3617	+ 2207	+ 1409	+ 920								
$H_{0\cdot0}(n.-n)$	− 171	− 470		+10273	+ 2929	+ 1457	+ 860	+ 544	+ 356	+ 237	+ 159							
$H^{1\cdot0}_{1\cdot0}\langle(n+1.-n)$		− 7922	(− 2262)	− 4544	− 1407	− 331	+ 151	+ 342	+ 400	+ 387								
$H^{1\cdot0}_{1\cdot0}\langle(n-1.-n)$	+ 480	+ 1669	(+ 3250)	+17358		[−172780]	− 45846	− 22667	− 13652	− 8939								
$H^{0\cdot1}_{0\cdot1}\langle(n.-n+1)$	−4400		+ 4017	− 350	− 1668	− 2067	− 2059	− 1862	− 1591	− 1307								
$H^{0\cdot1}_{0\cdot1}\langle(n.-n-1)$	− 726	− 2491	−18776		+234480	+ 62130	+ 31904	+ 19302	+ 12918	+ 8978								

Factor w

Factor w^2

[1] The terms enclosed by ( ) contain quantities which are functions of $W_1+[W_2]$. See Z 60.

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TABLE XXVI.[1] Unit=1″

	n	0	1	2	3	4	5	6	7	8	9	10
	$\bar{A}_{0\cdot0}(n.-n)$	− 86.282	− 293.33	− 237.94	+ 337.99	+ 102.73	+ 42.86	+ 20.42	+ 10.47	+ 5.50	+ 3.14	+ 1.79
	$\bar{A}_{1\cdot0}(n+1.-n)$	+ 139.1	− 468.0	− 105.1	− 5.8	+ 17.8	+ 20.4	+ 17.2	+ 13.1	+ 9.5	+ 6.6	+ 4.6
	$A_{1\cdot0}(n-1.-n)$	+ 139.1	+ 511.1	+ 1223.30	+ 6172.8	+ 1288.5	− 2217.1	− 772.0	− 364.3	− 193.9	− 109.7	− 64.4
	$\bar{A}_{0\cdot1}(n.-n+1)$	− 511.8	− 185.7	− 92.3	− 176.8	− 138.6	− 97.5	− 66.3	− 44.3	− 29.4	− 19.4	− 12.7
	$A_{0\cdot1}(n.-n-1)$	− 511.8	− 868.54	− 8375.6	− 1989.6	+ 3383.4	+ 1211.0	+ 583.3	+ 315.7	+ 181.2	+ 107.5	+ 65.3
	$\bar{A}_{2\cdot0}(n+2.-n)$	+ 225	+ 178	[+ 46]	+ 16	+ 10	+ 10	+ 11	+ 11	+ 11	+ 8	+ 7
	$\bar{A}_{2\cdot0}(n.-n)$	− 1136.6	− 5022	− 396	− 59	− 637	− 633	− 529	− 414	− 313	− 231	− 167
	$A_{2\cdot0}(n-2.-n)$	+ 225	+ 1133	+ 447	+ 8169	− 5845.9	− 31492	− 6657	+ 12392	+ 4626	+ 2342	+1335
	$\bar{A}_{1\cdot1}(n+1.-n+1)$	− 1744	[− 921]	− 252	− 95	− 66	− 64	− 62	− 56	− 48	− 40	− 31
	$\bar{A}_{1\cdot1}(n-1.-n+1)$	+ 3077	+ 2578.2	+ 988	− 1224	+ 4878	+ 2791	+ 1851	+ 1275	+ 889	+ 619	+ 433
	$\bar{A}_{1\cdot1}(n+1.-n-1)$	+ 3077	+ 1690	− 1309	+ 339	+ 642	+ 639	+ 547	+ 437	+ 336	+ 251	+ 184
	$A_{1\cdot1}(n-1.-n-1)$	− 1744	+ 928	−14498	+ 16091.9	+ 88785	+ 20253	− 37540	− 14329	− 7382	− 4270	−2614
	$\bar{A}_{0\cdot2}(n.-n+2)$	− 408	+ 1693	+ 376	+ 139	+ 100	+ 93	+ 85	+ 74	+ 62	+ 49	+ 38
	$\bar{A}_{0\cdot2}(n.-n)$	− 1731.8	− 5965	+ 930	− 5297	− 3545	− 2544	− 1842	− 1330	− 953	− 678	− 479
	$A_{0\cdot2}(n.-n-2)$	− 408	− 1709	−10613.2	− 61877	− 15453	+ 28418	+ 11105	+ 5832	+ 3427	+ 2126	+1358
	$\bar{A}_{0\cdot0}(n+1.-n+1)+\sigma$	+ 881	+ 1025	+ 593	+ 363	+ 227	+ 144	+ 92	+ 58	+ 38	+ 25	+ 15
	$\bar{A}_{0\cdot0}(n-1.-n-1)-\sigma$	+ 881	− 433	− 806	− 1276.2	− 3191	− 582	+ 932	+ 299	+ 133	+ 67	+ 36
	$\bar{A}_{0\cdot0}(n+1.-n-1)+\delta$	+ 1785	+ 1201	− 2422	− 899	− 442	− 243	− 141	− 84	− 52	− 33	− 20
	$\bar{A}_{0\cdot0}(n-1.-n+1)-\delta$	+ 1785	+ 1731.8	+ 3276	+ 725	− 1408	− 488	− 228	− 120	− 68	− 40	− 24
Factor w	$\bar{A}_{0\cdot0}(n.-n)$	+ 255.90	+ 913.5	+ 837.2	− 2314.7	− 657.0	− 284.2	− 144.0	− 79.1	− 45.5	− 26.9	− 16.3
	$\bar{A}_{1\cdot0}(n+1.-n)$	− 481	+ 2334	(+ 293)	+ 539	+ 93	− 33	− 67	− 70	− 61	− 48	− 37
	$A_{1\cdot0}(n-1.-n)$	− 481	− 2834	(− 2577.9)	− 20581	− 7990	+ 23970	+ 7069	+ 3255	+ 1770	+ 1042	+ 640
	$\bar{A}_{0\cdot1}(n.-n+1)$	+ 2222	+ 766	− 444	+ 546	+ 607	+ 513	+ 400	+ 299	+ 218	+ 157	+ 112
	$A_{0\cdot1}(n.-n-1)$	+ 2222	+ 3040.0	+20592	+ 10724	− 34098	− 10220	− 4804	− 2660	− 1592	− 995	− 636
	$\bar{A}_{2\cdot0}(n+2.-n)$	− 1185	− 1304	(− 370)	− 200	− 89	− 59	− 56	− 58	− 59	− 55	
	$\bar{A}_{2\cdot0}(n.-n)$	+ 6212	[+24269]	(+ 3109)	+ 7449	(+ 7038)	− 913	+ 1867	+ 2240	+ 2064	+ 1741	
	$A_{2\cdot0}(n-2.-n)$	− 1185	− 5173	(− 49)	[− 18308]	(+ 9397)	+154467	+ 58930	[−182738]	−55031	−26325	
	$\bar{A}_{1\cdot1}(n+1.-n+1)$	+ 6382	+ 4910	+ 1800	+ 652	+ 334	+ 290	+ 308	+ 322	+ 314	+ 288	
	$\bar{A}_{1\cdot1}(n-1.-n+1)$	−13156	−13963	−15732	[+ 2788]	− 31946	− 18174	− 12829	− 9506	− 7125	− 5339	
	$\bar{A}_{1\cdot1}(n+1.-n-1)$	−13156	(− 9387)	− 4	(− 5580)	[+ 4145]	− 1068	− 2294	− 2454	− 2236	− 1888	
	$A_{1\cdot1}(n-1.-n-1)$	+ 6382	(− 4309)	− 3080	(− 19838)	−357562	−162987	+526203	+160223	+77778	+44903	
	$\bar{A}_{0\cdot2}(n.-n+2)$	+ 1566	− 6307	− 2185	− 875	− 556	− 505	− 499	− 480	− 439	− 390	
	$\bar{A}_{0\cdot2}(n.-n)$	+ 8522	+27803	+ 534	+ 28647	+ 20067	+ 15674	+ 12376	+ 9708	+ 7530	+ 5770	
	$A_{0\cdot2}(n.-n-2)$	+ 1566	+26142	+ 8363	+192682	+111614	−377281	−116178	− 57314	−33545	−21282	

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	$\bar{A}_{2\cdot0}(n+1.-n+1)+\sigma$	− 3231	− 4485	− 3071	− 2131	− 1487	− 1038	− 724	− 502	− 348	− 239	
	$\bar{A}_{2\cdot0}(n-1.-n-1)-\sigma$	− 3231	+ 1914	+10207	+ 5633	+ 15590	+ 5433	− 14561	− 3854	− 1625	− 810	
	$\bar{A}_{2\cdot0}(n+1.-n-1)+\delta$	− 7865	− 6127	+20397	+ 7050	+ 3532	+ 2026	+ 1239	+ 786	+ 506	+ 333	
	$\bar{A}_{0\cdot0}(n-1.-n+1)-\delta$	− 7865	− 8522	−14815	− 4453	+ 13454	+ 4420	+ 2111	+ 1161	+ 685	+ 422	
Factor w^2	$\bar{A}_{0\cdot0}(n.-n)$	− 342	− 1400	− 1414	+ 9275	+ 2217	+ 949	+ 498	+ 288	+ 176	+ 111	+ 72
	$\bar{A}_{1\cdot0}(n+1.-n)$	+ 822	− 6260	(− 808)	− 3713	− 1063	− 301	+ 3	+ 112	+ 148	+ 146	
	$\bar{A}_{1\cdot0}(n-1.-n)$	+ 822	+ 7747	(+ 3863)	+ 86540	+ 25894	[−157278]	− 35253	− 15073	− 8107	− 4866	
	$\bar{A}_{0\cdot1}(n.-n+1)$	− 5126	− 1504	+ 3211	− 338	− 1089	− 1189	− 1077	− 916	− 743	− 584	
	$\bar{A}_{0\cdot1}(n.-n-1)$	− 5126	− 5952	−95930	− 31075	+214716	+ 47967	+ 20749	+ 11345	+ 6922	+ 4480	

[1] The terms enclosed by ( ) contain quantities which are functions of $W_2'+[W_3]$. See Z 69.

TABLE XXVII.[1]

Unit=1″

$A_{0\cdot0}(n.-n)$	− 86.282	− 293.33	− 237.94	+ 337.99	+ 102.73	+ 42.86	+ 20.42	+ 10.47	+ 5.50	+ 3.14	+ 1.79
$A_{1\cdot0}(n+1.-n)$	+ 225.4	− 174.7	+ 132.9	− 343.8	− 84.9	− 22.4	− 3.2	+ 2.6	+ 4.0	+ 3.5	+ 2.8
$A_{1\cdot0}(n-1.-n)$	+ 225.4	+ 804.4	+ 1461.24	+ 5834.8	+ 1185.8	− 2260.0	− 792.4	− 374.8	− 199.4	− 112.8	− 66.2
$A_{0\cdot1}(n.-n+1)$	− 511.8	− 185.7	− 92.3	− 176.8	− 138.6	− 97.5	− 66.3	− 44.3	− 29.4	− 19.4	− 12.7
$A_{0\cdot1}(n.-n-1)$	− 511.8	− 868.54	− 8375.6	− 1989.6	+ 3383.4	+ 1211.0	+ 583.3	+ 315.7	+ 181.2	+ 107.5	+ 65.3
$A_{2\cdot0}(n+2.-n)$	+ 86	+ 646	+ 151	+ 22	− 8	− 10	− 6	− 2	+ 1	+ 1	+ 2
$A_{2\cdot0}(n.-n)$	− 1415	− 5065	− 1514	− 6226	− 1943	+ 1564	+ 226	− 63	− 129	− 128	− 107
$A_{2\cdot0}(n-2.-n)$	+ 86	+ 622	− 776	+ 1996	− 7134.4	− 29275	− 5885	+ 12756	+ 4820	+ 2452	+1399
$A_{1\cdot1}(n+1.-n+1)$	− 1232	[− 735]	− 160	+ 82	+ 73	+ 34	+ 4	− 12	− 19	− 21	− 18
$A_{1\cdot1}(n-1.-n+1)$	+ 3589	+ 2764.2	+ 1080	− 1047	+ 5017	+ 2889	+ 1917	+ 1319	+ 918	+ 638	+ 446
$A_{1\cdot1}(n+1.-n-1)$	+ 3589	+ 2559	+ 7067	+ 2329	− 2741	− 572	− 36	+ 121	+ 155	+ 144	+ 119
$A_{1\cdot1}(n-1.-n-1)$	− 1232	+ 1797	− 6122	+ 18062	+ 85402	+ 19042	− 38123	− 14645	− 7563	− 4377	−2679
$A_{0\cdot2}(n.-n+2)$	− 406	+ 1693	+ 376	+ 139	+ 100	+ 93	+ 85	+ 74	+ 62	+ 49	+ 38
$A_{0\cdot2}(n.-n)$	− 1731.8	− 5965	+ 930	− 5297	− 3545	− 2544	− 1842	− 1330	− 953	− 678	− 479
$A_{0\cdot2}(n.-n-2)$	− 406	− 1709	−10613.2	− 61877	− 15453	+ 28418	+ 11105	+ 5832	+ 3427	+ 2126	+1358
$A_{0\cdot0}(n+1.-n+1)+\sigma$	+ 881	+ 1025	+ 593	+ 363	+ 227	+ 144	+ 92	+ 58	+ 38	+ 25	+ 15
$A_{0\cdot0}(n-1.-n-1)-\sigma$	+ 881	− 433	− 806	− 1276.2	− 3191	− 582	+ 932	+ 299	+ 133	+ 67	+ 36
$A_{0\cdot0}(n+1.-n-1)+\delta$	+ 1785	+ 1201	− 2422	− 899	− 442	− 243	− 141	− 84	− 52	− 33	− 20
$A_{0\cdot0}(n-1.-n+1)-\delta$	+ 1785	+ 1731.8	+ 3276	+ 725	− 1408	− 488	− 228	− 120	− 68	− 40	− 24

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TABLE XXVII[1]—Continued. Unit=1″

	n	0	1	2	3	4	5	6	7	8	9	10
Factor w	$A_{0\cdot0}(n.-n)$	+ 255.90	+ 913.5	+ 837.2	− 2314.7	− 657.0	− 284.2	− 144.0	− 79.1	− 45.5	− 26.9	− 16.3
	$A_{1\cdot0}(n+1.-n)$	− 737	+ 1421	(− 544)	+ 2854	+ 750	+ 251	+ 77	+ 9	− 15	− 21	− 21
	$A_{1\cdot0}(n-1.-n)$	− 737	− 3748	(− 3415.1)	− 18266	− 7333	+ 24254	+ 7213	+ 3334	+ 1815	+ 1069	+ 656
	$A_{0\cdot1}(n.-n+1)$	+ 2222	+ 766	− 444	+ 546	+ 607	+ 513	+ 400	+ 299	+ 212	+ 157	+ 112
	$A_{0\cdot1}(n.-n-1)$	+ 2222	+ 3040.0	+20592	+ 10724	− 34098	− 10220	− 4804	− 2660	− 1592	− 995	− 636
	$A_{2\cdot0}(n+2.-n)$	− 704	− 3638	(− 663)	− 739	− 182	− 26	+ 11	+ 12	+ 2	− 7	
	$A_{2\cdot0}(n.-n)$	+ 7174	[+24769]	(+ 5394)	+ 27491	(+ 14935)	− 24850	− 5135	− 945	+ 355	+ 747	
	$A_{2\cdot0}(n-2.-n)$	− 704	− 2389	(+ 2529)	[+ 2273]	[(+ 17387)]	+130497	+ 51861	[−185993]	−56801	−27367	
	$A_{1\cdot1}(n+1.-n+1)$	+ 4160	+ 4144	+ 2244	+ 106	− 273	− 223	− 92	+ 23	+ 102	+ 131	
	$A_{1\cdot1}(n-1.-n+1)$	−15378	−14729	−15288	[+ 2242]	− 32553	− 18687	− 13229	− 9805	− 7337	− 5496	
	$A_{1\cdot1}(n+1.-n-1)$	−15378	(−12427)	−20596	(− 16304)	[+ 38243]	+ 9152	+ 2510	+ 206	− 644	− 893	
	$A_{1\cdot1}(n-1.-n-1)$	+ 4160	(− 7349)	−23672	(− 30562)	−323464	−152767	+531007	+162883	+79370	+45798	
	$A_{0\cdot2}(n.-n+2)$	+ 1566	− 6307	− 2185	− 875	− 556	− 505	− 499	− 480	− 439	− 390	
	$A_{0\cdot2}(n.-n)$	+ 8522	+27803	+ 534	+ 28647	+ 20067	+ 15674	+ 12376	+ 9708	+ 7530	+ 5770	
	$A_{0\cdot2}(n.-n-2)$	+ 1566	+26142	+ 8363	+192682	+111614	−377281	−116178	− 57314	−33545	−21282	
	$A_{0\cdot0}(n+1.-n+1)+\sigma$	− 3231	− 4485	− 3071	− 2131	− 1487	− 1038	− 724	− 502	− 348	− 239	
	$A_{0\cdot0}(n-1.-n-1)-\sigma$	− 3231	+ 1914	+10207	+ 5633	+ 15590	+ 5433	− 14561	− 3854	− 1625	− 810	
	$A_{0\cdot0}(n+1.-n-1)+\delta$	− 7865	− 6127	+20397	+ 7050	+ 3532	+ 2026	+ 1239	+ 786	+ 508	+ 333	
	$A_{0\cdot0}(n-1.-n+1)-\delta$	− 7865	− 8522	−14815	− 4453	+ 13454	+ 4420	+ 2111	+ 1161	+ 685	+ 422	
Factor w^2	$A_{0\cdot0}(n.-n)$	− 342	− 1400	− 1414	+ 9275	+ 2217	+ 949	+ 498	+ 288	+ 176	+ 111	+ 72
	$A_{1\cdot0}(n+1.-n)$	+ 1164	− 4860	(+ 606)	− 12988	− 3280	− 1250	− 495	− 176	− 28	+ 35	
	$A_{1\cdot0}(n-1.-n)$	+ 1164	+ 9147	(+ 5277)	+ 77265	+ 23677	[−158227]	− 35751	− 15361	− 8283	− 4977	
	$A_{0\cdot1}(n.-n+1)$	− 5126	− 1504	+ 3211	− 338	− 1089	− 1189	− 1077	− 916	− 743	− 584	
	$A_{0\cdot1}(n.-n-1)$	− 5126	− 5952	−95930	− 31075	+214716	+ 47967	+ 20749	+ 11345	+ 6922	+ 4480	

[1] The terms enclosed by ( ) contain quantities which are functions of $W_1'+[W_1]$. See Z 69.

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TABLE XXVIII.

$[(1-e\cos\epsilon)(\overline{W}_2''+\overline{W}_3''+\overline{W}_4'')]$ Unit—4th decimal of a radian.

	Cos	w^0	w	w^1
η^2		− 4.1829	+ 12.406	− 16.58
η'^2		− 68.61	+ 347.9	
f^2		− 83.96	+ 413.1	
$\eta\,\eta'$		+ 83.96	− 413.1	
	Δ	+134.0	− 714.2	
η	$2\theta+2\Delta$	+ 70.842	[− 165.57]	[+255.8]
η'	$2\theta+\Delta$	− 42.107	+ 147.38	−288.6
η^2	$4\theta+4\Delta$	−345.88	[+ 843.0]	
$\eta\,\eta'$	$4\theta+3\Delta$	+876.64	[−1481.7]	
η'^2	$4\theta+2\Delta$	−514.54	+ 405.5	
f^2	$4\theta+3\Delta-\Sigma$	− 61.87	+ 273.1	
		m'		

TABLE XXVIIIa.

$2[T_4]$ Unit—4th decimal of a radian.

	Sin	w^{-2}		w^0	
		w^0	w	w^0	w
	$\phi+2\theta+2\Delta$	−0.00005	+0.00073	−0.0682	+0.4056
η	$2\theta+2\Delta$			−0.8324	+2.1665
η	ϕ			+0.3381	−2.5547
η	$\phi+4\theta+4\Delta$			+1.0220	−7.370
η'	$2\theta+\Delta$			+0.2654	−1.846
η'	$\phi+\Delta$			−0.3622	+2.472
η'	$\phi+4\theta+3\Delta$			−1.2106	+8.472
		m'^2		m'^2	

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TABLE XXVIIIb.

W_3 Unit—4th decimal of a radian.

	Cos	w^{-2}		w^{-1}			w	
		w^0	w	w^0	w	w^2	w^0	w
	$-\varepsilon+\phi$	−0.00004	+0.00038	−0.0032	−0.0005	−0.4803		
	$\varepsilon +2\theta+2\Delta$			+0.0237	−0.15101		+ 13.16	− 30.86
	$2\varepsilon-\phi+2\theta+2\Delta$	0.00000	+0.00004	+0.0050	−0.0318		− 0.81	+ 1.31
	$\phi+2\theta+2\Delta$	+0.00001	−0.00023	+0.0726	−0.4507			
	$2\varepsilon +4\theta+4\Delta$			+0.0153	−0.0770		+ 3.88	− 14.38
	$\varepsilon+\phi+4\theta+4\Delta$	+0.00004	−0.00038	+0.0181	−0.0814		− 16.23	+ 61.80
	$2\varepsilon+\phi+6\theta+6\Delta$			−0.0088	+0.0576		− 3.0	+ 15.7
η	$2\theta+2\Delta$			+0.5242	−3.3539		+ 13.16	− 30.86
η	$\varepsilon-\phi+2\theta+2\Delta$			+0.1384	−0.7747		+ 14.86	− 11.30
η	$-\varepsilon+\phi+2\theta+2\Delta$			−0.0508	+0.4660		+ 58.25	− 170.9
η	ε			+0.0749	−0.2385			
η	$\varepsilon +4\theta+4\Delta$			+0.1723	−0.8380		−154.6	+ 589.2
η	$\phi+4\theta+4\Delta$			−0.5378	+4.082		− 16.23	+ 61.80
η	$\varepsilon+\phi+2\theta+2\Delta$			−0.1801	+1.1267		− 7.71	− 6.66
η	$\varepsilon+\phi+6\theta+6\Delta$			−0.3275	+2.032		+178.4	− 983.0
η'	$2\theta+\Delta$			−0.6099	+3.634			
η'	$\varepsilon-\phi+2\theta+\Delta$			−0.1412	+0.6843		+ 10.77	− 49.04
η'	$-\varepsilon+\phi+2\theta+\Delta$			+0.1220	−0.8554		− 31.34	+ 118.9
η'	$\varepsilon +\Delta$			+0.0524	−0.4182			
η'	$\varepsilon +4\theta+3\Delta$			−0.0411	+0.2314		+221.0	− 694.3
η'	$\phi+4\theta+3\Delta$			+0.7660	−5.430			
η'	$\varepsilon+\phi+2\theta+3\Delta$			+0.0460	−0.3060		+ 14.12	− 26.01
η'	$\varepsilon+\phi+6\theta+5\Delta$			+0.3718	−2.0745		−287.1	+1309.3
	$(\theta-\theta_0)$ sin							
η	$2\theta+2\Delta$			+0.0490	−0.2949			
η	$\varepsilon-\phi+2\theta+2\Delta$			+0.0144	−0.0705			
η	$-\varepsilon+\phi+2\theta+2\Delta$			−0.0542	+0.3582			
η	ϕ			+0.5810	−4.7017			
η	$\varepsilon +4\theta+4\Delta$			−0.0618	+0.4650			
η	$\phi+4\theta+4\Delta$			−0.0453	+0.3389			
η	$\varepsilon+\phi+2\theta+2\Delta$			−0.0010	+0.0290			
η'	$2\theta+\Delta$			−0.0364	+0.2398			
η'	$\varepsilon-\phi+2\theta+\Delta$			−0.0107	+0.0585			
η'	$-\varepsilon+\phi+2\theta+\Delta$			+0.0402	−0.2889			
η'	$\phi +\Delta$			−0.7670	+5.4890			
η'	$\varepsilon +4\theta+3\Delta$			+0.0459	−0.3715			
η'	$\phi+4\theta+3\Delta$			+0.0336	−0.2709			
η'	$\varepsilon+\phi+2\theta+3\Delta$			+0.0007	−0.0220			
		m'^3		m'^2			m'	

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TABLE XXVIIIc.

$[(1-e\cos\epsilon)\overline{W}_3'']$

Unit—4th decimal of a radian.

	Cos	w	
		w^0	w
η	$2\theta+2\Delta$	+60.76	−152.6
η'	$2\theta+\Delta$	−20.57	+ 69.8
		m'	

TABLE XXIX.

$[(1-e\cos\epsilon)(\overline{W}_3-\overline{W}_3'')]$ Unit—4th decimal of a radian.

	Cos	w^{-1}		w	
		w^0	w	w^0	w
		−0.00004	+0.00038		
η	$2\theta+2\Delta$	−0.0032	−0.0005		
η'	$2\theta+\Delta$	+0.5106	−3.0290	+13.16	−30.9
		−0.6292	+3.463		
	$(\theta-\theta_0)$ sin				
η	$2\theta+2\Delta$	+0.0092	−0.0072		
η'	$2\theta+\Delta$	−0.0069	+0.0094		
		m'^2		m'	

These developments cover the function W within the extent of our tables. This does not mean that W is always inclusive of all these terms, but that these terms occur in one or more of the tables. With the exception of $[(1-e\cos\epsilon)\overline{W}]$, which contains $\overline{W}_3-\overline{W}_3''$, W is to be understood to mean

$$W=W_1+W_2'+[W_2]+(W_3''+W_3''+W_4'')$$

and

$$\overline{W}=\overline{W}_1+\overline{W}_2'+[\overline{W}_2]+(\overline{W}_3''+\overline{W}_3''+\overline{W}_4'').$$

The ascending powers of w, η, η', j^2 are selected independently in each function.

To avoid a long series which is analogous in construction to T_2, the function $W_3''+W_3''+W_4''$ is not tabulated. The sum of this function and Tables XVII, XVIII, XIX, XXII*a* gives W. Since W is so long and we only need $\overline{W}$, it is not tabulated. The function

$$\overline{W}=W_{\phi=0}$$

is given in Table XXIX*a*.

It is convenient to collect here $[(1-e\cos\epsilon)\overline{W}]$, which is required later. The function is given by the sum of Tables XVI, XX, XXI, XXVIII, and XXIX, and is tabulated in Table XXIX*b*.

We shall also need the function Ξ

$$\Xi=x+2\eta y=\Xi_1+\Xi_2'+[\Xi_2]+(\Xi_2''+\Xi_3''+\Xi_4'')$$

Evidently Ξ can be written by inspection if W is tabulated. If the double headings are retained in the construction of Ξ the mass factors and ranks are explicit as in the construction of W. If W is not given, we can write by inspection Ξ_1 (previously required in the computation), Ξ_2' and $[\Xi_2]$ from W_1, W_2', and $[W_2]$, respectively. The remainder, namely, $\Xi_2''+\Xi_3''+\Xi_4''$, can be written from $W_2''+W_3''+W_4''$, i. e., by inspection of Tables XXIII, XXIV, XXV. The function $\frac{1}{3}\Xi$ is given in Table XXIX*c*.

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TABLE XXIXa.

W.

Unit=1''.

	Cos	w^{-1}				w^{-3}				w^{-5}			
		w^0	w	w^2	w^3	w^0	w	w^2	w^3	w	w^2	w^3	w^5
	$\varepsilon+2\theta+2\phi$	+ 294.89	− 86.26	++ 255.90	− 342.2		+ 0.2108	− 1.059		− 0.2108	+ 1.059		+ 2.38
	$2\varepsilon+4\theta+4\phi$		− 978.6	++ 1571	− 1415		− 0.6139	++ 4.059		+ 0.6139	− 4.059		− 10.3
			+ 102.7	− 646.5	+ 2209.8		− 0.2108	++ 1.059		− 0.2108	+ 1.059		− 2.38
η	$2\theta+2\phi$	+1173.6	− 2906.1	+ 1735	+ 3127		− 6.330	+41.24					
	ε		+ 278	− 961	++ 1645		+ 0.738	− 3.71					
	$\varepsilon+4\theta+4\phi$	− 839.5	+ 4784	− 14212	+25911	−0.316	+ 3.54	−22.8					
	$2\varepsilon+2\theta+2\phi$		+ 190	− 449	− 58		+ 0.274	− 1.70					
	$2\varepsilon+6\theta+6\phi$		− 772	+ 7102	−35536		+ 1.800	−11.66					
η'	$2\theta+\phi$	− 318.2	+ 212	+ 1487	− 5950		+ 8.285	−48.64					
	$\varepsilon+\phi$		+ 188	− 766	− 1504		− 0.227	+ 1.34					
	$\varepsilon+4\theta+3\phi$	+1229.8	− 6069	++ 16395	−31079	+0.114	− 5.00	+34.3					
	$2\varepsilon+2\theta+3\phi$		− 177	++ 546	− 339		− 0.204	++ 1.38					
	$2\varepsilon+6\theta+5\phi$		+ 1211	− 10221	+47956		− 2.637	++ 15.35					
η^2	$4\theta+4\phi$		− 753	+ 4802		−0.95	+ 9.4						
	$\varepsilon+2\theta+2\phi$	−3358	+10660	+ − 25394		−1.27	++30.3						
	$\varepsilon+6\theta+6\phi$	+ 978	− 5286	++ 19864		+4.42	− 30.4						
	$\varepsilon+2\theta+2\phi$	++2940	−23666	+102458		++1.80	− 4.5						
	−	++ 396	+ 489	− 6259		− 2.62	+12.6						
	2ε		+ 66	− 959			− 0.2						
	$2\varepsilon+4\theta+4\phi$		− 2316	+ − 13688			− 1.5						
	$2\varepsilon+8\theta+8\phi$		+ 4626	− 55029			−11.8						
$\eta\eta'$	4		+ 2283	− 12753		+0.68	−15.0						
	$4\theta+3\phi$	+8609	−17128	+ − 35165		+0.79	−76.8						
	$\varepsilon+2\theta+\phi$	−2280	+12028	+ − 34202		−5.57	+36.0						
	$\varepsilon+2\theta+3\phi$	+1492	− 2193	− 9774		−1.91	+14.0						
	$\varepsilon+6\theta+5\phi$	−8658	+63021	−255722		−3.95	++ 7.1						
	$-\varepsilon+2\theta+\phi$	+2068	− 9588	− 43388		+6.18	−25.5						
	$2\varepsilon+4\theta+3\phi$		− 637	+ + 3699			+ 0.1						
	$2\varepsilon+4\theta+5\phi$		+ 2796	− 13717			++ 4.3						
	$2\varepsilon+8\theta+7\phi$		++ 2791	− 18175			− 1.6						
			−14329	+160224			+34.8						
η^3	$4\theta+2\phi$		− 1732	+ − 8521		−0.12	+ 6.8						
	$\varepsilon+2\theta+2\phi$	−5341	+ 3562	− 7700		−0.12	++50.3						
	$\varepsilon+6\theta+4\phi$	+ 861	+ 2864	+ + 22660		+2.12	− 15.3						
		+6349	−41206	+156926		+1.90	− 2.2						

XX second degree terms in this column were used.

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f^2	η^2		m'		m''	
			−1634		−4.04	
	$-\epsilon+2\delta$		+ 6264	− 13066		+13.8
	$2\epsilon+$ 2Δ		+ + 376	− − 2185		+ 0.4
	$2\epsilon+4\delta+4\Delta$		+ − 3544	+ − 20067		−25.5
	$2\epsilon+8\delta+6\Delta$		+11104	−116175		
			+ 1732	− 8521		− 3.8
	$4\delta+3\Delta-\Sigma$		+ + 203	− + 2250		+ 1.1
	$\epsilon+2\delta+2\Delta$		+ + +14806	− + 41509		
	$\epsilon+6\delta+5\Delta-\Sigma$		− 2257	+ − 10634	+0.07	− 0.1
	$-\epsilon+2\delta-4-2\Sigma$		− 3827	+ − 9074	−0.22	+ 1.2
	$2\epsilon+4+2\Sigma$		+ + 1025	− − 4486		
	$2\epsilon+4\delta+4\Delta$		+ − 1387	+ − 11469		+ 3.8
	$2\epsilon+8\delta+7\Delta-\Sigma$		+ 300	− 3954		− 0.7
			− 304			
			−2677			
			+ 260			
			+ − 866			

[Continued on next page.

In the construction of Table

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TABLE XXIX*a*—Continued.

$\overline{W}$.

Unit=1″.

	Cos	w^{-1}			w^{-2}		
		w^0	w	w^2	w^2	w	w^2
η^3	$\varepsilon+4\theta+4\Delta$	+ 2549	+ 2164		−11.9	+ 89	
	$-\varepsilon+4\theta+4\Delta$	− 3089	+ 8155		+ 3.9	− 25	
	$\varepsilon+8\theta+8\Delta$	−11300	+ 76250		− 8.9	+ 70	
$\eta^2\eta'$	$\varepsilon+4\theta+5\Delta$	−11449	+ 42212		+ 1.9	− 23	
	$\varepsilon+4\theta+3\Delta$	− 2661	− 27530		+36.4	−241	
	$-\varepsilon+4\theta+3\Delta$	+ 6865	− 4540		−20.3	+118	
	$\varepsilon+8\theta+7\Delta$	+50005	−304611		+33.8	−248	
$\eta\,\eta'^2$	$\varepsilon+4\theta+4\Delta$	+26091	− 71730		−10.1	+ 83	
	$\varepsilon+4\theta+2\Delta$	− 1356	+ 30293		−25.5	+153	
	$-\varepsilon+4\theta+2\Delta$	− 2204	− 20846		+28.0	−153	
	$\varepsilon+8\theta+6\Delta$	−73583	+400009		−41.9	+284	
η'^3	$\varepsilon+4\theta+3\Delta$	−13756	+ 22165		+10.1	− 65	
	$-\varepsilon+4\theta+\Delta$	− 3317	+ 18452		−12.4	+ 64	
	$\varepsilon+8\theta+5\Delta$	+36006	−172164		+16.6	−104	
$f^2\eta$	$\varepsilon+4\theta+3\Delta-\Sigma$	− 2011	+ 14604		− 1.9	+ 14	
	$-\varepsilon+4\theta+3\Delta-\Sigma$	+ 1808	− 13617		+ 1.9	− 14	
	$\varepsilon+8\theta+7\Delta-\Sigma$	− 2381	+ 18919		− 1.1	+ 10	
	$\varepsilon+4\theta+4\Delta$	+14204	− 88026		+ 5.7	− 42	
$f^2\,\eta'$	$\varepsilon+4\theta+4\Delta-\Sigma$	− 554	+ 140		+ 0.5	− 4	
	$-\varepsilon+4\theta+2\Delta-\Sigma$	− 3545	+ 22886		− 1.8	+ 14	
	$\varepsilon+8\theta+6\Delta-\Sigma$	+ 3827	− 27870		+ 1.3	− 11	
	$\varepsilon+4\theta+3\Delta$	−17503	+ 99584		− 3.7	+ 28	
	$(\theta-\theta_0)\sin$						
η	$2\theta+2\Delta$					− 2.19	+13.6
	ε	+ 767.7	− 2820.9	+ 5210	+ 1.265	− 5.34	+22.7
	$\varepsilon+4\theta+4\Delta$					+ 0.78	− 6.0
	$2\varepsilon+2\theta+2\Delta$					− 0.55	+ 3.4
η'	$2\theta+\Delta$					+ 1.63	−11.0
	$\varepsilon+\Delta$	− 570.0	+ 2421.1	− 4950	− 0.455	+ 5.94	−37.3
	$\varepsilon+4\theta+3\Delta$					− 0.58	+ 4.8
	$2\varepsilon+2\theta+3\Delta$					+ 0.41	− 2.8
η^3	$4\theta+4\Delta$					+ 10.93	
	$\varepsilon+2\theta+2\Delta$					− 2.19	
	$-\varepsilon+2\theta+2\Delta$					− 1.92	
	$2\varepsilon+4\theta+4\Delta$					+ 3.12	
$\eta\,\eta'$	Δ	− 570.0	+ 2421.1	− 4950	− 0.455	+ 5.94	− 7.2
	$4\theta+3\Delta$					− 23.00	
η^3	ε	+ 6624	− 47448		+23.8	−221.9	
$\eta^2\eta'$	$\varepsilon+\Delta$	−18540	+123024		−73.4	+572.4	
	$-\varepsilon+\Delta$	+ 8414	− 57880		+36.0	−282.2	
$\eta\,\eta'^2$	ε	+25564	−157424		+87.3	−652.8	
	$\varepsilon+2\Delta$	+10478	− 70250		+55.2	−374.8	
η'^3	$\varepsilon+\Delta$	−15678	+ 94846		−69.9	+498.6	
$f^2\eta$	ε	−25564	+157424	−511232	−23.1	+165.0	
	$\varepsilon+\Delta+\Sigma$	+22012	−121258	+359162	+ 9.9	− 77.0	
$f^2\,\eta'$	$\varepsilon+\Delta$	+23524	−150306	+498328	+14.8	−112.0	
	$\varepsilon+\Sigma$	−12048	+ 76364	−251640	− 5.2	+ 45.8	
	$(\theta-\theta_0)^2\cos$						
η	ε					− 0.356	+ 2.623
η'	$\varepsilon+\Delta$					+ 0.266	− 2.100
		m'			m'^2		

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TABLE XXIXa—Continued.

W. Unit=1″

	Cos	w^0	w	w^2
	$\frac{1}{2}\varepsilon+\theta+\Delta$	− 293.4	+ 913.5	− 1400.1
	$\frac{1}{2}\varepsilon+3\theta+3\Delta$	+ 338.1	− 2315	+ 9277
	$\frac{1}{2}\varepsilon+5\theta+5\Delta$	+ 42.9	− 284.3	+ 948.2
	$\frac{1}{2}\varepsilon+7\theta+7\Delta$	+ 10.5	− 79.2	+ 288.5
η	$\frac{1}{2}\varepsilon+3\theta+3\Delta$	+ 6172.8	− 20580	+ 86549
	$-\frac{1}{2}\varepsilon+\theta+\Delta$	+ 511.2	− 2834	+ 7746
	$\frac{1}{2}\varepsilon+\theta+\Delta$	− 467.9	+ 2335	− 6259
	$\frac{1}{2}\varepsilon+5\theta+5\Delta$	− 2217.1	+ 23971	−157808
	$\frac{1}{2}\varepsilon+3\theta+3\Delta$	− 5.8	+ 539	− 3713
	$\frac{1}{2}\varepsilon+7\theta+7\Delta$	− 364.3	+ 3259	− 15083
η'	$\frac{1}{2}\varepsilon+3\theta+2\Delta$	− 8375.5	+ 20591	− 95913
	$-\frac{1}{2}\varepsilon+\theta$	− 1023.4	+ 4443	− 10251
	$\frac{1}{2}\varepsilon+\theta+2\Delta$	− 92.3	− 444	+ 3212
	$\frac{1}{2}\varepsilon+5\theta+4\Delta$	+ 3385.4	− 34097	+214736
	$\frac{1}{2}\varepsilon+3\theta+4\Delta$	− 138.6	+ 608	− 1069
	$\frac{1}{2}\varepsilon+7\theta+6\Delta$	+ 583.3	− 4805	+ 20748
η^2	$\frac{1}{2}\varepsilon+\theta+\Delta$	− 5022	+ 24269	
	$\frac{1}{2}\varepsilon+5\theta+5\Delta$	−31492	+154465	
	$-\frac{1}{2}\varepsilon+3\theta+3\Delta$	+ 8169	− 18309	
	$\frac{1}{2}\varepsilon+3\theta+3\Delta$	− 59	+ 7449	
	$\frac{1}{2}\varepsilon+7\theta+7\Delta$	+12392	−182737	
	$-\frac{1}{2}\varepsilon+\theta+\Delta$	+ 1133	− 5174	
	$\frac{1}{2}\varepsilon+9\theta+9\Delta$	+ 2342	− 25879	
$\eta\,\eta'$	$\frac{1}{2}\varepsilon+\theta$	+ 6153	− 26311	
	$\frac{1}{2}\varepsilon+\theta+2\Delta$	+ 988	− 15732	
	$\frac{1}{2}\varepsilon+5\theta+4\Delta$	+88784	−357566	
	$-\frac{1}{2}\varepsilon+3\theta+2\Delta$	−14498	− 3083	
	$\frac{1}{2}\varepsilon+3\theta+2\Delta$	− 1309	− 5	
	$\frac{1}{2}\varepsilon+3\theta+4\Delta$	+ 4878	− 31947	
	$\frac{1}{2}\varepsilon+7\theta+6\Delta$	−37540	+526187	
	$-\frac{1}{2}\varepsilon+\theta$	− 3487	+ 12764	
	$\frac{1}{2}\varepsilon+9\theta+8\Delta$	− 7382	+ 77025	
η'^2	$\frac{1}{2}\varepsilon+\theta+\Delta$	− 5966	+ 27801	
	$\frac{1}{2}\varepsilon+5\theta+3\Delta$	−61877	+192684	
	$-\frac{1}{2}\varepsilon+3\theta+\Delta$	− 1709	+ 26144	
	$\frac{1}{2}\varepsilon-\theta+\Delta$	+ 1693	− 6306	
	$\frac{1}{2}\varepsilon+3\theta+3\Delta$	− 5297	+ 28649	
	$\frac{1}{2}\varepsilon+7\theta+5\Delta$	+28418	−377278	
f^2	$\frac{1}{2}\varepsilon+\theta+\Delta$	+ 6846	− 30542	
	$\frac{1}{2}\varepsilon+5\theta+4\Delta-\Sigma$	− 3191	+ 15590	
	$-\frac{1}{2}\varepsilon+3\theta+2\Delta-\Sigma$	− 806	+ 10210	
	$\frac{1}{2}\varepsilon+3\theta+3\Delta$	− 3829	+ 33852	
	$\frac{1}{2}\varepsilon+7\theta+6\Delta-\Sigma$	+ 932	− 14562	
	$-\frac{1}{2}\varepsilon+\theta\quad-\Sigma$	+ 1762	− 6460	
			m'	

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TABLE XXIXb.

$[(1-e\cos\epsilon)\,\overline{W}]$

Unit—4th decimal of a radian.

	Cos	w^{-5}	w^{-4}	w^{-3}	w^{-2}	w^{-1}	w^{0}	w	w^{2}
				−0.00004	+0.01060	− 0.054	− 4.0682	+ 12.406	− 16.59
η	$2\theta+2\Delta$		−0.000055	+0.00041	−0.2771	+ 45.203	− 72.26	+ 21.0	+164.8
η'	$2\theta+\Delta$		+0.000020	−0.00017	+0.4017	− 18.414	+ 19.67	+ 72.1	−288.5
η^2			+0.00081	−0.0533	+0.423	− 2.32	− 50.0	+ 279.4	+126.2
	$4\theta+4\Delta$		+0.00011	−0.0470	+1.294	− 131.22	+ 293.6	− 542	+111
$\eta\,\eta'$	Δ		−0.00145	+0.0457	−0.717	+ 4.83	+ 120.2	− 655.4	−120.0
	$4\theta+3\Delta$		−0.00003	+0.0336	−3.482	+ 381.92	− 561.9	+ 910	−162
η'^2			+0.00042	−0.0099	+0.333	− 2.572	− 83.96	+ 413.1	
	$4\theta+2\Delta$		−0.00027	−0.0043	+2.441	− 274.98	+ 192.5	− 378.5	
j^2					−0.186	+ 1.345	+ 79.15	− 413.1	
	$4\theta+3\Delta-\Sigma$		−0.00093	+0.0003	+0.052	− 15.295	+ 10.3	+ 109	
η^3	$2\theta+2\Delta$	+0.00030	−0.0022	+0.262	−1.70	+ 28.2	− 433		
	$6\theta+6\Delta$	+0.0001	−0.0008	+0.262	−1.70	+ 428	− 2295		
$\eta^2\eta'$	$2\theta+\Delta$	−0.00021	+0.0018	−0.767	+4.46	− 316.1	+ 1591.5		
	$2\theta+3\Delta$	−0.00011	+0.0009	−0.094	+0.69	+ 108.5	− 49		
	$6\theta+5\Delta$	−0.0001	+0.001	−0.86	+5.2	−1889	+ 8900		
$\eta\,\eta'^2$	2θ	+0.00004	−0.0004	+0.555	−2.87	+ 237.6	− 1030		
	$2\theta+2\Delta$	+0.00008	−0.0009	+0.276	−1.85	− 125.3	− 552		
	$6\theta+4\Delta$	+0.00004		+0.83	−4.7	+2770	−11240		
η'^3	$2\theta+\Delta$	−0.00001		−0.290	+1.21	− 168.7	+ 867		
	$6\theta+3\Delta$	−0.00005		−0.200	+1.21	−1946	+ 4580		
$j^2\eta$	$2\theta+\Delta-\Sigma$			+0.032	−0.23	+ 126.0	− 620		
	$2\theta+2\Delta$					− 389.4	+ 1844	−4500	
	$6\theta+5\Delta-\Sigma$			+0.032	−0.23	+ 113	− 730		
$j^2\,\eta'$	$2\theta+\Delta$					+ 362.4	− 1749		
	$2\theta+2\Delta-\Sigma$			−0.011	+0.09	− 7.7	+ 144		
	$6\theta+4\Delta-\Sigma$			−0.011	+0.09	− 187	+ 1080		
	$(\theta-\theta_0)\sin$								
η	$2\theta+2\Delta$		−0.000176	+0.00132	−0.1098	+ 0.6675	− 1.920		
η'	$2\theta+\Delta$		+0.000063	−0.00053	+0.0805	− 0.5404	+ 1.692		
η^2	$4\theta+4\Delta$		+0.000100	−0.00090	+0.492	− 3.760	+ 8.04		

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$\eta\,\eta'$	$\frac{A}{4\theta+3A}$			-0.0000	-0.00001	-0.000		$-$	0.39	$-$	5.97		
				-0.00168	$+0.01469$	-1.086		$+$	8.037	$-$	17.28		
η'^2	$4\theta+2A$			$+0.00048$	-0.00444	$+0.536$		$-$	4.046	$+$	9.024		
	$(\theta-\theta_0)^2\cos$												
η^3				$+0.000171$	-0.00149	$+0.0520$		$-$	0.382	$+$	1.055		
$\eta^2\eta'$	A			-0.000188	$+0.00177$	-0.0771		$+$	0.610	$-$	1.798		
η'^3				$+0.000046$	-0.000464	$+0.0286$		$-$	0.243	$+$	0.765		
			m'^2	m^2	m^2, m'^2	m^2, m^2		m'^2, m'		m'^2, m'		m'	

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TABLE XXIXc.

$\frac{1}{3^{\Sigma}}$

Unit=$1''$

	Cos	w^{-1}				w^{-2}		
		w^0	w	w^2	w^3	w^0	w	w^2
	$\varepsilon+2\theta+2\Delta$		− 90.5	+ 302.7	− 478.2			
	$2\varepsilon+4\theta+4\Delta$		− 26.6	+ 125.5	− 270.3			
η	$2\theta+2\Delta$	+ 589.8	− 1571.9	+ 1680			− 1.82	+12.00
	ε						+ 0.42	− 2.10
	$\varepsilon+4\theta+4\Delta$		+ 616	− 3638	+11175		− 0.42	+ 2.10
	$2\varepsilon+2\theta+2\Delta$		+ 23	− 161	+ 439			
	$2\varepsilon+6\theta+6\Delta$		+ 219	− 1451	+ 4616			
η'	$2\theta+\Delta$	− 106.1	+ 360	− 517			+ 1.81	−10.64
	$\varepsilon+\Delta$		− 43	+ 161	− 269		− 0.08	+ 0.45
	$\varepsilon+4\theta+3\Delta$		− 760	+ 3906	−10778		+ 0.08	− 0.45
	$2\varepsilon+2\theta+3\Delta$		+ 52	− 143	+ 87			
	$2\varepsilon+6\theta+5\Delta$		− 314	+ 1874	− 5403			
η^2						−0.317	+ 1.63	
	$4\theta+4\Delta$	−1679	+ 7272	−13527		−0.633	+10.02	
	$\varepsilon+2\theta+2\Delta$		+ 274	− 63			− 1.20	
	$\varepsilon+6\theta+6\Delta$		− 3474	+29267			+ 3.60	
	$-\varepsilon+2\theta+2\Delta$		+ 1156	− 2171			− 2.40	
	2ε						− 0.21	
	$2\varepsilon+4\theta+4\Delta$		+ 180	+ 113			+ 0.21	
	$2\varepsilon+8\theta+8\Delta$		− 1375	+11897				
$\eta\,\eta'$	Δ					+0.227	− 1.30	
	$4\theta+3\Delta$	+3690	−12966	+19401		+0.340	−19.92	
	$\varepsilon+2\theta+\Delta$		+ 222	− 1234			+ 1.96	
	$\varepsilon+2\theta+3\Delta$		− 769	+ 2197			+ 0.01	
	$\varepsilon+6\theta+5\Delta$		+ 9240	−70866			− 7.25	
	$-\varepsilon+2\theta+\Delta$		− 646	+ 1806			+ 5.27	
	$2\varepsilon+\Delta$		+ 99	− 444			+ 0.04	
	$2\varepsilon+4\theta+3\Delta$		− 109	− 922			− 0.04	
	$2\varepsilon+4\theta+5\Delta$		− 846	+ 4256				
	$2\varepsilon+8\theta+7\Delta$		+ 4012	−31827				
η'^2						−0.039	+ 0.24	
	$4\theta+2\Delta$	−1780	+ 4725	− 5354		−0.039	+10.42	
	$\varepsilon+2\theta+2\Delta$		+ 499	− 649			− 0.32	
	$\varepsilon+6\theta+4\Delta$		− 5930	+40905			+ 2.86	
	$-\varepsilon+2\theta$						− 2.54	
	$2\varepsilon+2\Delta$		− 55	+ 285				
	$2\varepsilon+4\theta+4\Delta$		+ 980	− 4150				
	$2\varepsilon+8\theta+6\Delta$		− 2890	+20791				
j^2	$4\theta+3\Delta-\Sigma$	− 101	+ 493	− 1128			+ 0.11	
	$\varepsilon+2\theta+2\Delta$		+ 587	− 2983				
	$\varepsilon+6\theta+5\Delta-\Sigma$		− 193	+ 1759			+ 0.14	
	$-\varepsilon+2\theta+\Delta-\Sigma$						− 0.14	
	$2\varepsilon+\Delta+\Sigma$		− 192	+ 705				
	$2\varepsilon+4\theta+4\Delta$		+ 298	− 1876				
	$2\varepsilon+8\theta+7\Delta-\Sigma$		− 65	+ 616				
		m'				m'^2		

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TABLE XXIXc—Continued.

$\frac{1}{3}\Sigma$ Unit=1″

	Cos	w^0	w	w^2
	$\frac{1}{2}\epsilon+\theta+\Delta$	− 31.4	+ 131.0	− 255
	$\frac{1}{2}\epsilon+3\theta+3\Delta$	− 48.0	+ 193.6	− 360
	$\frac{1}{2}\epsilon+5\theta+5\Delta$	− 15.2	+ 81.7	− 201
	$\frac{1}{2}\epsilon+7\theta+7\Delta$	− 5.2	+ 34.7	− 107
η	$\frac{1}{2}\epsilon+3\theta+3\Delta$	+ 1304	− 8173	+30282
	$-\frac{1}{2}\epsilon+\theta+\Delta$	− 196	+ 506	− 598
	$\frac{1}{2}\epsilon+\theta+\Delta$	+ 34	− 146	+ 292
	$\frac{1}{2}\epsilon+5\theta+5\Delta$	+ 356	− 2212	+ 6781
	$\frac{1}{2}\epsilon+3\theta+3\Delta$	+ 1	− 67	+ 294
	$\frac{1}{2}\epsilon+7\theta+7\Delta$	+ 138	− 999	+ 3437
η'	$\frac{1}{2}\epsilon+3\theta+2\Delta$	− 1361	+ 7468	−25691
	$-\frac{1}{2}\epsilon+\theta$			
	$\frac{1}{2}\epsilon+\theta+2\Delta$	+ 29	− 12	− 155
	$\frac{1}{2}\epsilon+5\theta+4\Delta$	− 482	+ 2635	− 7209
	$\frac{1}{2}\epsilon+3\theta+4\Delta$	+ 52	− 197	+ 280
	$\frac{1}{2}\epsilon+7\theta+6\Delta$	− 207	+ 1348	− 4176
η^2	$\frac{1}{2}\epsilon+\theta+\Delta$	− 625	+ 3058	
	$\frac{1}{2}\epsilon+5\theta+5\Delta$	− 7151	+ 70387	
	$-\frac{1}{2}\epsilon+3\theta+3\Delta$	+ 5478	− 5874	
	$\frac{1}{2}\epsilon+3\theta+3\Delta$	+ 18	+ 1924	
	$\frac{1}{2}\epsilon+7\theta+7\Delta$	− 2111	+ 17665	
	$-\frac{1}{2}\epsilon+\theta+\Delta$	+ 187	− 590	
	$\frac{1}{2}\epsilon+9\theta+9\Delta$	− 771	+ 6931	
$\eta\,\eta'$	$\frac{1}{2}\epsilon+\theta+2\Delta$	− 231	− 1142	
	$\frac{1}{2}\epsilon+5\theta+4\Delta$	+17640	−159928	
	$-\frac{1}{2}\epsilon+3\theta+2\Delta$	− 9842	+ 1346	
	$\frac{1}{2}\epsilon+3\theta+4\Delta$	− 892	+ 3699	
	$\frac{1}{2}\epsilon+3\theta+2\Delta$	+ 106	− 2494	
	$\frac{1}{2}\epsilon+7\theta+6\Delta$	+ 5918	− 45149	
	$-\frac{1}{2}\epsilon+\theta$			
	$\frac{1}{2}\epsilon+9\theta+8\Delta$	+ 2513	− 20914	
η'^2	$\frac{1}{2}\epsilon+\theta+\Delta$	− 507	+ 2729	
	$\frac{1}{2}\epsilon+5\theta+3\Delta$	−10202	+ 84314	
	$-\frac{1}{2}\epsilon+3\theta+\Delta$	+ 1055	− 678	
	$\frac{1}{2}\epsilon-\theta+\Delta$	− 100	+ 387	
	$\frac{1}{2}\epsilon+3\theta+3\Delta$	+ 871	− 2817	
	$\frac{1}{2}\epsilon+7\theta+5\Delta$	− 4065	+ 27951	
f^2	$\frac{1}{2}\epsilon+\theta+\Delta$	+ 601	− 3122	
	$\frac{1}{2}\epsilon+5\theta+4\Delta-\Sigma$	− 423	+ 4435	
	$-\frac{1}{2}\epsilon+3\theta+2\Delta-\Sigma$	+ 285	− 356	
	$\frac{1}{2}\epsilon+3\theta+3\Delta$	+ 426	− 2410	
	$\frac{1}{2}\epsilon+7\theta+6\Delta-\Sigma$	− 108	+ 988	
	$-\frac{1}{2}\epsilon+\theta\quad-\Sigma$	− 106	+ 402	
		m'		

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TABLE XXIXc—Continued.

$\frac{1}{3}\Xi$

Unit=1″

	Cos	w^{-1}			w^{-2}		
		w^0	w	w^2	w^0	w	w^2
η^3	$2\theta+2\Delta$	+ 1568	− 8912		+3.6	−23.3	
	$6\theta+6\Delta$	+ 5879	− 31559		+3.6	−23.3	
$\eta^2\eta'$	$2\theta+\Delta$	− 2385	+ 11662		−4.7	+26.6	
	$2\theta+3\Delta$	+ 2238	− 1015		−2.6	+18.4	
	$6\theta+5\Delta$	−21644	+102003		−9.9	+59.1	
$\eta\,\eta'^2$	2θ				−0.2	+ 1.7	
	$2\theta+2\Delta$	− 1723	− 7588		+4.0	−27.1	
	$6\theta+4\Delta$	+25396	−103013		+7.6	−43.2	
η'^3	$2\theta+\Delta$	− 1160	+ 5960		−1.4	+ 8.3	
	$6\theta+3\Delta$	− 9257	+ 31500		−1.4	+ 8.3	
$j^2\eta$	$6\theta+5\Delta-\Sigma$	+ 1040	− 6697		+0.3	− 2.1	
	$2\theta+2\Delta$	− 5354	+ 25370	−61855			
$j^2\ \eta'$	$2\theta+\Delta$	+ 2492	− 12023				
	$2\theta+2\Delta-\Sigma$	− 53	+ 989		−0.1	+ 0.6	
	$6\theta+4\Delta-\Sigma$	− 1285	+ 7413		−0.1	+ 0.6	
	$(\theta-\theta_0)$ sin						
η	$2\theta+2\Delta$					− 0.55	+3.40
η'	$2\theta+\Delta$					+ 0.41	−2.74
η^3	$4\theta+4\Delta$					− 3.12	
	$\epsilon+2\theta+2\Delta$					− 1.10	
	$-\epsilon+2\theta+2\Delta$					− 1.10	
$\eta\,\eta'$	Δ	− 569.95	+ 2421.1	− 4950	+0.45	+ 5.94	
	$4\theta+3\Delta$					− 5.75	
	$\epsilon+2\theta+\Delta$					+ 0.20	
	$\epsilon+2\theta+3\Delta$					+ 0.82	
	$-\epsilon+2\theta+\Delta$					+ 1.01	
η'^2	$4\theta+2\Delta$					+ 2.55	
	$\epsilon+2\theta+2\Delta$					− 0.15	
	$-\epsilon+2\theta$					− 0.15	
	$(\theta-\theta_0)^3$ cos						
$\eta\,\eta'$	Δ					− 0.26	
η'^2						+ 0.20	
		m'			m'^2		

COMPARISON OF TABLES.

As a computer would discover in constructing tables, and as will be evident from an application of the method to a planet, the coefficients in Table II and others of the same form are given with unnecessary accuracy. Although so many digits would never be required, except in a much more exhaustive development, they are given, for completeness, as they resulted from computation.

In all the tables whose constructions involve the multiplication of trigonometric series, the errors are difficult or impossible to determine. Although v. Zeipel's manuscript, which the author generously furnished for comparison, is of assistance, the computations are not entirely parallel, and comparison is not always possible. Many of the computations are so long and

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complicated that the origin of certain discrepancies is obscure. Aside from possible errors of calculation, differences are due to the independent adoption of the highest powers of m', w, η, η', j^2, and the number of arguments in a given series or product of series. In most cases our series are more complete than v. Zeipel's. Whether or not the extension of the tables increases the accuracy of the result remains to be seen from future applications of the theory.

Tables II–XV.—The discrepancies seem to be due to v. Zeipel's errors of calculation and to their subsequent effects. The larger number of these errors have been traced in the manuscript.

Tables XVI, XVII check satisfactorily.

Table XVIII.—The bracketed quantities in the first three columns are in error through previous discrepancies. We did not discover the source of the general disagreement in terms of the third degree, second order in the mass. These terms do not affect v. Zeipel's subsequent tables, since they are of order higher than have been included.

Tables XIX, XX agree satisfactorily.

Table XXI.—The discrepancies are numerous and their origin is obscure because of the very long computation involved. In addition to performing a complete duplicate computation, the terms of first degree and a part of the computation of second degree terms were checked by the solution of the differential equation in the form given in Z 64. With the exception of three or four single instances, the discrepancies occur in two groups, having the following probable explanations. The neglect of the term

$$\frac{3}{4}[\overline{W}_1^2]\frac{d\phi_1}{d\theta}$$

in Z 65, eq. (109), by v. Zeipel accounts for one group of differences. The other group can be fairly well explained by an error in the addition of second order terms in $+\frac{w}{2}\phi_1$ to $\phi_2 - \frac{w}{2}\phi_1$. Assuming that for these terms he added $-w\phi$, and, correcting his table, three discrepancies are removed and two others are improved.

Table XXII.—Considering the disagreements in Table XXI, Table XXII checks satisfactorily.

Table XXIII–XXVII.—These tables, like II–XV, are simple in construction, and the discrepancies are due to errors of calculation, or they are the result of previous ones, with the exception that some quantities have different numerical values because they are more inclusive. The latter have been indicated by ( ).

Table XXVIII.—The discrepancies arise from the quantities in parentheses in Table XXVII. The omission of the term depending upon the inclination is justifiable in view of its magnitude.

Table XXIX.—The discrepancies are numerous and striking, but, since v. Zeipel does not give the formulæ of computation, they remain unexplained. The remark is made (Z 77), "Die Berechnung der Funktion $[(1-e\cos\varepsilon)(\overline{W}_2 - \overline{W}_2'')]$, welche eine sehr komplicirte war, wird hier nicht im Einzelnen mitgetheilt." For this reason the development of the formulæ which we used has been included and the auxiliary functions $2[T_1]$, W_2, $[(1-e\cos\varepsilon)\overline{W}_2'']$ have been tabulated. The differences are not serious because of the high rank of the function. Our table is deficient in certain terms whose computation would be long and the omission of which is justifiable in view of their magnitude.

PERTURBATIONS OF THE MEAN ANOMALY.

For clearness some of v. Zeipel's developments will be amplified and repeated in an order which we found more convenient.

The determination of the disturbed mean anomaly is accomplished with the integration of Z 9, eq. (47), (which implicitly contains Z 8, eq. (38)). By Z 9, eq. (43),

$$\theta = \tfrac{1}{2}(\epsilon - e\sin\epsilon) - g' = \tfrac{1}{2}g - g'$$

The differential equation is repeated in Z 78, eq. (124), in which is emphasized the fact that the arguments are functions of both ϵ and θ, as is the case for $\frac{dW}{d\epsilon}$.

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If we observe the character of θ as it is expressed in the definition and recall that we have admitted trigonometric terms in θ, multiplied by t, it is evident that this argument, which is a function of the disturbed positions of the planet and Jupiter, is not periodic, but varies continuously with the time. In the foregoing equation g and g' can not be regarded as angles which are always less than 360°. θ contains, therefore, a nontrigonometric secular part in ϵ and a periodic part in θ and ϵ.

If we write

$$\theta = (\theta - [\theta]) + [\theta]$$

$\theta - [\theta]$ contains the secular term in ϵ as well as periodic terms. The segregation of terms of different type can be made explicit by the introduction of

$$\theta = \vartheta + \theta_1(\vartheta,\epsilon) + \theta_2(\vartheta,\epsilon) + \theta_3(\vartheta,\epsilon) + \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \qquad Z\ 78,\ \text{eq. (125)}$$

where ϑ is a function of ϵ and θ_1, θ_2, θ_3 $\cdot\ \cdot\ \cdot$ are the periodic parts of $\theta - [\theta]$, i. e., they are entirely trigonometric functions of ϵ. This covers the condition that θ_i can not include trigonometric secular terms in ϵ. By definition of ϑ and θ_i

$$\frac{d\vartheta}{d\epsilon} = \left[\frac{d\theta}{d\epsilon}\right] = [F(\theta,\epsilon)] - \frac{d[n'\delta z']}{d\epsilon}$$

$$\Sigma \frac{d\theta_i}{d\epsilon} = (F(\theta,\epsilon) - [F(\theta,\epsilon)]) - \frac{d}{d\epsilon}(n'\delta z' - [n'\delta z'])$$

where $[n'\delta z']$ is the long period term between Jupiter and Saturn.

The derivative of (125) is

$$\frac{d\theta}{d\epsilon} = \frac{\partial\theta}{\partial\epsilon} + \frac{\partial\theta}{\partial\vartheta}\frac{d\vartheta}{d\epsilon}$$

$$= \left(\frac{\partial\theta_1}{\partial\epsilon} + \frac{\partial\theta_2}{\partial\epsilon} + \frac{\partial\theta_3}{\partial\epsilon} + \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\right) + \left(1 + \frac{\partial\theta_1}{\partial\vartheta} + \frac{\partial\theta_2}{\partial\vartheta} + \frac{\partial\theta_3}{\partial\vartheta} + \cdot\ \cdot\ \cdot\ \cdot\ \cdot\right)\frac{d\vartheta}{d\epsilon}$$

Expanding $F(\theta,\epsilon)$, eq. (124) in a Taylor's series in ascending powers of θ_i and making the above substitution for $\frac{d\theta}{d\epsilon}$, (124) becomes (126), in which

$$F(\vartheta,\epsilon) = \frac{1}{2}(1 - e\cos\epsilon)\left\{w + (1-w)\,\overline{W} - \frac{3}{4}(1-w)\left(\overline{W} - \frac{1}{3}\Xi\right)\left(\overline{W} + \frac{1}{9}\Xi\right) + \cdots\right\}; \qquad \theta = \vartheta$$

From the Taylor's series $\frac{d\vartheta}{d\epsilon}$ is written in (127). This is the differential equation for ϑ, the right-hand side of which can be computed.

Substituting $\frac{d\vartheta}{d\epsilon}$ in (126) and equating functions of equal rank, we have the differential equations ($128_1 - 128_3$) for θ_i, which can be integrated in succession.

Before integration we convert eqs. (128) into differential equations for $n\delta z$ as follows:

Let

$$n\delta z = (n\delta z - [n\delta z]) + [n\delta z]$$
$$= n\delta z_1 + n\delta z_2 + n\delta z_3 + \cdots + [n\delta z] \qquad Z\ 88,\ \text{eq. (144)},$$

where $n\delta z_i$ is not only a function of first and higher orders in m', in which the lowest rank is i, but is entirely trigonometric or periodic. Then

Z 9, eq.(46) gives $n\delta z - [n\delta z] = \frac{2}{1-w}\left\{\theta_1(\vartheta,\epsilon) + \theta_2(\vartheta,\epsilon) + \theta_3(\vartheta,\epsilon) + \cdots\cdots + w\eta\sin\epsilon + (n'\delta z' - [n'\delta z'])\right\}$

and

$$[n\delta z] = \frac{2}{1-w}\left\{\vartheta - \frac{w}{2}\epsilon + [n'\delta z'] + c' - \mu c\right\} \qquad Z\ 88,\ \text{eq. (145)},$$

where it is to be noticed that $[n\delta z]$, unlike $[W]$, is not free from terms in ϵ. Subdividing the first of these two equations according to rank, we have Z 79, eqs. (130), in which $-n'\delta z' + [n'\delta z']$ can be neglected.

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Differentiating eqs. (130) partially with respect to ε, substituting in eqs. (128), evaluating the right-hand sides of eqs. (128), we have eqs. (131_1-131_3), in which the superscript indicates that only terms of first order in the mass are included, and where the argument ϑ replaces the argument θ.

For purposes of calculation, the integrations are arranged as follows:

In

$$\overline{W} = \overline{W}_1 + \overline{W}_2' + [\overline{W}_2] + (\overline{W}_2'' + \overline{W}_3'' + \overline{W}_4'')$$

consider first only $\overline{W}_2'' + \overline{W}_3'' + \overline{W}_4''$ in the integration of eqs. (131). The integrations will concern only part of the terms of first order in $n\delta z_1 + n\delta z_2 + n\delta z_3$. It is shown by v. Zeipel that the integration for all three ranks can be performed conveniently at the same time. Let this part of the function be indicated by enclosing it in ( ). The integral

$$(n\delta z_1^{(1)}) + (n\delta z_2^{(1)}) + (n\delta z_3^{(1)})$$

which is a trigonometric series, is given by Z 80, eq. (135), in which the coefficients $\overline{L}_{p,q}$ are defined by (136) and are easily derived from Table XXVII. The coefficients $\overline{L}_{p,q}$ are tabulated in Table XXX.

The remaining terms of rank one which are of first order only, namely, $n\delta z_1^{(1)} - (n\delta z_1^{(1)})$, are given by the first of Z 81, eqs. (137), in which $\overline{W}_1$, $\overline{W}_2$, $[\overline{W}_2]$, can be written by inspection from Tables XVII, XVIII, XIX, XXII*a*. The function is tabulated in Table XXXI.

The remaining terms of first order in $n\delta z_2$ and $n\delta z_3$ are given by the sum of Z 82, eqs. (139) and (140). The function

$$n\delta z_2^{(1)} - (n\delta z_2^{(1)}) + n\delta z_3^{(1)} - (n\delta z_3^{(1)})$$

is given in Table XXXII.

These developments complete $n\delta z^{(1)}$ within the limits of the tables, and we next consider $n\delta z^{(2)}$. We shall limit ourselves to functions in which the lowest rank is the first or second. Consequently, $n\delta z_3$ contributes nothing.

Any function of second order in the mass and first rank must contain the factor $\frac{m'^2}{w^2}$ and in the given $F(\vartheta, \varepsilon)$ this factor occurs only in $\overline{W}_1^{(2)}$. We have, therefore, by Z 80, eq. (131_1), for one part of $n\delta z_1^{(2)}$, indicated by parentheses,

$$(n\delta z_1^{(2)}) = \int \{(1 - e\cos\varepsilon)\,\overline{W}_1^{(2)} - [(1 - e\cos\varepsilon)\,\overline{W}_1^{(2)}]\}\,d\varepsilon$$

This function is tabulated in Table XXXIII.

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TABLE XXX. Unit=1″

	n	0	1	2	3	4	5	6	7	8	9	10
	$\bar{L}_{0\cdot0}(n.-n)$		− 586.66	− 237.94	+ 225.33	+ 51.36	+ 17.14	[+ 6.81]	+ 2.99	+ 1.38	+ 0.70	+ 0.36
	$\bar{L}_{1\cdot0}(n+1.-n)$	+ 225.4	+ 116.5	+ 66.4	− 137.5	− 28.3	− 6.5	− 0.8	+ 0.6	+ 0.8	+ 0.6	+ 0.5
	$\bar{L}_{1\cdot0}(n-1.-n)$	− 225.4	− 1608.8		+ 11669.6	+ 1185.8	+ 1506.7	+ 396.2	− 149.9	− 66.5	− 32.3	− 16.5
	$\bar{L}_{0\cdot1}(n.-n+1)$	− 1023.6	− 185.7	− 61.5	− 88.4	− 55.4	− 32.5	− 18.9	− 11.1	− 6.5	− 3.9	− 2.4
	$\bar{L}_{0\cdot1}(n.-n-1)$	+ 1023.6		− 16751.2	− 1989.6	+ 2255.6	+ 605.5	+ 233.3	+ 105.2	+ 51.8	+ 26.9	+ 14.5
	$\bar{L}_{2\cdot0}(n+2.-n)$	+ 43	+ 258	+ 50	+ 6	− 2	− 2	− 1	0	0	0	0
	$\bar{L}_{2\cdot0}(n.-n)$		−10130	− 1514	− 4151	− 972	+ 626	+ 75	− 18	− 32	− 28	− 21
	$\bar{L}_{2\cdot0}(n-2.-n)$	− 43	− 415	+ 776	− 3992		− 58550	− 5885	+ 8504	+ 2410	+ 981	+466
	$\bar{L}_{1\cdot1}(n+1.-n+1)$	− 821	− 368	− 64	+ 27	+ 21	+ 8	+ 1	− 2	− 4	− 3	− 3
	$\bar{L}_{1\cdot1}(n-1.-n+1)$	− 7178		+ 2160	− 1047	+ 3345	+ 1444	+ 767	+ 440	+ 262	+ 160	+ 99
	$\bar{L}_{1\cdot1}(n+1.-n-1)$	+ 7178	+ 2559	+ 4711	+ 1164	− 1096	− 191	− 10	+ 30	+ 34	+ 29	+ 22
	$\bar{L}_{1\cdot1}(n-1.-n-1)$	+ 821	− 1797	+ 12244		+ 170804	+ 19042	− 25415	− 7322	− 3025	− 1459	−765
	$\bar{L}_{0\cdot2}(n.-n+2)$	− 408	+ 1129	+ 188	+ 56	+ 33	+ 27	+ 21	+ 16	+ 12	+ 9	+ 6
	$\bar{L}_{0\cdot2}(n.-n)$		−11930	+ 930	− 3531	− 1772	− 1018	− 614	− 379	− 238	− 151	− 96
	$\bar{L}_{0\cdot2}(n.-n-2)$	+ 408	+ 3418		− 123754	− 15453	+ 18945	+ 5553	+ 2333	+ 1142	+ 608	+340
	$\bar{L}_{0\cdot0}(n+1.-n+1)+\sigma$	+ 587	+ 512	+ 237	+ 121	+ 65	+ 36	+ 20	+ 12	+ 7	+ 4	+ 2
	$\bar{L}_{0\cdot0}(n-1.-n-1)-\sigma$	− 587	+ 433	+ 1612		− 6382	− 582	+ 621	+ 150	+ 53	+ 22	+ 10
	$\bar{L}_{0\cdot0}(n+1.-n-1)+\delta$	+ 3570	+ 1201	− 1615	− 450	− 177	− 81	− 40	− 21	− 12	− 7	− 4
	$\bar{L}_{0\cdot0}(n-1.-n+1)-\delta$	− 3570		+ 6552	+ 725	− 939	− 244	− 91	− 40	− 19	− 10	− 5
Factor w	$\bar{L}_{0\cdot0}(n.-n)$		+ 2413.6	+ 1075.1	− 1768.5	− 379.8	− 130.8	− 54.8	− 25.6	− 12.8	− 6.7	− 3.6
	$\bar{L}_{1\cdot0}(n+1.-n)$	− 737	+ 1084	− (187)	+ 1021	+ 200	+ 46	+ 4	− 7	− 8	− 7	− 5
	$\bar{L}_{1\cdot0}(n-1.-n)$	+ 737	+ 6180		− 70526	− 9499	+ 18752	+ 4232	+ 1558	+ 701	+ 351	+187
	$\bar{L}_{0\cdot1}(n.-n+1)$	+ 3420	+ 766	− 275	+ 317	+ 275	+ 193	+ 128	+ 83	+ 54	+ 35	+ 22
	$\bar{L}_{0\cdot1}(n.-n-1)$	− 3420		+ 91438	+ 14703	− 26491	− 6018	− 2248	− 1027	− 521	− 282	−159
	$\bar{L}_{2\cdot0}(n+2.-n)$	− 352	− 1472	− (282)	− 66	− 2	+ 8	+ 5	0	− 2	− 4	
	$\bar{L}_{2\cdot0}(n.-n)$		+58689	[(+ 5580)]	+ 16299	+ (7168)	− 8328	− 998	[+ 126]	+ 324	+ 288	
	$\bar{L}_{2\cdot0}(n-2.-n)$	+ 352	+ 1153	[(− 3214)]	− 34026		+542444	+ 67140	−144713	−33619	−12916	
	$\bar{L}_{1\cdot1}(n+1.-n+1)$	+ 2329	+ 2072	+ 929	+ 85	− 27	− 11	+ 16	+ 32	+ 40	+ 36	
	$\bar{L}_{1\cdot1}(n-1.-n+1)$	+37422		− 32828	+ 3112	[− 25186]	− 10886	− 6124	− 3752	− 2386	− 1553	
	$\bar{L}_{1\cdot1}(n+1.-n-1)$	−37422	[(−14117)]	− 10066	− (7326)	+ 13010	+ 2031	+ 144	− 295	− 358	− 315	
	$L_{1\cdot1}(n-1.-n-1)$	− 2329	[(+ 6421)]	+109204		−1484032	−206260	+414666	+ 96717	+37531	+17877	

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	$\overline{L}_{0,2}(n.-n+2)$	+ 1159	− 3828	− 1093	− 361	− 196	− 156	− 135	− 116	− 95	− 77				
	$\overline{L}_{0,2}(n.-n)$		+67536	− 396	+ 22629	+ 11805	+ 7238	+ 4739	+ 3153	+ 2120	+ 1433				
	$\overline{L}_{0,2}(n.-n-2)$	− 1159	−42030		+1004134	+ 157973	−295725	− 69195	− 27124	−13065	− 7034				
	$\overline{L}_{0,0}(n+1.-n+1)+\sigma$	− 1958	− 2242	− 1276	− 751	− 453	− 278	− 172	− 108	− 68	− 42				
	$\overline{L}_{0,0}(n-1.-n-1)-\sigma$	+ 1958	− 1481	− 15578		+ 63070	+ 7179	− 11156	− 2227	− 745	− 307				
	$\overline{L}_{0,0}(n+1.-n-1)+\delta$	−19300	− 7328	+ 15213	+ 3975	+ 1590	+ 756	+ 400	+ 218	+ 125	+ 74				
	$\overline{L}_{0,0}(n-1.-n+1)-\delta$	+19300		− 36180	− 5178	+ 9908	+ 2454	+ 935	+ 427	+ 215	+ 116				
Factor w^2	$L_{0,0}(n.-n)$		− 5214	− 2489	+ 7952	+ 1488	+ 510	+ 221	+ 106	+ 56	+ 31	+ 18			
	$\overline{L}_{1,0}(n+1.-n)$	+ 1164	− 3601	+ (397)	− 5808	− 1227	− 390	− 127	− 34	+ 1	+ 12				
	$\overline{L}_{1,0}(n-1.-n)$	− 1164	−12114		+ 366108	+ 42675	[−136738]	− 24224	− 8325	− 3696	− 1873				
	$\overline{L}_{0,1}(n.-n+1)$	− 6331	− 1504	+ 2232	− 328	− 601	− 525	− 399	− 291	− 207	− 145				
	$\overline{L}_{0,1}(n.-n-1)$	+ 6331		−466174	− 60481	+ 187295	+ 33010	+ 11447	+ 5151	+ 2643	+ 1472				

The coefficients in parentheses are functions of the coefficients in Table XXVII.

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TABLE XXXI.

$n\delta z_1^{(1)} - (n\delta z_1^{(1)})$ Unit $= 1''$

	sin	w^{-1}		
		w^0	w	w^2
	$\varepsilon+2\vartheta+2\Delta$	+ 294.89	− 740.6	+ 734
η	ε			
	$\varepsilon+4\vartheta+4\Delta$	− 839.5	+ 3495	− 6224
	$2\varepsilon+2\vartheta+2\Delta$	− 147.4	+ 517	− 737
η'	$\varepsilon \quad + \Delta$			
	$\varepsilon+4\vartheta+3\Delta$	+ 1229.8	− 4069	+ 5671
η^2	$-\varepsilon+2\vartheta+2\Delta$	+ 784	(− 3570)	(+ 10522)
	$\varepsilon+2\vartheta+2\Delta$	− 202	(− 1657)	(+ 13183)
	$\varepsilon+6\vartheta+6\Delta$	+ 2940	(− 17009)	[(+ 43527)]
	$2\varepsilon+4\vartheta+4\Delta$	+ 415	(− 2587)	(+ 6440)
	2ε		− 192	+ 705
$\eta\,\eta'$	$-\varepsilon+2\vartheta+\Delta$	− 2386	(+ 11567)	(+ 37527)
	$\varepsilon+2\vartheta+3\Delta$	+ 1492	(− 968)	(− 12562)
	$\varepsilon+2\vartheta+\Delta$	− 1962	(+ 9257)	(− 23263)
	$\varepsilon+6\vartheta+5\Delta$	− 8658	(+ 42767)	(− 92732)
	$2\varepsilon+4\vartheta+3\Delta$	− 615	(+ 3264)	(− 6905)
	$2\varepsilon \quad + \Delta$		+ 142	− 605
η'^2	$-\varepsilon+2\vartheta$	+ 1634	− 7081	+ 16199
	$\varepsilon+2\vartheta+2\Delta$	− 861	− 3794	+ 22127
	$\varepsilon+6\vartheta+4\Delta$	+ 6349	− 25753	+ 45318
f^2	$-\varepsilon+2\vartheta+\Delta-\Sigma$	+ 866	− 4260	+ 10988
	$\varepsilon+6\vartheta+5\Delta-\Sigma$	+ 260	− 1674	+ 5101
	$\varepsilon+2\vartheta+2\Delta$	− 2677	+ 12681	− 30930
η^3	$\varepsilon+4\vartheta+4\Delta$	+ 5907	− 11149	
	$-\varepsilon+4\vartheta+4\Delta$	− 269	+ 5158	
	$\varepsilon+8\vartheta+8\Delta$	−11300	+ 76249	
$\eta^2\eta'$	$\varepsilon+4\vartheta+5\Delta$	−11449	+ 42212	
	$\varepsilon+4\vartheta+3\Delta$	−11270	+ 951	
	$-\varepsilon+4\vartheta+3\Delta$	+ 1744	− 23941	
	$\varepsilon+8\vartheta+7\Delta$	+50005	−304611	
$\eta\,\eta'^2$	$\varepsilon+4\vartheta+4\Delta$	+26091	− 71730	
	$\varepsilon+4\vartheta+2\Delta$	+ 3985	+ 16118	
	$-\varepsilon+4\vartheta+2\Delta$	− 3137	+ 35021	
	$\varepsilon+8\vartheta+6\Delta$	−73583	+400009	
η'^3	$\varepsilon+4\vartheta+3\Delta$	−13756	+ 22165	
	$-\varepsilon+4\vartheta+\Delta$	+ 3317	− 18452	
	$\varepsilon+8\vartheta+5\Delta$	+36006	−172164	
$f^2\eta$	$\varepsilon+4\vartheta+3\Delta-\Sigma$	− 1707	+ 13125	
	$-\varepsilon+4\vartheta+3\Delta-\Sigma$	− 2112	+ 15096	
	$\varepsilon+8\vartheta+7\Delta-\Sigma$	− 2381	+ 18919	
	$\varepsilon+4\vartheta+4\Delta$	+14204	− 88026	
$f^2\ \eta'$	$\varepsilon+4\vartheta+4\Delta-\Sigma$	− 554	+ 140	
	$-\varepsilon+4\vartheta+2\Delta-\Sigma$	+ 3545	− 22885	
	$\varepsilon+8\vartheta+6\Delta-\Sigma$	+ 3827	− 27870	
	$\varepsilon+4\vartheta+3\Delta$	−17503	+ 99584	
	$+(\vartheta-\vartheta_0)\cos$			
η	ε	− 767.7	+ 2821	− 5210
η'	$\varepsilon+\Delta$	+ 570.0	− 2421	+ 4950
η^2	2ε	− 384	+ 1410	− 2605
$\eta\,\eta'$	$2\varepsilon+\Delta$	+ 285	− 1211	+ 2475
η^3	ε	− 6624	+ 47448	
$\eta^2\eta'$	$\varepsilon+\Delta$	[+17970]	[−120603]	
	$-\varepsilon+\Delta$	+ 8984	− 60301	
$\eta\,\eta'^2$	$\varepsilon+2\Delta$	−10478	+ 70250	
	ε	−25564	+157424	
η'^3	$\varepsilon+\Delta$	+15678	− 94846	
$f^2\eta$	$\varepsilon+\Delta \quad +\Sigma$	−22012	+121258	−359162
	ε	+25565	−157424	[+511232]
$f^2\ \eta'$	$\varepsilon \quad +\Sigma$	+12048	− 76364	+251640
	$\varepsilon+\Delta$	−23524	+150306	−498328
		m'		

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TABLE XXXII.

$n\delta z_2^{(1)} - (n\delta z_2^{(1)})$

$+ n\delta z_3^{(1)} - (n\delta z_3^{(1)})$ Unit=1″.

	Sin	w^0	w
	$\varepsilon+2\vartheta+2\Delta$	− 294.9	+ 1036
η	ε	+ 384	− 1410
	$\varepsilon+4\vartheta+4\Delta$	+ 1679	− 10348
	$2\varepsilon+2\vartheta+2\Delta$	− 74	+ 149
η'	$\varepsilon \quad + \Delta$	− 285	+ 1211
	$\varepsilon+4\vartheta+3\Delta$	− 2460	+ 13057
η^2	$-\varepsilon+2\vartheta+2\Delta$	− 101	− 883
	$\varepsilon+2\vartheta+2\Delta$	− 978	+ 6459
	$\varepsilon+6\vartheta+6\Delta$	− 8820	(+ 77487)
	$2\varepsilon+4\vartheta+4\Delta$	+ 424	− 1332
	2ε	[+ 96]	[− 352]
$\eta\,\eta'$	$-\varepsilon+2\vartheta+\Delta$	− 2068	+ 8418
	$\varepsilon+2\vartheta+3\Delta$	− 1492	+ 2460
	$\varepsilon+2\vartheta+\Delta$	+ 2280	− 12618
	$\varepsilon+6\vartheta+5\Delta$	+ 25974	(−206223)
	$2\varepsilon+4\vartheta+3\Delta$	− 615	+ 1420
	$2\varepsilon \quad + \Delta$	[− 71]	[+ 303]
η'^2	$-\varepsilon+2\vartheta$	+ 1634	− 5447
	$\varepsilon+2\vartheta+2\Delta$	+ 861	+ 2933
	$\varepsilon+6\vartheta+4\Delta$	− 19047	[+134400]
j^2	$-\varepsilon+2\vartheta+\Delta-\Sigma$	+ 866	− 3394
	$\varepsilon+6\vartheta+5\Delta-\Sigma$	− 780	+ 7362
	$\varepsilon+2\vartheta+2\Delta$	+ 2677	− 15358
η^3	$\varepsilon+4\vartheta+4\Delta$	− 5098	
	$-\varepsilon+4\vartheta+4\Delta$	+ 4499	
	$\varepsilon+8\vartheta+8\Delta$	+ 45200	
$\eta^2\eta'$	$\varepsilon+4\vartheta+5\Delta$	+ 22898	
	$\varepsilon+4\vartheta+3\Delta$	+ 5322	
	$-\varepsilon+4\vartheta+3\Delta$	− 11270	
	$\varepsilon+8\vartheta+7\Delta$	−200020	
$\eta\,\eta'^2$	$\varepsilon+4\vartheta+4\Delta$	− 52182	
	$\varepsilon+4\vartheta+2\Delta$	+ 2712	
	$-\varepsilon+4\vartheta+2\Delta$	+ 4408	
	$\varepsilon+8\vartheta+6\Delta$	+294332	
η'^3	$\varepsilon+4\vartheta+3\Delta$	+ 27512	
	$-\varepsilon+4\vartheta+\Delta$	+ 6634	
	$\varepsilon+8\vartheta+5\Delta$	−144024	
$j^2\eta$	$\varepsilon+4\vartheta+3\Delta-\Sigma$	+ 4022	
	$-\varepsilon+4\vartheta+3\Delta-\Sigma$	− 3616	
	$\varepsilon+8\vartheta+7\Delta-\Sigma$	+ 9524	
	$\varepsilon+4\vartheta+4\Delta$	− 28408	
$j^2\,\eta'$	$\varepsilon+4\vartheta+4\Delta-\Sigma$	+ 1108	
	$-\varepsilon+4\vartheta+2\Delta-\Sigma$	+ 7090	
	$\varepsilon+8\vartheta+6\Delta-\Sigma$	− 15308	
	$\varepsilon+4\vartheta+3\Delta$	+ 35006	
		m'	

The coefficients in parentheses differ from v. Zeipel's values because they contain additional terms. See p. 134.

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The remaining terms in the differential equation for $n\delta z_1^{(2)}$ are, by eq. (143),

$$\frac{\delta}{\delta\varepsilon}\{n\delta z_1^{(2)} - (n\delta z_1^{(2)})\} = (1-e\cos\varepsilon)\left\{\overline{W}_2'^{(2)} + [\overline{W}_2]^{(2)} - \frac{3}{4}\left(\overline{W}_1^{(1)} - \frac{1}{3}\Xi_1^{(1)}\right)\left(\overline{W}_1^{(1)} + \frac{1}{9}\Xi_1^{(1)}\right)\right\}$$
$$-\left[(1-e\cos\varepsilon)\left\{\overline{W}_2'^{(2)} + [\overline{W}_2]^{(2)} - \frac{3}{4}\left(\overline{W}_1^{(1)} - \frac{1}{3}\Xi_1^{(1)}\right)\left(\overline{W}_1^{(1)} + \frac{1}{9}\Xi_1^{(1)}\right)\right\}\right]$$

all the terms of which are of the second order whose lowest rank is the second. They therefore contain the factor $\frac{m'^2}{w^2}$.

To obtain $n\delta z_2^{(2)}$ it is necessary to return to eqs. (124)–(130) and make developments for terms of the second order similar to those for first order. The resulting differential equation is:

$$\frac{\delta}{\delta\varepsilon}n\delta z_2^{(2)} = \frac{(1-e\cos\varepsilon)}{2}\{n\delta z_1^{(1)} - (n\delta z_1^{(1)})\}\frac{\delta}{\delta\vartheta}\overline{W}_1^{(1)} - (1-e\cos\varepsilon)\eta\ w\ \sin\varepsilon\frac{\delta}{\delta\vartheta}\overline{W}_1^{(2)}$$
$$-\left[\frac{(1-e\cos\varepsilon)}{2}\{n\delta z_1^{(1)} - (n\delta z_1^{(1)})\}\frac{\delta}{\delta\vartheta}\overline{W}_1^{(1)} - (1-e\cos\varepsilon)\eta\ w\ \sin\varepsilon\frac{\delta}{\delta\vartheta}\overline{W}_1^{(2)}\right]$$
$$-\frac{1}{2}[(1-e\cos\varepsilon)\overline{W}_1^{(1)}]\frac{\delta}{\delta\vartheta}\{n\delta z_1 - (n\delta z_1^{(1)})\} - \frac{w}{2}\frac{\delta}{\delta\vartheta}(n\delta z_1^{(2)}).$$

The sum of the last two equations, when integrated, gives the terms of second order having the factor $\frac{m'^2}{w^2}$. It has been shown by v. Zeipel through computation and we have shown analytically that

$$\overline{W}_2'^{(2)} + \frac{1}{2}\{n\delta z_1^{(1)} - (n\delta z_1^{(1)})\}\frac{\delta}{\delta\vartheta}\overline{W}_1^{(1)} = \eta\ w\ \sin\varepsilon\frac{\delta}{\delta\vartheta}\overline{W}_1^{(2)}$$

and

$$[(1-e\cos\varepsilon)\overline{W}_1^{(1)}]\frac{\delta}{\delta\vartheta}\{n\delta z_1^{(1)} - (n\delta z_1^{(1)})\} + w\frac{\delta}{\delta\vartheta}(n\delta z_1^{(2)}) = 0.$$

Therefore,

$$\frac{\delta}{\delta\varepsilon}\{n\delta z_1^{(2)} - (n\delta z_1^{(2)}) + n\delta z_2^{(2)}\} = (1-e\cos\varepsilon)\left\{[\overline{W}_2]^{(2)} - \frac{3}{4}\left(\overline{W}_1^{(1)} - \frac{1}{3}\Xi_1^{(1)}\right)\left(\overline{W}_1^{(1)} + \frac{1}{9}\Xi_1^{(1)}\right)\right\}$$
$$-\left[(1-e\cos\varepsilon)\left\{[\overline{W}_2]^{(2)} - \frac{3}{4}\left(\overline{W}_1^{(1)} - \frac{1}{3}\Xi_1^{(1)}\right)\left(\overline{W}_1^{(1)} + \frac{1}{9}\Xi_1^{(1)}\right)\right\}\right]$$

The integral is tabulated in Table XXXIV.

Summarizing, we have included first order terms in

$$n\delta z_1 + n\delta z_2 + n\delta z_3$$

given by tables XXX, XXXI, XXXII and second order terms in

$$n\delta z_1 + n\delta z_2$$

given by Tables XXXIII and XXXIV. The addition of Tables XXX–XXXIV gives the short period terms in $n\delta z$, or, the function

$$n\delta z - [n\delta z]$$

which is tabulated in Table XXXV.

Returning now to the differential equation for ϑ, the evaluation of $F(\vartheta, \varepsilon)$ and its derivatives in *Z* 78, eq. (127) gives *Z* 91, eq. (146). The variable does not appear; $\frac{d\vartheta}{d\varepsilon}$ is a function of ϑ alone Therefore the function is of long period. The integration is one step in the determination of $[n\delta z]$, the long period terms in the perturbations of the mean anomaly.

The function $[(1-e\cos\varepsilon)\overline{W}]$ is tabulated in Table XXIX*b*.

The function $\left[(1-e\cos\varepsilon)\left(\overline{W} - \frac{1}{3}\Xi\right)\left(\overline{W} + \frac{1}{9}\Xi\right)\right]$, computed from Tables XXIX*a* and XXIX*c*, is given in Table XXXVI.

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The function $\left[(1-e\cos\varepsilon)(\theta_1+\theta_2+\theta_3)\frac{\delta\overline{W}}{\delta\vartheta}\right]$ is computed as follows:

First, $$\theta-\vartheta=\theta_1+\theta_2+\theta_3$$

is given by *Z* 93, eq. (150) by means of Table XXXV, and $\frac{\delta\overline{W}}{\delta\vartheta}$ is readily written by inspection of Table XXIX*a*. Performing the indicated multiplications and retaining only the terms which are independent of ε, we have the required function as tabulated in Table XXXVII.

By eq. (146), the sum of Tables XXIX*b*, XXXVI, and XXXVII, multiplied by the factor $\frac{1-w}{w}$, gives $\Phi(\vartheta)$, tabulated in Table XXXVIII.

TABLE XXXIII.

$(n\partial z_1^{(2)})$ Unit=1″

	Sin	w^{-3}		
		w^0	w	w^2
η	$\varepsilon+4\vartheta+4\Delta$	− 0.316	+ 1.59	− 3.6
η'	$\varepsilon+4\vartheta+3\Delta$	+ 0.114	− 0.67	+ 1.8
η^2	$-\varepsilon+2\vartheta+2\Delta$	+ 2.62	− 16.8	
	$\varepsilon+2\vartheta+2\Delta$	+ 4.42	− 28.4	
	$\varepsilon+6\vartheta+6\Delta$	+ 1.80	− 11.7	
	$2\varepsilon+4\vartheta+4\Delta$	+ 0.16	− 0.8	+ 1.8
$\eta\,\eta'$	$-\varepsilon+2\vartheta+\Delta$	− 6.18	+ 36.9	
	$\varepsilon+2\vartheta+3\Delta$	− 1.90	+ 13.6	
	$\varepsilon+2\vartheta+\Delta$	− 5.57	+ 32.8	
	$\varepsilon+6\vartheta+5\Delta$	− 3.95	+ 23.6	
	$2\varepsilon+4\vartheta+3\Delta$	− 0.06	+ 0.3	− 0.9
η'^2	$-\varepsilon+2\vartheta$	+ 4.04	− 21.4	
	$\varepsilon+2\vartheta+2\Delta$	+ 2.12	− 14.4	
	$\varepsilon+6\vartheta+4\Delta$	+ 1.90	− 10.8	
f^2	$-\varepsilon+2\vartheta+\Delta-\Sigma$	+ 0.22	− 1.6	
	$\varepsilon+6\vartheta+5\Delta-\Sigma$	+ 0.07	− 0.5	
	$+(\vartheta-\vartheta_0)\cos$			
η	ε	− 1.265	+ 6.35	−14.3
η'	$\varepsilon+\Delta$	+ 0.455	−. 2.69	+ 7.2
η^2	2ε	+ 0.63	− 3.2	+ 7.2
$\eta\,\eta'$	$2\varepsilon+\Delta$	− 0.23	+ 1.3	− 3.6
η^2	ε	−23.8	+222	
$\eta^2\eta'$	$\varepsilon+\Delta$	+72.9	−569	
	$-\varepsilon+\Delta$	+36.5	−285	
$\eta\,\eta'^2$	$\varepsilon+2\Delta$	−55.2	+375	
	ε	−87.3	+653	
η'^3	$\varepsilon+\Delta$	+69.9	−439	
$f^2\eta$	$\varepsilon+\Delta+\Sigma$	− 9.9	+ 77	
	ε	+23.1	−166	
$f^2\,\eta'$	$\varepsilon\quad+\Sigma$	+ 5.2	− 45	
	$\varepsilon+\Delta$	−14.8	+112	
		m'^2		

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TABLE XXXIV.

$\{n\delta z_1^{(2)} - (n\delta z_1^{(2)})\} + n\delta z_2^{(2)}$ Unit=1″

	Sin	w^{-2}		
		w^0	w	w^1
	$\varepsilon+2\vartheta+2\Delta$	− 0.614	+ 4.06	−10.3
	$2\varepsilon+4\vartheta+4\Delta$	− 0.079	+ 0.40	
η	ε	− 0.74	+ 3.7	
	$\varepsilon+4\vartheta+4\Delta$	+ 1.74	−18.1	
	$2\varepsilon+2\vartheta+2\Delta$	+ 0.31	− 2.0	
	$2\varepsilon+6\vartheta+6\Delta$	+ 0.45	− 2.9	
η'	$\varepsilon+\Delta$	+ 0.30	− 1.8	
	$\varepsilon+4\vartheta+3\Delta$	− 4.26	+32.0	
	$2\varepsilon+6\vartheta+5\Delta$	− 0.66	+ 3.8	
η^2	$-\varepsilon+2\vartheta+2\Delta$	− 6.4		
	$\varepsilon+2\vartheta+2\Delta$	+ 6.4		
	$\varepsilon+6\vartheta+6\Delta$	+ 5.1		
	$2\varepsilon+4\vartheta+4\Delta$	− 1.4		
	$2\varepsilon+8\vartheta+8\Delta$	− 2.2		
$\eta\,\eta'$	$-\varepsilon+2\vartheta+\Delta$	+12.0		
	$\varepsilon+2\vartheta+3\Delta$	− 0.9		
	$\varepsilon+2\vartheta+\Delta$	− 8.5		
	$\varepsilon+6\vartheta+5\Delta$	−11.8		
	$2\varepsilon+4\vartheta+3\Delta$	+ 3.4		
	$2\varepsilon+4\vartheta+5\Delta$	− 0.8		
	$2\varepsilon+8\vartheta+7\Delta$	+ 6.5		
η'^2	$-\varepsilon+2\vartheta$	− 5.1		
	$\varepsilon+2\vartheta+2\Delta$	+ 1.3		
	$\varepsilon+6\vartheta+4\Delta$	+ 6.4		
f^2	$-\varepsilon+2\vartheta+\Delta-\Sigma$	− 0.3		
	$\varepsilon+6\vartheta+5\Delta-\Sigma$	+ 0.3		
	$2\varepsilon+4\vartheta+4\Delta$	+ 1.4		
	$+(\vartheta-\vartheta_0)\cos$			
η	ε	− 1.02	− 8.4	
	$\varepsilon+4\vartheta+4\Delta$	− 0.78	+ 6.0	
	$2\varepsilon+2\vartheta+2\Delta$	+ 0.41	− 2.5	
η'	$\varepsilon+\Delta$	− 3.25	+30.1	
	$\varepsilon+4\vartheta+3\Delta$	+ 0.58	− 4.8	
	$2\varepsilon+2\vartheta+3\Delta$	− 0.31	+ 2.1	
η^2	$-\varepsilon+2\vartheta+2\Delta$	+ 3.6		
	$\varepsilon+2\vartheta+2\Delta$	+ 1.1		
	2ε	+ 0.5		
	$2\varepsilon+4\vartheta+4\Delta$	− 0.8		
$\eta\,\eta'$	$-\varepsilon+2\vartheta+\Delta$	− 3.4		
	$2\varepsilon+\Delta$	+ 1.6		
	$(\vartheta-\vartheta_0)^2\sin$			
η	ε	− 0.36	+ 2.6	
η'	$\varepsilon+\Delta$	+ 0.27	− 2.1	
		m'^2		

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TABLE XXXV.

Logarithmic. $n\delta z - [n\delta z]$ Unit=1″.

	Sin	w^{-3}	w^{-2}	w^{-1}	w^0	w	w^2
$\eta\,\eta'$	$\frac{1}{2}\varepsilon+\vartheta$				4.1570	4.8741_n	
	$\frac{1}{2}\varepsilon+\vartheta+\Delta$				2.7684$_n$	3.3827	3.7172$_n$
η^2	$\frac{1}{2}\varepsilon+\vartheta+\Delta$				4.0056$_n$	4.7686	
η'^2	$\frac{1}{2}\varepsilon+\vartheta+\Delta$				4.0766$_n$	4.8295	
j^2	$\frac{1}{2}\varepsilon+\vartheta+\Delta$				4.1365	4.8738_n	
$\eta\,\eta'$	$\frac{1}{2}\varepsilon+\vartheta+2\Delta$				3.3345	4.5162_n	
η'	$\frac{1}{2}\varepsilon+3\vartheta+2\Delta$				4.2240$_n$	4.9611	5.6685$_n$
η	$\frac{1}{2}\varepsilon+3\vartheta+3\Delta$				4.0671	4.8483_n	5.5636
η'^2	$\frac{1}{2}\varepsilon+5\vartheta+3\Delta$				5.0926$_n$	6.0018	
$\eta\,\eta'$	$\frac{1}{2}\varepsilon+5\vartheta+4\Delta$				5.2325	6.1714_n	
η^2	$\frac{1}{2}\varepsilon+5\vartheta+5\Delta$				4.7675$_n$	5.7344	
j^2	$\frac{1}{2}\varepsilon+5\vartheta+4\Delta-\Sigma$				3.8050$_n$	4.7998	
η'	$-\frac{1}{2}\varepsilon+\vartheta$				3.3112	3.8350_n	4.1355
η	$-\frac{1}{2}\varepsilon+\vartheta+\Delta$				3.2065$_n$	3.7910	4.0833$_n$
η'^2	$-\frac{1}{2}\varepsilon+3\vartheta+\Delta$				3.5338	4.6236_n	
$\eta\,\eta'$	$-\frac{1}{2}\varepsilon+3\vartheta+2\Delta$				4.0879	5.0382	
η^2	$-\frac{1}{2}\varepsilon+3\vartheta+3\Delta$				3.6012$_n$	4.5318_n	
j^2	$-\frac{1}{2}\varepsilon+3\vartheta+2\Delta-\Sigma$				3.2074	4.1925_n	
η	ε		9.868$_n$	0.5689	2.922	3.4600_n	3.3670
η'	$\varepsilon+\Delta$		9.482	0.2533$_n$	2.673$_n$	3.2959	3.1772$_n$
$\eta\,\eta'$	$\varepsilon+2\vartheta+\Delta$	0.746$_n$	[1.384]	3.2927$_n$	[4.14906]	[4.6990$_n$]	
	$\varepsilon+2\vartheta+2\Delta$		9.788$_n$	2.47560	3.10847$_n$	3.4540	[3.3960$_n$]
η^2	$\varepsilon+2\vartheta+2\Delta$	0.645	[1.342$_n$]	2.305$_n$	[3.6179$_n$]	[4.4018]	
η'^2	$\varepsilon+2\vartheta+2\Delta$	0.326	1.119$_n$	2.935$_n$	3.3017$_n$	[4.39206]	
j^2	$\varepsilon+2\vartheta+2\Delta$			3.4276$_n$	4.23764	4.76933$_n$	
$\eta\,\eta'$	$\varepsilon+2\vartheta+3\Delta$	0.28$_n$	1.102	3.1738	[3.5449$_n$]	[3.8446$_n$]	
$\eta\,\eta'^2$	$\varepsilon+4\vartheta+2\Delta$			3.6004	4.27485		
η'	$\varepsilon+4\vartheta+3\Delta$	9.057	0.692$_n$	3.10161	3.9302$_n$	4.52415	[4.78162$_n$]
$\eta^2\eta'$	$\varepsilon+4\vartheta+3\Delta$			4.0519$_n$	3.7975		
η'^2	$\varepsilon+4\vartheta+3\Delta$			4.1385$_n$	4.6961		
$j^2\,\eta'$	$\varepsilon+4\vartheta+3\Delta$			4.2431$_n$	5.1290		
η	$\varepsilon+4\vartheta+4\Delta$	9.500$_n$	0.522	2.9351$_n$	3.8035	4.41616_n	4.63017
η^2	$\varepsilon+4\vartheta+4\Delta$			3.7714	4.2108$_n$		
$\eta\,\eta'^2$	$\varepsilon+4\vartheta+4\Delta$			4.4165	5.0931$_n$		
$j^2\eta$	$\varepsilon+4\vartheta+4\Delta$			4.1524	5.0661$_n$		
$\eta^2\eta'$	$\varepsilon+4\vartheta+5\Delta$			4.0588$_n$	4.8136		
$j^2\eta$	$\varepsilon+4\vartheta+3\Delta-\Sigma$			3.2322$_n$	4.2342		
$j^2\,\eta'$	$\varepsilon+4\vartheta+4\Delta-\Sigma$			2.744$_n$	[3.0962]		
η'^2	$\varepsilon+6\vartheta+4\Delta$	0.28	[0.64$_n$]	3.8027	4.77998$_n$	[5.52852]	
$\eta\,\eta'$	$\varepsilon+6\vartheta+5\Delta$	0.596$_n$	[1.070]	3.9374$_n$	[4.94342]	[5.70347$_n$]	
η^2	$\varepsilon+6\vartheta+6\Delta$	0.255	[0.8$_n$]	3.4684	[4.50125$_n$]	[5.27451]	
j^2	$\varepsilon+6\vartheta+5\Delta-\Sigma$	8.8	[9.3$_n$]	2.415	3.4823$_n$	[4.2931]	
η'^2	$\varepsilon+8\vartheta+5\Delta$			4.5564	5.4999_n		
$\eta\,\eta'^2$	$\varepsilon+8\vartheta+6\Delta$			4.8668$_n$	5.8416		
$\eta^2\eta'$	$\varepsilon+8\vartheta+7\Delta$			4.6990	5.7030$_n$		
η^3	$\varepsilon+8\vartheta+8\Delta$			4.0531$_n$	5.0844		
$j^2\,\eta'$	$\varepsilon+8\vartheta+6\Delta-\Sigma$			3.5829	4.6352$_n$		
$j^2\eta$	$\varepsilon+8\vartheta+7\Delta-\Sigma$			3.3768$_n$	4.4540		
η'^2	$-\varepsilon+2\vartheta$	0.606	[1.422$_n$]	3.2132	3.6657$_n$	3.9260	
$\eta\,\eta'$	$-\varepsilon+2\vartheta+\Delta$	0.791$_n$	[1.690]	3.3777$_n$	3.8866	4.72168	
η^2	$-\varepsilon+2\vartheta+2\Delta$	0.418	[1.365$_n$]	2.894	[3.4616$_n$]	3.8078	
j^2	$-\varepsilon+2\vartheta+\Delta-\Sigma$	9.34	0.28$_n$	2.938	3.4714$_n$	3.7862	
η'^2	$-\varepsilon+4\vartheta+\Delta$			3.5208	4.07255		
$\eta\,\eta'^2$	$-\varepsilon+4\vartheta+2\Delta$			3.4965$_n$	4.59582		
$\eta^2\eta'$	$-\varepsilon+4\vartheta+3\Delta$			3.2416	4.5467$_n$		
η^3	$-\varepsilon+4\vartheta+4\Delta$			2.430$_n$	3.9848		
$j^2\,\eta'$	$-\varepsilon+4\vartheta+2\Delta-\Sigma$			3.5496	4.19852$_n$		
$j^2\eta$	$-\varepsilon+4\vartheta+3\Delta-\Sigma$			3.3247$_n$	4.05994		

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TABLE XXXV—Continued.

Logarithmic. $n\delta z-[n\delta z]$ Unit=1".

	Sin	w^{-3}	w^{-2}	w^{-1}	w^0	w	w^2
$\eta\ \eta'$	$\frac{1}{2}\varepsilon+3\vartheta+2\Delta$				3. 6731	4.0029_n	
	$\frac{1}{2}\varepsilon+3\vartheta+3\Delta$				2. 3528	3.2475_n	3. 9005
η^2	$\frac{1}{2}\varepsilon+3\vartheta+3\Delta$				3.6181_n	4. 2122	
j^2	$\frac{1}{2}\varepsilon+3\vartheta+3\Delta$				3.4072_n	4. 4000	
$\eta\ \eta'$	$\frac{1}{2}\varepsilon+3\vartheta+4\Delta$				3. 5244	4.4012_n	
η'	$\frac{1}{2}\varepsilon+5\vartheta+4\Delta$				3. 3533	4.4231_n	5. 2725
η	$\frac{1}{2}\varepsilon+5\vartheta+5\Delta$				3.1780_n	4. 2730	5.1359_n
η'^2	$\frac{1}{2}\varepsilon+7\vartheta+5\Delta$				4. 2775	5.4708_n	
$\eta\ \eta'$	$\frac{1}{2}\varepsilon+7\vartheta+6\Delta$				4.4051_n	5. 6177	
η^2	$\frac{1}{2}\varepsilon+7\vartheta+7\Delta$				3.9296	5.1605_n	
η	$2\varepsilon+2\vartheta+2\Delta$		9. 486	2.1744_n	2. 708	[2. 889_n]	2.599_n
η'	$2\varepsilon+2\vartheta+3\Delta$				1.946_n	2. 501	2.516_n
$\eta\ \eta'$	$2\varepsilon+4\vartheta+3\Delta$	8.8_n	[0. 561]	2.789_n	[3. 5813]	[4. 1074_n]	
	$2\varepsilon+4\vartheta+4\Delta$		8.90_n	9. 599	1. 711	2.5795_n	3. 1726
η^2	$2\varepsilon+4\vartheta+4\Delta$	9. 2	[0. 34_n]	2. 618	[3. 4962_n]	[4. 0890]	
η'	$2\varepsilon+6\vartheta+5\Delta$		9.819_n	0. 5840	2. 7821	3.7794_n	4. 51865
η	$2\varepsilon+6\vartheta+6\Delta$		9. 653	0.4645_n	2.5979_n	3. 6265	4.38424_n
	$\frac{1}{2}\varepsilon+5\vartheta+5\Delta$				1. 2340	2.1166_n	2. 7076
η'	$\frac{1}{2}\varepsilon+7\vartheta+6\Delta$				2. 3679	3.3518_n	4. 0587
η	$\frac{1}{2}\varepsilon+7\vartheta+7\Delta$				2.1758_n	3. 1926	3.9204_n
	$(\vartheta-\vartheta_0)$ cos						
η	ε	0.1021_n	0. 728	2.8978_n	3. 4504	3.7168_n	
η^2	ε	1.377_n	[2. 346]	3.8211_n	4. 6762		
$\eta\ \eta'^2$	ε	1.941_n	2. 815	4.4076_n	5. 1971		
$j^2\eta$	ε	1. 364	2.220_n	4. 4076	5.1971_n	5. 7086	
η'	$\varepsilon+\Delta$	9. 658	0.774_n	2. 7836	3.3840_n	3. 6946	
$\eta^2\eta'$	$\varepsilon+\Delta$	1. 863	2.755_n	4. 2546	5.0814_n		
η'^3	$\varepsilon+\Delta$	1. 844	2.642_n	4. 1953	4.9770_n		
$j^2\ \eta'$	$\varepsilon+\Delta$	1.170_n	2. 049	4.3715_n	5. 1770	5.6975_n	
$\eta\ \eta'^2$	$\varepsilon+2\Delta$	1.742_n	2. 574	4.0203_n	4. 8466		
$j^2\ \eta'$	$\varepsilon+\Sigma$	0. 716	1.65_n	4. 0809	4.8829_n	5. 4008	
$j^2\eta$	$\varepsilon+\Delta+\Sigma$	1.00_n	1. 89	4.3427_n	5. 0837	5.5553_n	
$\eta^2\eta'$	$-\varepsilon+\Delta$	1. 562	2.455_n	3. 9535	4.7803_n		
η^2	2ε	9. 801	[0. 43_n]	2. 5842	3.1493_n	3. 4158	
$\eta\ \eta'$	$2\varepsilon+\Delta$	9.357_n	[0. 473]	2.4548_n	3. 0830	3.3936_n	
	$(\vartheta-\vartheta_0)^2$ sin						
η	ε		9.56_n	0. 42			
η'	$\varepsilon+\Delta$		9. 43	0.32_n			

$n\delta z-[n\delta z]=\Sigma w^a\eta^p\eta'^q j^{2r}C_1 \sin \text{Arg.}+(\vartheta-\vartheta_0)\Sigma w^a\eta^p\eta'^q j^{2r}C_2 \cos \text{Arg.}+(\vartheta-\vartheta_0)^2\Sigma w^a\eta^p\eta'^q j^{2r}C_3 \sin \text{Arg.}$
where C_1, C_2, C_3, represent the respective coefficients.

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TABLE XXXVI.

$$-\frac{3}{4}\left[(1-e\cos\epsilon)(\overline{W}-\frac{1}{3}\mathbb{Z})(\overline{W}+\frac{1}{9}\mathbb{Z})\right]$$

Unit = 4th decimal of a radian.

	Cos	w^{-4}	w^{-3}	w^{-2}	w^{-1}	w^{0}	w
			+0.000032	−0.0080	+0.0493	− 0.176	+0.52
η^2		−0.00028	+0.0037	−0.133	+1.10	− 8.8	
$\eta\eta'$		−0.00014	+0.0026	−0.095	+1.27	−14.4	
j^2			−0.0003	+0.139	−1.20	+ 5.9	
$\eta\eta'$	Δ	+0.00047	−0.0070	+0.252	−2.51	+22.8	
η	$2\vartheta+2\Delta$	+0.000017	−0.00042	+0.0437	−0.366	+ 2.10	
$\eta\eta'$	$2\vartheta+\Delta$	−0.000006	+0.00045	−0.0639	+0.508	− 2.79	
η^2	$4\vartheta+4\Delta$	+0.00004	+0.0006	−0.194	+1.64	−11.4	
$\eta\eta'$	$4\vartheta+3\Delta$	−0.00012	−0.0012	+0.372	−3.59	+32.2	
$\eta\eta'^2$	$4\vartheta+2\Delta$	+0.00011	+0.0003	−0.252	+2.40	−19.8	
j^2	$4\vartheta+3\Delta-\Sigma$	+0.00001	−0.0001	+0.032	−0.19		
	$+(\vartheta-\vartheta_0)$ sin						
$\eta\eta'$	Δ		−0.00004		+0.010	− 0.08	
η	$2\vartheta+2\Delta$	+0.000066	−0.00060	+0.0399	−0.275	+ 0.94	
$\eta\eta'$	$2\vartheta+\Delta$	−0.000024	+0.00047	−0.0296	+0.221	− 0.81	
η^2	$4\vartheta+4\Delta$	−0.00023	+0.0028	−0.114	+1.02	− 4.7	
$\eta\eta'$	$4\vartheta+3\Delta$	+0.00039	−0.0053	+0.251	−2.20	+ 9.9	
η'^2	$4\vartheta+2\Delta$	−0.00011	+0.0024	−0.124	+1.11	− 5.1	
	$(\vartheta-\vartheta_0)^2$ cos						
η^2		−0.00017	+0.0014	−0.052	+0.38	− 1.4	
$\eta\eta'$	Δ	+0.00019	−0.0021	+0.077	−0.61	+ 2.4	
η'^2		−0.00005	+0.0008	−0.029	+0.24	− 1.0	
		m'^3	m'^3	m'^3, m'^2	m'^2	m'^2	m'^2

TABLE XXXVII.

$$\left[(\theta-\vartheta)(1-e\cos\epsilon)\frac{\partial\overline{W}}{\partial\vartheta}\right]$$

Unit = 4th decimal of a radian.

	Cos	w^{-4}	w^{-3}	w^{-2}	w^{-1}	w^{0}	w	w^{2}
			+0.000042	−0.01071	+ 0.0883	− 0.402	+ 1.31	− 3.9
η^2		−0.00043	+0.0056	−0.189	+ 2.73	− 51.3	+ 299	
$\eta\eta'^2$		−0.00021	+0.0048	−0.296	+ 4.47	− 59.8	+ 416	
j^2			−0.0004	+0.186	− 2.00	+ 11.7	− 40	
$\eta\eta'$	Δ	+0.00076	−0.0110	+0.530	− 7.59	+104.2	− 682	
η	$2\vartheta+2\Delta$	+0.000055	−0.00086	+0.1005	− 1.153	+ 21.86	− 81.5	+217
$\eta\eta'$	$2\vartheta+\Delta$	−0.000020	+0.00090	−0.1377	+ 1.463	− 9.50	+ 44.2	−176
η^2	$4\vartheta+4\Delta$	−0.00031	+0.0041	−0.477	+ 6.49	−133.8	+ 708	
$\eta\eta'$	$4\vartheta+3\Delta$	+0.00068	−0.0084	+1.295	−17.43	+261.3	−1266	
$\eta\eta'^2$	$4\vartheta+2\Delta$	−0.00030	+0.0041	−0.921	+11.58	− 95.2	+ 452	
j^2	$4\vartheta+3\Delta-\Sigma$	−0.00001	+0.0001	−0.036	+ 0.58	− 5.3	+ 25	
	$+(\vartheta-\vartheta_0)$ sin							
$\eta\eta'$	Δ	0.00000	−0.0004	+0.052	− 0.44	+ 2.0		
η	$2\vartheta+2\Delta$	+0.000044	−0.00052	+0.0266	− 0.212	+ 0.83		
$\eta\eta'$	$2\vartheta+\Delta$	−0.000016	+0.00038	−0.0197	+ 0.170	− 0.70		
η^2	$4\vartheta+4\Delta$	−0.00031	+0.0037	−0.153	+ 1.62	− 7.9		
$\eta\eta'$	$4\vartheta+3\Delta$	+0.00052	−0.0072	+0.335	− 3.40	+ 16.9		
η'^2	$4\vartheta+2\Delta$	−0.00015	+0.0032	−0.165	+ 1.68	− 8.6		
		m'^3	m'^3	m'^3, m'^2	m'^2	m'^2, m'	m'^2, m'	m'^2, m'

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TABLE XXXVIII.

Logarithmic. $\Phi(\vartheta)$ Unit=1 radian.

	Cos	w^{-6}	w^{-5}	w^{-4}	w^{-3}	w^{-2}	w^{-1}	w^{0}	w	w^{2}
				1.5	[3.909$_n$]	4.960	6.6748$_n$	7.2764	7.540$_n$	7.31
η^3			2.0	[4.644$_n$]	[5.160]	[6.150]	[8.048$_n$]	8.838	8.655$_n$	8.100$_n$
$\eta\,\eta'^2$			1.9	[3.41$_n$]	[4.75$_n$]	[6.509]	[8.2077$_n$]	[8.994]	8.919$_n$	
f^2				2.83$_n$	5.146	[6.299$_n$]	[7.994]	8.740$_n$	[8.656]	
$\eta\,\eta'$	Δ		2.34$_n$	[4.446]	[4.57]	[6.728$_n$]	[8.4022]	9.1999$_n$	9.0854	8.079
$\eta\,\eta'^2$	2ϑ	1.6	[2.6$_n$]	5.744	6.535$_n$	8.3811	9.1031$_n$	9.0128		
η'	$2\vartheta+\Delta$		0.8$_n$	[3.068]	[5.2988]	7.2212$_n$	[7.3772]	[8.0372]	[8.764$_n$]	8.668
$\eta^2\eta'$	$2\vartheta+\Delta$	2.32$_n$	3.30	5.886$_n$	6.718	8.5059$_n$	9.2804	9.2017$_n$		
η'^2	$2\vartheta+\Delta$			5.301$_n$	6.149	8.2302$_n$	9.0154	8.938$_n$		
$f^2\,\eta'$	$2\vartheta+\Delta$					8.5592	9.3245$_n$	9.2428		
η^3	$2\vartheta+2\Delta$	2.48	3.40$_n$	5.422	6.292$_n$	7.476	8.664$_n$	8.636		
η	$2\vartheta+2\Delta$		1.22	[2.94$_n$]	[5.1206$_n$]	7.6416	[7.9638$_n$]	[7.083$_n$]	[8.645]	8.582$_n$
$\eta\,\eta'^2$	$2\vartheta+2\Delta$	1.9	[3.0$_n$]	5.442	6.328$_n$	8.0915$_n$	8.630$_n$	8.742		
$f^2\eta$	$2\vartheta+2\Delta$					8.5904$_n$	9.3489	9.8024$_n$	9.6532	
$\eta^2\eta'$	$2\vartheta+3\Delta$	2.04$_n$	3.00	4.98$_n$	5.89	8.0326	8.1973$_n$	7.69		
$f^2\eta$	$2\vartheta+\Delta-\Sigma$			4.51	5.42$_n$	8.1011	8.873$_n$	8.792		
$f^2\,\eta'$	$2\vartheta+2\Delta-\Sigma$			4.04$_n$	5.00	6.89$_n$	8.182	8.158$_n$		
η'^2	$4\vartheta+2\Delta$		[2.66$_n$]	[2.7]	6.1031	[8.4188$_n$]	[8.5297]	[6.0]	7.90$_n$	
$\eta\,\eta'$	$4\vartheta+3\Delta$		[2.72]	[4.369]	6.2526$_n$	8.5594	[8.7988$_n$]	[7.94$_n$]	8.287	8.210
η^3	$4\vartheta+4\Delta$		[2.20$_n$]	[4.624$_n$]	[5.824]	[8.0924$_n$]	[8.4338]	[7.24]	7.74$_n$	8.044$_n$
f^2	$4\vartheta+3\Delta-\Sigma$		1.5$_n$	2.45	4.68	7.1747$_n$	7.301	8.111	8.127$_n$	
η'^2	$6\vartheta+3\Delta$			5.301$_n$	6.149	9.1294$_n$	9.7728	9.6609$_n$		
$\eta\,\eta'^2$	$6\vartheta+4\Delta$			5.92	6.74$_n$	9.4432	0.14644$_n$	0.05077		
$\eta^2\eta'$	$6\vartheta+5\Delta$	2.0$_n$	3.0	5.93$_n$	6.79	9.2774$_n$	0.03298	9.9494$_n$		
η^3	$6\vartheta+6\Delta$	2.0	3.0$_n$	5.420	6.292$_n$	8.634	9.4351$_n$	9.3608		
$f^2\,\eta'$	$6\vartheta+4\Delta-\Sigma$			4.04$_n$	5.00	8.272$_n$	9.1028	9.0334$_n$		
$f^2\eta$	$6\vartheta+5\Delta-\Sigma$			4.51	5.42$_n$	8.0554	8.926$_n$	8.864		
	$(\vartheta-\vartheta_0)$ sin									
$\eta\,\eta'$	Δ			[2.60$_n$]	4.71	5.94$_n$	6.507$_n$	6.606		
η'	$2\vartheta+\Delta$		1.36	[2.48]	4.49	[5.255$_n$]	[5.51]	5.25$_n$		
η	$2\vartheta+2\Delta$		1.82$_n$	[2.42]	4.64$_n$	5.350	[5.51$_n$]	[5.16]		
η'^2	$4\vartheta+2\Delta$		2.34	[3.00]	5.392	[6.179$_n$]	6.528$_n$	6.665		
$\eta\,\eta'$	$4\vartheta+3\Delta$		2.89$_n$	[3.46]	5.702$_n$	[6.467]	6.851	6.979$_n$		
η^3	$4\vartheta+4\Delta$		2.66	[3.459$_n$]	[5.357]	[6.127$_n$]	6.530$_n$	6.653		
	$(\vartheta-\vartheta_0)^2$ cos									
η^2				2.08$_n$	2.08		5.546$_n$	5.546		
η'^2				2.54	2.54$_n$		5.396$_n$	5.396		
$\eta\,\eta'$	Δ			2.5$_n$	2.5		5.776	5.776$_n$		
		m'^3	m'^3	m'^3, m'^2	m'^3, m'^2	m'^2, m'	m'^2, m'	m'^2, m'	m'^2, m'	m'^2, m'

$$\Phi(\vartheta)=\Sigma w^s\eta p_\eta' q_{j^2}{}^t C_1 \cos \text{Arg.} + (\vartheta-\vartheta_0)\Sigma w^s\eta p_\eta' q_{j^2}{}^t C_2 \sin \text{Arg.} + (\vartheta-\vartheta_0)^2\Sigma w^s\eta p_\eta' q_{j^2}{}^t C_3 \cos \text{Arg.}$$

where C_1, C_2, C_3, represent the respective coefficients.

COMPARISON OF TABLES.

Table XXX.—With the aid of the manuscript the source of all the discrepancies indicated by brackets has been traced. Coefficients in parentheses are functions of coefficients in parentheses in Table XXVII.

Table XXXI.—The function was computed by the first of *Z* 81, eqs. (137), which is more rigid than the one following it, which v. Zeipel used. Aside from the addition of omitted terms, the bracketed coefficients are more accurate by reason of the errors in v. Zeipel's Table XVIII.

Table XXXII.—The computation was performed according to *Z* 82, eqs. (139) and (140), in place of eq. (141) which is less rigid. Besides the discrepancies due to the addition of omitted terms, four bracketed coefficients are of opposite sign. These discrepancies may be due either

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to a numerical error or to the number of terms included. The remaining discrepancy is due to slight inaccuracies of v. Zeipel's computation.

Table XXXIII.—The discrepancy in this table follows from one in Table XVIII. Third degree terms in Table XVIII were not integrated because, in the aggregate, they amount to very little.

Table XXXIV.—Our table is more extensive. Second degree terms are, however, not complete, for they do not include second degree terms in

$$[y_2]\cos\epsilon+[z_2]\sin\epsilon$$

The discrepancies are of no importance.

The integration of eq. (146) is best performed individually for each planet. The analytical developments are as follows:

The differential equation can be written

$$\frac{d\vartheta}{d\epsilon}=\Phi(\vartheta)\frac{d}{d\epsilon}\left(\frac{w}{2}\epsilon\right)+\frac{d}{d\epsilon}\left(\frac{w}{2}\epsilon-[n'\delta z']\right)$$

By a change of variable

$$\frac{d\vartheta}{d\left(\frac{w}{2}\epsilon-[n'\delta z']\right)}=1+\Phi(\vartheta)\frac{d\left(\frac{w}{2}\epsilon\right)}{d\left(\frac{w}{2}\epsilon-[n'\delta z']\right)}$$

Writing

$$\frac{w}{2}\epsilon=\left(\frac{w}{2}\epsilon-[n'\delta z']\right)+[n'\delta z']$$

we have Z 96, eq. (152), in which the last term can be neglected.

For a given planet the factors w, η, j^2 and the argument A are known constants. Therefore $1+\Phi(\vartheta)$ can be expressed as in eq. (153), as a Fourier series of sines and cosines of multiples of 2ϑ, in which the nontrigonometrical term is designated by σ.

Expressing eq. (153) in terms of exponentials and solving for $d\left(\frac{w}{2}\epsilon-[n'\delta z']\right)$ by the expansion of $\{1+\Phi(\vartheta)\}^{-1}$, and reintroducing the trigonometric functions, we have the equation following eq. (153), in which the nontrigonometrical part is taken outside the brackets as a common factor. The brackets in this equation do not have the special significance which they have had previously.

The variables ϵ and ϑ are now separate and the integration can be performed. Transferring the common factor to the left-hand side of the equation, performing the integration and adding

$$\frac{n'}{n}c-c'$$

as the constant of integration, we have the argument ζ expressed as a function of ϑ in eq. (154), where ζ is defined by eq. (155).

The reversion of the series gives ϑ as a function of ζ. We have by eq. (154)

$$\vartheta=\zeta+\Sigma C\begin{matrix}\sin\\\cos\end{matrix}n\vartheta$$

where ΣC is a small quantity. Given

$$z=w+\alpha\Phi(z),\text{ where }\alpha\text{ is small,}$$

we have, by a theorem of Lagrange,

$$F(z)=F(w)+\alpha\Phi(w)\,F'(w)+\frac{\alpha^2}{1\cdot2}\frac{\partial}{\partial w}[\{\Phi(w)\}^2F'(w)]+\cdots\frac{\alpha^{n+1}}{n+1}\frac{\partial^n}{\partial w^n}[\{\Phi(w)\}^{n+1}F'(w)]+\cdots$$

By means of this theorem eqs. (156), (157) can be derived, where it is to be noticed that $(\zeta-\frac{1}{2}c+c')$ is an approximation for $(\zeta-\zeta_0)$. In our developments we have used $(\zeta-\zeta_0)$.

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If in Z eq. (155) we add and subtract $\left(\frac{w}{2}\epsilon-[n'\delta z']\right)$

$$\zeta=\frac{\sigma-\frac{1}{2}(A_2^2+B_2^2)}{1+\frac{1}{2}(A_2^2+B_2^2)}\left(\frac{w}{2}\epsilon-[n'\delta z']\right)+\frac{n'}{n}c-c'+\left(\frac{w}{2}\epsilon-[n'\delta z']\right)$$

Substituting this value of ζ in eq. (156),

$$\vartheta-\frac{w}{2}\epsilon+[n'\delta z']-\frac{n'}{n}c+c'=\frac{\sigma-\frac{1}{2}(A_2^2+B_2^2)}{1+\frac{1}{2}(A_2^2+B_2^2)}\left(\frac{w}{2}\epsilon-[n'\delta z']\right)+\text{Series}$$

Substituting the last equation in eq. (145), we obtain *Z* 98, eqs. (159), (160), and (161). In eq. (160) the factor $(\epsilon-c)$ is an approximation for $\frac{2}{w}(\zeta-\zeta_0)$; in our work we have used the latter.

Since $[n\delta z]_1$ is the series in eq. (156) multiplied by the factor $\frac{2}{1-w}$,

$$\vartheta=\frac{1-w}{2}[n\delta z]_1+\zeta$$

Table XXXV.—With the exception of the two coefficients under the heading w^2, all the bracketed quantities are functions of other coefficients in parentheses or brackets, or they are functions of additional terms. The two coefficients excepted seem to be in disagreement through some numerical error by v. Zeipel.

Table XXXVI.—Since the mass factors have not been kept explicit, it may be well to remark that only the zero degree term of third order has been included under the heading w^{-2}.

The bracketed quantities are numerous. Aside from the accumulation of discrepancies already discussed, the disagreements are to be attributed, in general, to the relative extent of the computations. It is found from computation that as the number of terms included in a product is increased the resulting coefficient for a given argument is numerically larger. For the most part our values are larger than v. Zeipel's. Hence the discrepancies are explained by assuming that our computation is more extensive. On the other hand, the function is computed much more accurately than is necessary, and many of our disagreements are less important than they appear to be.

Table XXXVII.—The comparison of Tables XXXVII is similar to that for Tables XXXVI with the exception that our values are not, in general, numerically larger. Some are larger and some are smaller. Below are brief tables showing to what extent we used the necessary series. The 0, 1, 2 signify the degrees of the terms included.

$$\frac{1-w}{2}\left\{n\delta z-[n\delta z]\right\}$$

w^{-1}				w^{-2}		
w^0	w	w^2	w^3	w^0	w	w^2
0 1 2	0 1 2	0 1 2	0 1	0 1 2	0 1 2	0
m'				m'^2		

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$$\frac{\partial}{\partial\vartheta}\left\{(1-e\cos\epsilon)\overline{W}-[(1-e\cos\epsilon)\overline{W}]\right\}$$

w^{-1}				w^{-2}			w^{-3}			
w^0	w	w^2	w^3	w^0	w	w^2	w^0	w	w^2	w^3
0 1 2	0 1 2	0 1 2	0 1 2	0 1 2	0 1 2	0		0 1	0 1	0 1
m'				m'^2			m'^3			

Table XXXVIII.—All the bracketed quantities probably contain only the accumulation of the discrepancies in Tables XXIX*b*, XXXVI, and XXXVII. This is a very important table, and it is from differences in Φ (ϑ) that the perturbations may be expected to differ most.

PERTURBATIONS OF THE RADIUS VECTOR.

If $\overline{W}$ and $\frac{1}{3}\overline{Z}$ are tabulated and the computation is performed in duplicate, it is not necessary to make the long developments and the auxiliary tables in *Z* §6, 99–114. For this reason the formulae in §6 have not been checked and the list of errata does not cover this section.

The essential formulae are given in *Z* 99. By *Z* 7, eq. (36),

$$\nu=-\frac{1}{2}\overline{W}-\frac{1}{6}\overline{Z}+\frac{1}{4}\overline{W}^2+\ \ldots\ldots$$

In order to parallel the form of $n\delta z$, we write

$$\nu=f(\theta)=f(\vartheta+\theta_1+\theta_2+\theta_3+\ \ldots\ .)=f(\vartheta)+\frac{\partial f(\vartheta)}{\partial\vartheta}\ (\theta_1+\theta_2+\theta_3+\ \ldots\ )+\ \ldots\ .$$

where $(\theta_1+\theta_2+\theta_3)$ is given by *Z* 93, eq. (150).

Hence the computation proceeds as follows: the perturbation is computed by eq. (36), the argument θ is replaced by ϑ, and a corrective term which is the product of $(\theta_1+\theta_2+\theta_3)$ and the derivative of the function with respect to ϑ is added. The perturbation ν is then expressed as a function of ϑ. It is tabulated in Table XLIII.

Table XLIII.—If there are no errors of calculation in the construction of the table, all the discrepancies are due to the accumulation of other discrepancies previously discussed.

The perturbation $\nu=f(\theta)$ includes

w^{-1}				w^{-2}		
w^0	w	w^2	w^3	w^0	w	w^2
0 1 2 3	0 1 2 3	0 1 2	0 1	0 1 2 3	0 1 2 3	0 1
m'				m'^2		

where the tabulated numbers signify the degrees of the terms included and where only $\overline{W}_1$ and $\overline{Z}_1$ are inclusive of third degree.

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TABLE XLIII.

Logarithmic. ν Unit=1″.

	Cos	w^{-2}	w^{-1}	w^{0}	w^{1}	w^{2}	w^{3}
			8.72	[9.88$_n$]	1.6349	2.1070$_n$	2.2333
η^2		9.80	[0.212$_n$]		2.759	3.4922$_n$	
η'^2		8.9	9.23		2.937	3.6295$_n$	
f^2					2.937$_n$	3.6295	
$\eta\,\eta'$	Δ	9.66$_n$	9.78		3.1136$_n$	3.8440	
$\eta\,\eta'^2$	2ϑ	0.556$_n$	1.204	3.2111$_n$	3.7970		
η'	$2\vartheta+\Delta$		0.504$_n$	2.3472	2.456$_n$	2.686$_n$	3.4735
$\eta^2\eta'$	$2\vartheta+\Delta$	0.997	1.711$_n$	3.6559	4.3103$_n$		
η'^2	$2\vartheta+\Delta$	0.438	1.220$_n$	3.3654	4.0763$_n$		
$f^2\,\eta'$	$2\vartheta+\Delta$			3.6975$_n$	4.3810		
η	$2\vartheta+2\Delta$		0.438	2.952$_n$	3.2529	[3.0689$_n$]	3.3979$_n$
η^3	$2\vartheta+2\Delta$	0.732$_n$	1.497	3.2410$_n$	4.0643		
$\eta\,\eta'^2$	$2\vartheta+2\Delta$	0.772$_n$	1.589	3.4136	4.0723		
$f^2\eta$	$2\vartheta+2\Delta$			3.9048	[4.5649$_n$]	4.9808	
$\eta^2\eta'$	$2\vartheta+3\Delta$	0.505	1.344$_n$	3.4757$_n$	[2.783]		
$f^2\eta$	$2\vartheta+\Delta-\Sigma$	9.33$_n$	0.15	2.938$_n$	[3.5830]		
$f^2\,\eta'$	$2\vartheta+2\Delta-\Sigma$	9.20	0.10$_n$	2.0251	3.2961$_n$		
η'^2	$4\vartheta+2\Delta$	8.9	1.2819$_n$	3.5514	3.6173$_n$	3.8147	
$\eta\,\eta'$	$4\vartheta+3\Delta$	9.75$_n$	[1.5024]	3.7885$_n$	[4.1394]	4.3110$_n$	
η^3	$4\vartheta+4\Delta$	9.98	[1.1342$_n$]	3.4007	[3.9091$_n$]	[4.1480]	
f^2	$4\vartheta+3\Delta-\Sigma$		9.64$_n$	2.305	2.542$_n$	[2.749$_n$]	
η'^2	$6\vartheta+3\Delta$	0.438	1.220$_n$	4.2675	4.7993$_n$		
$\eta\,\eta'^2$	$6\vartheta+4\Delta$	1.125$_n$	1.862	4.6479$_n$	[5.2324]		
$\eta^2\eta'$	$6\vartheta+5\Delta$	1.198	1.947$_n$	4.5397	[5.1768$_n$]		
η^3	$6\vartheta+6\Delta$	0.732$_n$	1.508	3.9457$_n$	[4.6328]		
$f^2\,\eta'$	$6\vartheta+4\Delta-\Sigma$	9.20	0.10$_n$	3.4099	4.1710$_n$		
$f^2\eta$	$6\vartheta+5\Delta-\Sigma$	9.70$_n$	0.56	3.2601$_n$	[4.0542]		
$\eta\,\eta'$	$\frac{1}{2}\varepsilon+\vartheta$				3.4878$_n$	4.1106	
	$\frac{1}{2}\varepsilon+\vartheta+\Delta$			8.3$_n$	2.2106	2.7179$_n$	2.919
f^2	$\frac{1}{2}\varepsilon+\vartheta+\Delta$				3.5709$_n$	4.2261	
η^2	$\frac{1}{2}\varepsilon+\vartheta+\Delta$				3.4507	4.1296$_n$	
η'^2	$\frac{1}{2}\varepsilon+\vartheta+\Delta$				3.5100	4.1837$_n$	
$\eta\,\eta'$	$\frac{1}{2}\varepsilon+\vartheta+2\Delta$				2.579$_n$	3.9270	
η'	$\frac{1}{2}\varepsilon+3\vartheta+2\Delta$			0.08	3.6873	4.1471$_n$	4.7839
η	$\frac{1}{2}\varepsilon+3\vartheta+3\Delta$			9.5	3.5727$_n$	4.1511	4.7545$_n$
η'^2	$\frac{1}{2}\varepsilon+5\vartheta+3\Delta$				4.5568	5.1414$_n$	
$\eta\,\eta'$	$\frac{1}{2}\varepsilon+5\vartheta+4\Delta$				4.7261$_n$	[5.4067]	
η^3	$\frac{1}{2}\varepsilon+5\vartheta+5\Delta$				4.2862	[5.0418$_n$]	
f^2	$\frac{1}{2}\varepsilon+5\vartheta+4\Delta-\Sigma$				3.2570	4.0005$_n$	
η'	$-\frac{1}{2}\varepsilon+\vartheta$			1.086$_n$	2.7090	3.3467$_n$	3.7098
η	$-\frac{1}{2}\varepsilon+\vartheta+\Delta$			0.88	2.1967$_n$	3.0952	3.5836$_n$
η'^2	$-\frac{1}{2}\varepsilon+3\vartheta+\Delta$				2.514	4.1049$_n$	
$\eta\,\eta'$	$-\frac{1}{2}\varepsilon+3\vartheta+2\Delta$				4.0853	[3.9122]	
η^3	$-\frac{1}{2}\varepsilon+3\vartheta+3\Delta$				3.8341$_n$	[3.8118]	
f^2	$-\frac{1}{2}\varepsilon+3\vartheta+2\Delta-\Sigma$				2.416	3.6926$_n$	
η	ε		9.62	0.58$_n$	2.143$_n$	2.682	2.9151$_n$
η'	$\varepsilon+\Delta$		9.04$_n$	9.9	2.061	2.666$_n$	2.9477
$\eta\,\eta'$	$\varepsilon+2\vartheta+\Delta$	0.444	1.1661$_n$	3.0588	3.8035$_n$	[4.2554]	
	$\varepsilon+2\vartheta+2\Delta$		9.487	2.1744$_n$	2.7280	2.972$_n$	2.976
η^2	$\varepsilon+2\vartheta+2\Delta$	0.344$_n$	1.1143	2.692$_n$	[3.5334]	[4.0772$_n$]	
η'^2	$\varepsilon+2\vartheta+2\Delta$	0.025$_n$	0.828	2.634	3.0726	4.0416$_n$	
f^2	$\varepsilon+2\vartheta+2\Delta$			3.1265	3.8806$_n$	4.3473	
$\eta\,\eta'$	$\varepsilon+2\vartheta+3\Delta$	9.98	0.811$_n$	2.873$_n$	3.1697	[3.5856]	
$\eta\,\eta'^2$	$\varepsilon+4\vartheta+2\Delta$	1.105	1.89$_n$	2.864	4.3477$_n$		
η'	$\varepsilon+4\vartheta+3\Delta$	8.8$_n$	0.398	2.8000$_n$	3.5327	4.0065$_n$	4.3207
$\eta^2\eta'$	$\varepsilon+4\vartheta+3\Delta$	1.260$_n$	2.083	3.0931	4.4160		
η'^3	$\varepsilon+4\vartheta+3\Delta$			3.8375	4.0446$_n$		
$f^2\,\eta'$	$\varepsilon+4\vartheta+3\Delta$	0.267	1.15$_n$	3.9421	4.6972$_n$		
η	$\varepsilon+4\vartheta+4\Delta$	9.19	0.248$_n$	[2.6356]	3.4317$_n$	3.9469	4.2558$_n$
η^3	$\varepsilon+4\vartheta+4\Delta$	0.774	1.66$_n$	3.0934$_n$	[3.7866$_n$]		
$\eta\,\eta'^2$	$\varepsilon+4\vartheta+4\Delta$			4.1154$_n$	4.5547		
$f^2\eta$	$\varepsilon+4\vartheta+4\Delta$	0.455$_n$	1.32	3.8518$_n$	4.6436		
$\eta^2\eta'$	$\varepsilon+4\vartheta+5\Delta$			3.7579	4.3244$_n$		

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TABLE XLIII—Continued.

Logarithmic. ν Unit=1″.

	Cos	w^{-3}	w^{-2}	w^{-1}	w^0	w	w^2
$j^2\eta$	$\varepsilon+4\vartheta+3\Delta-\Sigma$			3.0030	3.8869_n		
$j^2\ \eta'$	$\varepsilon+4\vartheta+4\Delta-\Sigma$			2.4425	1.85_n		
$\eta\eta'^2$	$\varepsilon+6\vartheta+4\Delta$	9.98_n	0.480	3.5016$_n$	4.3723	4.9952_n	
$\eta\ \eta'$	$\varepsilon+6\vartheta+5\Delta$	0.296	0.823_n	3.6369	[4.5582$_n$]	5.2093	
η^2	$\varepsilon+6\vartheta+6\Delta$	9.95_n	0.538	3.1685$_n$	[4.1334]	[4.8131_n]	
j^2	$\varepsilon+6\vartheta+5\Delta-\Sigma$	8.5_n	[9.15]	2.114$_n$	3.0881	[3.7886_n]	
η'^3	$\varepsilon+8\vartheta+5\Delta$			4.2554$_n$	4.9349		
$\eta\eta'^2$	$\varepsilon+8\vartheta+6\Delta$	1.320	2.152_n	4.5657	5.3010_n		
$\eta^2\eta'$	$\varepsilon+8\vartheta+7\Delta$	1.228_n	2.093	4.3995$_n$	5.1827		
η^3	$\varepsilon+8\vartheta+8\Delta$	0.648	1.54_n	3.7543	4.5812_n		
$j^2\ \eta'$	$\varepsilon+8\vartheta+6\Delta-\Sigma$			3.2818$_n$	4.1442		
$j^2\eta$	$\varepsilon+8\vartheta+7\Delta-\Sigma$			3.0763	3.9759_n		
η'^2	$-\varepsilon+2\vartheta$	0.305	[1.1007_n]	2.912	3.4958_n	[3.8151]	
$\eta\eta'$	$-\varepsilon+2\vartheta+\Delta$	0.490_n	[1.3330]	3.0166$_n$	[3.7273]	[4.3119]	
η^2	$-\varepsilon+2\vartheta+2\Delta$	0.117	0.982_n	2.288$_n$	[3.2375_n]	[3.7892]	
j^2	$-\varepsilon+2\vartheta+\Delta-\Sigma$	9.04	9.96_n	2.636	3.2817_n	[3.6568]	
η'^2	$-\varepsilon+4\vartheta+\Delta$			3.2197	3.9650_n		
$\eta\eta'^2$	$-\varepsilon+4\vartheta+2\Delta$	1.146_n	1.89	3.0204	4.2441		
$\eta^2\eta'$	$-\varepsilon+4\vartheta+3\Delta$	1.005	1.78_n	3.5247$_n$	4.0012_n		
η^3	$-\varepsilon+4\vartheta+4\Delta$	0.290_n	1.15	3.1793	2.982		
$j^2\ \eta'$	$-\varepsilon+4\vartheta+2\Delta-\Sigma$			3.2486	4.0585_n		
$j^2\eta$	$-\varepsilon+4\vartheta+3\Delta-\Sigma$	9.98_n	0.8	2.957$_n$	3.8580		
η	$\frac{1}{2}\varepsilon+\vartheta+\Delta$			9.0	2.3363	[3.0704_n]	[3.5111]
η'	$\frac{1}{2}\varepsilon+\vartheta+2\Delta$			9.5	1.500	2.3585	3.1842_n
$\eta\eta'$	$\frac{1}{2}\varepsilon+3\vartheta+2\Delta$				2.779	3.7820_n	
	$\frac{1}{2}\varepsilon+3\vartheta+3\Delta$			9.28	2.1614$_n$	3.0257	3.6491_n
η^3	$\frac{1}{2}\varepsilon+3\vartheta+3\Delta$				1.32	2.966	
η'^2	$\frac{1}{2}\varepsilon+3\vartheta+3\Delta$				3.3450	4.1111_n	
j^2	$\frac{1}{2}\varepsilon+3\vartheta+3\Delta$				3.2309	4.1965_n	
$\eta\eta'$	$\frac{1}{2}\varepsilon+3\vartheta+4\Delta$				3.2994$_n$	4.1520	
η'	$\frac{1}{2}\varepsilon+5\vartheta+4\Delta$			1.017	3.1617$_n$	4.1967	5.0160_n
η	$\frac{1}{2}\varepsilon+5\vartheta+5\Delta$			0.88$_n$	2.9688	4.0380_n	4.8781
η'^2	$\frac{1}{2}\varepsilon+7\vartheta+5\Delta$				4.0855_n	5.2422	
$\eta\eta'$	$\frac{1}{2}\varepsilon+7\vartheta+6\Delta$				4.1991	5.3823_n	
η^2	$\frac{1}{2}\varepsilon+7\vartheta+7\Delta$				3.7114_n	4.9188	
j^2	$\frac{1}{2}\varepsilon+7\vartheta+6\Delta-\Sigma$				2.615$_n$	3.8317	
$\eta\eta'$	$-\frac{1}{2}\varepsilon+\vartheta$				3.2411	3.7872_n	
η^2	$-\frac{1}{2}\varepsilon+\vartheta+\Delta$				2.819$_n$	3.4476	
j^2	$-\frac{1}{2}\varepsilon+\vartheta\quad-\Sigma$				2.9181_n	3.4813	
η^2	2ε				2.364_n	3.0737	
$\eta\eta'$	$2\varepsilon+\Delta$				2.624	3.3489_n	
η'^2	$2\varepsilon+2\Delta$				2.207$_n$	2.978	
j^2	$2\varepsilon+\Delta+\Sigma$				2.620$_n$	3.2765	
η	$2\varepsilon+2\vartheta+2\Delta$			9.8$_n$	1.63	2.362_n	2.873
η'	$2\varepsilon+2\vartheta+3\Delta$			9.5	1.796	2.303_n	2.1007
$\eta\eta'$	$2\varepsilon+4\vartheta+3\Delta$				1.92_n	2.700	
	$2\varepsilon+4\vartheta+4\Delta$		8.7	[8.8]	1.5802_n	2.4158	2.9867_n
η^2	$2\varepsilon+4\vartheta+4\Delta$				2.330	3.1764_n	
η'^2	$2\varepsilon+4\vartheta+4\Delta$				3.1079	3.9008_n	
j^2	$2\varepsilon+4\vartheta+4\Delta$				2.786	3.6809_n	
$\eta\eta'$	$2\varepsilon+4\vartheta+5\Delta$				2.9881_n	3.8425	
η'	$2\varepsilon+6\vartheta+5\Delta$		9.64	[0.53]	2.652_n	3.6204	4.3279_n
η	$2\varepsilon+6\vartheta+6\Delta$		9.48_n	[0.36_n]	2.4419	3.4512_n	4.1892
η'^2	$2\varepsilon+8\vartheta+6\Delta$				3.6135$_n$	4.6784	
$\eta\eta'$	$2\varepsilon+8\vartheta+7\Delta$				3.7124	4.8075_n	
η^2	$2\varepsilon+8\vartheta+8\Delta$				3.2109_n	4.3338	
j^2	$2\varepsilon+8\vartheta+7\Delta-\Sigma$				2.068$_n$	3.2092	
η'	$\frac{1}{2}\varepsilon+5\vartheta+5\Delta$			9.3$_n$	1.140_n	2.0056	2.5727_n
η	$\frac{1}{2}\varepsilon+7\vartheta+6\Delta$			0.5$_n$	2.2749_n	3.2377	3.9184_n
	$\frac{1}{2}\varepsilon+7\vartheta+7\Delta$			0.3	2.0542	3.0565$_n$	3.7710
	$\frac{1}{2}\varepsilon+7\vartheta+7\Delta$			8.1	0.43_n	1.346	1.959_n

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TABLE XLIII—Continued.

Logarithmic. ν Unit=1″

	Cos	w^{-3}	w^{-2}	w^{-1}	w^0	w	w^2
	$(\vartheta-\vartheta_0)$ sin						
$\eta\ \eta'$	Δ	9.66	0.810_n	2.7559	3.3840_n	3.6946	
η'	$2\vartheta+\Delta$		9.79_n	0.54			
η	$2\vartheta+2\Delta$		9.92	[0.63_n]			
η	ε	9.801_n	0.425	2.5970_n	3.1493	3.4158_n	
η^3	ε	1.075_n	2.045	3.5201_n	4.3751		
$\eta\ \eta'^2$	ε	1.640_n	2.514	4.1066_n	4.8961		
$j^2\eta$	ε	1.063	1.916_n	4.1066	4.8961_n	5.4076	
η'	$\varepsilon+\Delta$	9.36	0.471_n	2.4824	3.0830_n	3.3936	
$\eta^2\eta'$	$\varepsilon+\Delta$	1.565	2.456_n	3.9671	4.7890_n		
η'^3	$\varepsilon+\Delta$	1.543	2.341_n	3.8942	4.6760_n		
$j^2\ \eta'$	$\varepsilon+\Delta$	0.87_n	1.75	4.0705_n	4.8759	5.3965_n	
$\eta\ \eta'^2$	$\varepsilon+2\Delta$	1.441_n	2.273	3.7192_n	4.5456		
$j^2\ \eta'$	$\varepsilon+\Sigma$	0.42	1.36_n	3.7799	4.5819_n	5.0998	
$j^2\eta$	$\varepsilon+\Delta+\Sigma$	0.695_n	1.585	4.0417_n	4.7827	5.2543_n	
η^2	$\varepsilon+4\vartheta+4\Delta$		9.59_n	0.45			
η'	$\varepsilon+4\vartheta+3\Delta$		9.46	0.34_n			
η	$2\varepsilon+2\vartheta+2\Delta$		9.45	[0.11_n]			
η'	$2\varepsilon+2\vartheta+3\Delta$		9.32_n	[0.04]			
$\eta^2\eta'$	$-\varepsilon+\Delta$	1.255_n	2.149	3.6240_n	4.4615		
	$(\vartheta-\vartheta_0)^2$ cos						
η	ε		9.25	0.117_n			
η'	$\varepsilon+\Delta$		9.12_n	0.02			
		m'^2	m'^2	m'^2, m'	m'	m'	m'

$$r=\Sigma w^s\eta^p\eta'^q j^{2t}C_1\cos\text{Arg.}+(\vartheta-\vartheta_0)\Sigma w^s\eta^p\eta'^q j^{2t}C_2\sin\text{Arg.}+(\vartheta-\vartheta_0)^2\Sigma w^s\eta^p\eta'^q j^{2t}C_3\cos\text{Arg.}$$

where C_1, C_2, C_3 represent the respective coefficients.

PERTURBATIONS OF THE THIRD COORDINATE.

For the third coordinate the developments are limited to perturbations of the first order and of the first degree with the exception of some secular terms of second degree. We can therefore use osculating elements in this section, and use θ and ϑ without distinction.

By *Z* 8 eq. (39), 41, eq. (83) and 8, eq. (41) the equations *Z* 115, (192) are given, in which Σ is defined.

Since

$$\frac{dS}{d\varepsilon}=\frac{\partial S}{\partial\varepsilon}+\frac{\partial S}{\partial\theta}\frac{d\theta}{d\varepsilon}=\Sigma$$

By Z 9, eq. (45) we have, with sufficient accuracy, *Z* 115, eqs. (193). Within these limits,

$$\frac{d\theta}{d\varepsilon}=\frac{w}{2}(1-e\cos\varepsilon).$$

Substituting this relation in the above equation and in eq. (192) in turn, the differential equation to be integrated is (194).

Since F, G, H are power series in w, it is evident from eqs. (192) that

$$\frac{dS}{d\varepsilon}=\Sigma_0+\Sigma_1 w+\Sigma_2 w^2+\ \ldots\ldots$$

where

$$-\Sigma_t=F_{t.p.q}+G_{t.p.q}+H_{t.p.q}$$

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Therefore, eq. (194) becomes

$$\frac{\partial S}{\partial \varepsilon}+\frac{w}{2}(1-e\cos\varepsilon)\frac{\partial S}{\partial \theta}=\Sigma_0+\Sigma_1 w+\Sigma_2 w^2+\ \ldots\ldots$$

Comparing the coefficients of like powers of w on either side of the equation, it is evident that the integral must be of the form

$$S=S_{-1}\frac{1}{w}+S_0+S_1 w+S_2 w^2+\ \ldots\ldots$$

Substituting this relation in the preceding equation and equating like powers of w, the system of equations $(195_{-1})-(195_1)$ follows.

Within the extent of the following developments one more equation should be written by analogy.

This system of equations is integrated in a manner similar to that for $\frac{dW}{d\varepsilon}$ (see p. 81). Each equation is broken up into two equations, one a function of ε and one independent of ε. The differential equation (194) is then replaced by eight differential equations, the integrals of which can be obtained in the order,

$$S_{-1},\ (S_0-[S_0]),\ [S_0],\ (S_1-[S_1]),\ [S_1],$$

As in the case of $\frac{dW}{d\varepsilon}$, the condition is imposed that

$$\left[\frac{\partial S_i}{\partial \varepsilon}\right]=0$$

The equivalent equations are (196)–(200).

A comparison of the differential equations for $(S_i-[S_i])$ with the expressions for $\frac{dW_2''}{d\varepsilon}$ $\frac{dW_2''}{d\varepsilon}$ leads to an analogous form of integration for certain terms. Within the extent of our developments,

$$(\Sigma_0-[\Sigma_0])+w(\Sigma_1-[\Sigma_1])$$

and

$$-\frac{1}{2}(1-e\cos\varepsilon)\frac{\partial}{\partial\theta}\int(\Sigma_0-[\Sigma_0])d\varepsilon-[(1-e\cos\varepsilon)\frac{\partial}{\partial\theta}\int(\Sigma_0-[\Sigma_0])d\varepsilon]$$

take the place of $\frac{\partial W_2''}{\partial \varepsilon}$ and $\frac{\partial W_2''}{\partial \varepsilon}$, respectively. Without change of notation for the third coordinate, $(S-[S])$ is given by eqs. (201), (202), where $\tilde{F}, \tilde{G}, \tilde{H}$ are computed from F, G, H in Tables XII–XIV, by means of eqs. (118) and (119). The coefficients $\tilde{F}, \tilde{G}, \tilde{H}$ are tabulated in Tables L to LII.

The function $[S]$ is obtained from the integration of eq. (203). A constant of integration is added, which is the same in form as Hansen's constant of integration for the perturbation of the third coordinate, namely,

$$c_1(\cos\psi-e)+c_2\sin\psi \qquad \text{Z eq. (204)}$$

where c_1 and c_2 are undetermined.

By eqs. (192), the pertubation $\frac{u}{\cos i}$ is derived from

$$\frac{u}{\cos i}=\bar{S}$$

The perturbation comprises the computed value of eq. (202), the trigonometric sine series given by Tables L to LII (which can be written by inspection with the aid of Table XV*b*), the series forming Table LIII, and the constant of integration (204), in all of which

$$\psi=\varepsilon$$

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TABLE L.

Unit=1″.

	n	0	1	2	3	4	5
	$\tilde{F}_{1\cdot0}(n+1.-n+1)+\pi'$	+ 52.7	+ 96.0	+ 57.0	+ 33.8	+ 20.1	+ 12.0
	$\tilde{F}_{1\cdot0}(n-1.-n+1)+\pi'$	+158.2		− 285.0	−101.4	− 47.0	− 24.0
	$\tilde{F}_{1\cdot0}(n+1.-n-1)-\pi'$	−158.2	−191.9	− 95.0	− 50.7	− 28.2	− 16.0
	$\tilde{F}_{1\cdot0}(n-1.-n-1)-\pi'$	− 52.7	−191.9	− 285.0		+ 140.9	+ 48.1
Factor w	$\tilde{F}_{1\cdot0}(n+1.-n+1)+\pi'$	−201	−352	− 253	−176	− 119	− 80
	$\tilde{F}_{1\cdot0}(n-1.-n+1)+\pi'$	−812		+1495	+594	+ 305	+172
	$\tilde{F}_{1\cdot0}(n+1.-n-1)-\pi'$	+812	+897	+ 498	+297	+ 183	+114
	$\tilde{F}_{1\cdot0}(n-1.-n-1)-\pi'$	+201	+513	+ 355		−1478	−439

TABLE LI.

Unit=1″.

	$\tilde{G}_{0\cdot0}(n.-n+1)+\pi'$	− 26.37	− 47.98	− 28.50	− 16.91	− 10.06	− 6.02
	$\tilde{G}_{0\cdot0}(n.-n-1)-\pi'$	+ 79.10	+ 95.96	+ 47.50	+ 25.36	+ 14.09	+ 8.02
	$\tilde{G}_{1\cdot0}(n+1.-n+1)+\pi'$	+ 90.3	+ 112.3	+ 58.5	+ 29.0	+ 13.6	+ 5.8
	$\tilde{G}_{1\cdot0}(n-1.-n+1)+\pi'$	+ 530.8	+ 720.8	+ 468.9	+ 311.7	+ 207.7	+ 138.0
	$\tilde{G}_{1\cdot0}(n+1.-n-1)-\pi'$	− 124.2	− 120.5	− 53.3	− 21.8	− 7.4	− 1.2
	$\tilde{G}_{1\cdot0}(n-1.-n-1)-\pi'$	+ 609.9		− 1549.1	− 674.1	− 369.6	− 219.0
	$\tilde{G}_{0\cdot1}(n.-n+2)+\pi'$	− 162.4	− 211.6	− 103.8	− 47.7	− 19.7	− 6.4
	$\tilde{G}_{0\cdot1}(n.-n)+\pi'$	− 166.5	− 352.6	− 298.2	− 229.0	− 167.2	− 118.3
	$\tilde{G}_{0\cdot1}(n.-n)-\pi'$	+ 166.5	+ 96.7	+ 13.2	− 14.4	− 20.7	− 19.3
	$\tilde{G}_{0\cdot1}(n.-n-2)-\pi'$		+ 1825.5	+ 881.4	+ 516.6	+ 321.2	+ 204.1
Factor w	$\tilde{G}_{0\cdot0}(n.-n+1)+\pi'$	+ 100.4	+ 176.3	+ 126.8	+ 87.8	+ 59.6	+ 39.9
	$\tilde{G}_{0\cdot0}(n.-n-1)-\pi'$	− 406.6	− 448.6	− 249.2	− 148.6	− 91.5	− 57.2
	$\tilde{G}_{1\cdot0}(n+1.-n+1)+\pi'$	− 432	− 592	− 370	− 218	− 122	− 64
	$\tilde{G}_{1\cdot0}(n-1.-n+1)+\pi'$	−2047	− 3188	− 2412	−1811	−1342	− 982
	$\tilde{G}_{1\cdot0}(n+1.-n-1)-\pi'$	+ 718	+ 821	+ 440	+ 225	+ 107	+ 44
	$\tilde{G}_{1\cdot0}(n-1.-n-1)-\pi'$	−2401		+12134	+4939	+2788	+1744
	$\tilde{G}_{0\cdot1}(n.-n+2)+\pi'$	+ 693	+ 951	+ 568	+ 314	+ 158	+ 68
	$\tilde{G}_{0\cdot1}(n.-n)+\pi'$	+ 893	+ 1773	+ 1607	+1356	+1089	+ 844
	$\tilde{G}_{0\cdot1}(n.-n)-\pi'$	− 893	− 747	− 254	− 27	+ 68	+ 98
	$\tilde{G}_{0\cdot1}(n.-n-2)-\pi'$		−13263	− 5889	−3549	−2336	−1586

TABLE LII.

Unit=1″.

	$\tilde{H}_{0\cdot0}(n.-n+1)+\pi'$	− 79.10		+ 142.49	+ 50.72	+ 23.48	+ 12.03
	$\tilde{H}_{0\cdot0}(n.-n-1)-\pi'$	+ 26.37	+ 95.96	+ 142.49		− 70.45	− 24.07
	$\tilde{H}_{1\cdot0}(n+1.-n+1)+\pi'$	− 609.9	− 528.9	− 231.4	− 108.8	− 52.7	− 25.7
	$\tilde{H}_{1\cdot0}(n-1.-n+1)+\pi'$	+ 124.2	+ 528.9	+1121.6		− 897.4	− 365.9
	$\tilde{H}_{1\cdot0}(n+1.-n-1)-\pi'$	− 530.8		+ 551.7	+ 166.9	+ 64.3	+ 26.5
	$\tilde{H}_{1\cdot0}(n-1.-n-1)-\pi'$	− 90.3	− 312.4	− 421.4	− 572.6	− 967.8	
	$\tilde{H}_{0\cdot1}(n.-n+2)+\pi'$		+1057.8	+ 311.5	+ 111.3	+ 39.4	+ 11.6
	$\tilde{H}_{0\cdot1}(n.-n)+\pi'$	− 166.5	−1057.8		+1145.2	+ 501.5	+ 276.0
	$\tilde{H}_{0\cdot1}(n.-n)-\pi'$	+ 166.5	+ 290.1		+ 71.9	+ 62.1	+ 44.9
	$\tilde{H}_{0\cdot1}(n.-n-2)-\pi'$	+ 162.4	+ 608.5	+ 881.4	+1551.0		− 1020.6
Factor w	$\tilde{H}_{0\cdot0}(n.-n+1)+\pi'$	+ 406.6		− 747.8	− 297.2	− 152.4	− 85.8
	$\tilde{H}_{0\cdot0}(n.-n-1)-\pi'$	− 100.4	− 256.6	− 177.8		+ 739.1	+ 219.8
	$\tilde{H}_{1\cdot0}(n+1.-n+1)+\pi'$	+2402	+2483	+1362	+ 740	+ 406	+ 222
	$\tilde{H}_{1\cdot0}(n-1.-n+1)+\pi'$	− 717	−2483	−4550		+8048	+ 3120
	$\tilde{H}_{1\cdot0}(n+1.-n-1)-\pi'$	+2046		−4336	−1406	− 697	− 298
	$\tilde{H}_{1\cdot0}(n-1.-n-1)-\pi'$	+ 432	+1214	+1481	+1901	+1186	
	$\tilde{H}_{0\cdot1}(n.-n+2)+\pi'$		−3908	−1705	− 753	− 325	− 126
	$\tilde{H}_{0\cdot1}(n.-n)+\pi'$	+ 893	+3908		−9529	−3936	− 2233
	$\tilde{H}_{0\cdot1}(n.-n)-\pi'$	− 893	−1855		− 39	− 287	− 270
	$\tilde{H}_{0\cdot1}(n.-n-2)-\pi'$	− 693	−1987	−2363	− 310		+13643

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TABLE LIII.

$[S]-\{c_1(\cos\phi-e)+c_2\sin\phi\}$ Unit=1″.

	Sin	w^{-1}	w^0	w
	$\phi+4\theta+3\Delta-\Pi'$	− 25.36	+ 123.2	− 281.8
η	$4\theta+3\Delta-\Pi'$	+ 50.7	− 246.5	+ 563.6
	$-\phi+2\theta+\Delta-\Pi'$	− 816.8	+ 3636	−8548
	$\phi+2\theta+3\Delta+\Pi'$	− 521.8	+ 2851	−7663
	$\phi+2\theta+\Delta-\Pi'$	+ 432.9	− 2034	+5237
	$\phi+6\theta+5\Delta-\Pi'$	+ 129.9	− 861	+2770
η'	$\phi-2\theta+\Pi'$	− 649.4	+ 3096	−7475
	$\phi+2\theta+2\Delta+\Pi'$	+ 596.4	− 2916	+7216
	$\phi+2\theta+2\Delta-\Pi'$	− 26.5	+ 494	−2266
	$\phi+6\theta+4\Delta-\Pi'$	− 214	+ 1236	−3395
	$(\theta-\theta_0)\cos$			
	$\phi+\Delta+\Pi'$	+ 191.93	− 705.2	+1302.6
η	$\Delta+\Pi'$	− 383.8	+ 1410	−2605
η^2	$\phi+\Delta+\Pi'$	+6584	−40060	
	$\phi-\Delta-\Pi'$	−5312	+29610	
$\eta\,\eta'$	$\phi+2\Delta+\Pi'$	−5742	+36970	
	$\phi+\Pi'$	−6024	+38180	
	$\phi-\Pi'$	+6024	−38180	
η'^2	$\phi+\Delta+\Pi'$	+6584	−40060	
	$\phi+\Delta-\Pi'$	−1656	[+11860]	
j^2	$\phi+\Delta+\Pi'$	−3002	+18970	
			m'	

and by eq. (193),

$$\theta-\theta_0=\frac{w}{2}nt$$

By inspection it is clear that the periodic part of $\overline{S}$ is of the form

$$\Sigma U_{p\cdot q}\eta^p\eta'^q\sin A$$

and the secular terms are of the form

$$\Sigma U_{p\cdot q}\eta^p\eta'^q\frac{w}{2}nt\cos\{(A-\varepsilon)+\varepsilon\}+\frac{w}{2}nt.\ \eta U_{1\cdot 0}\cos A$$

Expanding $\cos\{(A-\varepsilon)+\varepsilon\}$, and collecting coefficients of $\sin\varepsilon$ and $\cos\varepsilon$, the secular terms can be written

$$nt\{K_1(\cos\varepsilon-e)+K_2\sin\varepsilon\}$$

where

$$K_1=\Sigma U_{p\cdot q}\eta^p\eta'^q\frac{w}{2}\cos(A-\varepsilon)-\frac{w}{4}U_{1\cdot 0}\cos A$$

$$K_2=-\Sigma U_{p\cdot q}\eta^p\eta'^q\frac{w}{2}\sin(A-\varepsilon)$$

Introducing this notation, the perturbation can be written in the form of eq. (205).

The coefficients $U_{p\cdot q}$ are given in Table LIV. K_1 and K_2, which are constants, are tabulated in Tables LV_I and LV_{II}, respectively. For a given planet the factors and arguments are known. Therefore K_1 and K_2 reduce each to a single numerical quantity.

Since the Bohlin-v.Zeipel method is based on the fundamental principles of Hansen, the constants of integration are determined by the condition which must be satisfied when the

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perturbations are developed on the basis of osculating elements, namely, that the perturbations and their first derivatives shall be zero at the time $t=0$. The relations to be satisfied are

$$\left.\begin{aligned} u &= 0 \\ \frac{du}{dt} &= 0 \end{aligned}\right\} t=0$$

and the following equations are equivalent relations:

$$\left.\begin{aligned} \frac{u}{\iota \cos i} &= 0 \\ \frac{d}{d\varepsilon}\left(\frac{u}{\iota \cos i}\right) &= 0 \end{aligned}\right\} t=0$$

TABLE LIV

Logarithmic. $\Sigma U_{p\cdot q}\eta^p\eta'^q \sin$ Arg. Unit=1″

	Sin	w^{-1}	w^0	w
η	$-\Delta-\Pi'$		3.0621_m	3.7258
η'	$-\Pi'$		2.8235	3.5528_m
	$2\theta+\Delta-\Pi'$		2.2831	2.8483_m
η	$4\theta+3\Delta-\Pi'$	1.705	3.1591_m	3.8608
η'	$4\theta+2\Delta-\Pi'$		3.2462	3.9166_m
η	$\frac{1}{2}\varepsilon+\theta\quad-\Pi'$		3.2112_m	3.8544
η'	$\frac{1}{2}\varepsilon+\theta+\Delta-\Pi'$		2.5875	3.4153_m
	$\frac{1}{2}\varepsilon+3\theta+2\Delta-\Pi'$		2.2787	2.6304_m
η'	$\frac{1}{2}\varepsilon+5\theta+3\Delta-\Pi'$		3.3155	3.5865_m
η	$\frac{1}{2}\varepsilon+5\theta+4\Delta-\Pi'$		3.0779_m	3.3972
η	$-\frac{1}{2}\varepsilon-\theta-2\Delta-\Pi'$		3.1158_m	3.7378
η'	$-\frac{1}{2}\varepsilon-\theta-\Delta-\Pi'$		3.1493	3.7544_m
	$-\frac{1}{2}\varepsilon+\theta\quad-\Pi'$		2.3242	3.0060_m
η'	$-\frac{1}{2}\varepsilon+3\theta+\Delta-\Pi'$		3.3863	4.1833_m
η	$-\frac{1}{2}\varepsilon+3\theta+2\Delta-\Pi'$		3.3532_m	4.1452
η	$\varepsilon+2\theta+\Delta-\Pi'$	2.6364	3.3704_m	3.8423
η'	$\varepsilon+2\theta+2\Delta-\Pi'$	1.423_m	2.706	3.4014_m
	$\varepsilon+4\theta+3\Delta-\Pi'$	1.4042_m	2.1720	2.6339_m
η'	$\varepsilon+6\theta+4\Delta-\Pi'$	2.3306_m	3.1922	3.7582_m
η	$\varepsilon+6\theta+5\Delta-\Pi'$	2.1137	3.0138_m	3.6101
η	$-\varepsilon-2\theta-3\Delta-\Pi'$	2.7175	3.4858_m	3.9484
η'	$-\varepsilon-2\theta-2\Delta-\Pi'$	2.7756_m	3.5070	3.9456_m
	$-\varepsilon\quad-\Delta-\Pi'$		1.6810	2.2463_m
η'	$-\varepsilon+2\theta\quad-\Pi'$	2.8125	3.4427_m	3.7846
η	$-\varepsilon+2\theta+\Delta-\Pi'$	2.9121_m	3.4958	3.8338_m
η	$\frac{1}{2}\varepsilon+3\theta+2\Delta-\Pi'$		2.6058	3.5312_m
η'	$\frac{1}{2}\varepsilon+3\theta+3\Delta-\Pi'$		1.760	1.82_m
	$\frac{1}{2}\varepsilon+5\theta+4\Delta-\Pi'$		1.7510_m	2.8113
η'	$\frac{1}{2}\varepsilon+7\theta+5\Delta-\Pi'$		2.9120_m	4.0613
η	$-\frac{1}{2}\varepsilon-3\theta-4\Delta-\Pi'$		2.8673	3.8458_m
η'	$-\frac{1}{2}\varepsilon-3\theta-3\Delta-\Pi'$		2.9620_m	3.9124
	$-\frac{1}{2}\varepsilon-\theta-2\Delta-\Pi'$		2.0569_m	2.7932
η'	$-\frac{1}{2}\varepsilon+\theta-\Delta-\Pi'$		2.9275_m	3.4708
η	$-\frac{1}{2}\varepsilon+\theta\quad-\Pi'$		2.9702	3.5487_m
η	$2\varepsilon+4\theta+3\Delta-\Pi'$		1.640	2.731_m
η'	$2\varepsilon+4\theta+4\Delta-\Pi'$		1.617	2.340_m
	$2\varepsilon+6\theta+5\Delta-\Pi'$		1.206_m	2.2110
η	$-2\varepsilon-4\theta-5\Delta-\Pi'$		2.4012	3.3634_m
η'	$-2\varepsilon-4\theta-4\Delta-\Pi'$		2.5241_m	3.4544
	$-2\varepsilon-2\theta-3\Delta-\Pi'$		1.5290_m	2.3210
η'	$-2\varepsilon\quad-2\Delta-\Pi'$		2.3174_m	3.0558
η	$-2\varepsilon\quad-\Delta-\Pi'$		2.3514	3.0737_m
		m'		

$$\frac{u}{\iota \cos i} = \Sigma U_{p\cdot q}\eta^p\eta'^q \sin A + nt\{K_1(\cos\varepsilon - e) + K_2 \sin\varepsilon\} + c_1(\cos\varepsilon - e) + c_2 \sin\varepsilon$$

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TABLE $LV_{,I}$.

Logarithmic K_1 Unit=1″.

	Cos	w^0	w	w^2
η'^2	$\Delta-\Pi'$	2.9180_n	3.7732	2.8138
	$\Delta+\Pi'$	1.9821	2.5473_n	
η^2	$\Delta+\Pi'$	2.8035	3.7182_n	
η'^2	$\Delta+\Pi'$	3.5175	4.3017_n	
j^2	$\Delta+\Pi'$	3.1764_n	3.9772	
$\eta\,\eta'$	$2\Delta+\Pi'$	3.4580_n	4.2668	
		m'		

$$K_1 = \Sigma w^s \eta^p \eta'^q j^{\prime\prime\prime} \cos \text{Arg.}$$

TABLE $LV_{,II}$.

Logarithmic K_2 Unit=1″.

	Sin	w^0	w	w^2
η'^2	$\Delta-\Pi'$	2.9180	3.7732_n	
$\eta\,\eta'$	Π'	3.7799	4.5819_n	
	$\Delta+\Pi'$	1.9821_n	2.5473	2.8138_n
η^2	$\Delta+\Pi'$	3.7744_n	4.5420	
η'^2	$\Delta+\Pi'$	3.5175_n	4.3017	
$\eta\,\eta'$	$2\Delta+\Pi'$	3.4580	4.2668_n	
j^2	$\Delta+\Pi'$	3.1764	3.9772_n	
		m'		

$$K_2 = \Sigma w^s \eta^p \eta'^q j^{\prime\prime\prime} \sin \text{Arg.}$$

$$\frac{u}{\epsilon \cos i} = \Sigma U_{p,q} \eta^p \eta'^q \sin A + nt\left\{K_1(\cos\varepsilon - e) + K_2 \sin\varepsilon\right\} + c_1(\cos\varepsilon - e) + c_2 \sin\varepsilon$$

By eq. (205), at the date of osculation,

$$t = 0, \qquad \theta = \theta_0$$

$$\frac{u}{\epsilon \cos i} = \Sigma U_{p,q} \eta^p \eta'^q \sin A + c_1(\cos\varepsilon - e) + c_2 \sin\varepsilon \tag{A}$$

By Hansen,[1]

$$\frac{d}{d\varepsilon}\left(\frac{u}{\epsilon \cos i}\right) = \frac{d}{d\psi}\left(\frac{U}{\epsilon \cos i}\right) = \frac{dS}{d\psi} = 0 \tag{B}$$

in which v. Zeipel's notation is adopted.

From the various parts of S, enumerated above, $\frac{dS}{d\psi}$ can be computed. Since S contains the constants of integration

$$c_1(\cos\psi - e) + c_2 \sin\psi$$

the derivative, $\frac{dS}{d\psi}$, contains the constants

$$-c_1 \sin\varepsilon + c_2 \cos\varepsilon$$

The solution of eqs. (A) and (B) gives c_1 and c_2. But there is a better way of determining the derivative of the perturbation. The exposition of this second method is postponed until a particular example is considered, for the perturbations are not yet in a form which leads to the development of the equations.

[1] Auseinandersetzung einer zweckmässigen Methode zur Berechnung der Absoluten Störungen der kleinen Planeten, Erste Abhandlung, § 5, p. 8

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COMPARISON OF TABLES.

Tables L, LI, LII check satisfactorily.

Table LIII.—With one exception, the agreement is satisfactory. The bracketed coefficient contains a misprint in sign in v. Zeipel's table. That it is a misprint is evident from Table LV_I, in which the correct sign is given to the corresponding coefficient.

The terms included in the last column are computed from the additional tables, $XII w^3$, $XIII w^3$, $XIV w^3$ and from first degree terms in Z 116, eq. (200). The latter part, namely,

$$\frac{1}{2}\left[e\cos\varepsilon\left(\frac{\delta}{\delta\theta}\int(\Sigma_1-[\Sigma_1])d\varepsilon-\frac{1}{2}\,\frac{\delta}{\delta\theta}\int\left\{\frac{\delta}{\delta\theta}\int(\Sigma_0-[\Sigma_0])d\varepsilon\right\}d\varepsilon\right)\right]$$

is added to both eq. (200) and eq. (203).

Table LIV.—Our table is more extensive. The one bracketed quantity includes an additional term from Table LIII.

Tables LV_I, LV_{II} check satisfactorily.

CONSTANTS OF INTEGRATION IN $n\delta z$ AND ν.

The constants in $\frac{u}{\cos i}$ were treated in the preceding section by the familiar Hansen method. It is the purpose of this section to modify the similar treatment of the constants in the perturbations $n\delta z$ and ν so as to incorporate them in the elements a_0, e_0, π_0, ψ_0. Through the constants of integration, the constant elements, which have been used from the beginning without definition, are to be explained.

Since the group method of developing perturbations is built upon the fundamental principles of Hansen, his conditions for the determination of the constants of integration must be fulfilled. These conditions depend upon the choice of initial osculating or mean elements. Osculating elements are used here. The corresponding conditions are that the perturbations and their first derivatives, at the date of osculation, $(t=0)$, shall be zero.

Consider the relation of the constants of integration to the elements. There are two constants in each perturbation since the differential equations are of the second order. The constant added in the first integration is a velocity; the one added in the second integration is a displacement, or, a perturbation. Now, recalling that the position and velocity of a body for any time t can be transformed into the constants which are ordinarily called the elements of the orbit, it is evident, by analogy, that a displacement of the body and the velocity of the displacement can be transformed similarly into changes in the elements. The four constants in $n\delta z$ and ν are related to the four elements, a, e, π, c, defining the shape and size of the orbit and the position in the orbit, and the two constants in the perturbation which is measured perpendicular to the plane of the orbit are related to the elements Ω, i, which determine the position of the plane of the orbit. It is possible therefore to modify all six elements, but it is v. Zeipel's preference to make the transformations only for the first four constants.

It is not necessary to compute

$$\left.\begin{matrix} n\delta z & \nu \\ \frac{dn\delta z}{d\varepsilon} & \frac{d\nu}{d\varepsilon}\end{matrix}\right\}t=0$$

for the following developments perform the transformation analytically, and the changes in the elements can be computed from auxiliary functions.

Let a_0, e_0, π_0, c_0 be osculating elements; let a, e, π, c be the osculating elements modified by the constants of integration in the manner indicated above.

For undisturbed motion,

$$\varepsilon-e_0\sin\varepsilon=c_0+n_0t \qquad tg(v-\pi_0)=\sqrt{\frac{1+e_0}{1-e_0}}\,tg\,\tfrac{1}{2}\varepsilon$$

$$r\cos(v-\pi_0)=a(\cos\varepsilon-e_0) \qquad r\sin(v-\pi_0)=a_0\sqrt{1-e_0^2}\sin\varepsilon$$

Hansen's choice of ideal coordinates demands that the coordinates and their velocities shall have the same form of expression for disturbed and undisturbed motion. The ideal polar

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coordinates are designated by $\bar{\varepsilon}$ or $\bar{f}$ and $\bar{r}$. The relations which are analogous to the above are

$$\bar{\varepsilon} - e_0 \sin \bar{\varepsilon} = c_0 + n_0 t + n_0 \delta z = c_0 + n_0 (t + \delta z) \qquad tg(v - \pi_0) = \sqrt{\frac{1+e_0}{1-e_0}}\, tg \tfrac{1}{2} \bar{\varepsilon}$$

$$\bar{r} \cos \bar{f} = a_0 (\cos \bar{\varepsilon} - e_0) \qquad \bar{r} \sin \bar{f} = a_0 \sqrt{1 - e_0^2} \sin \bar{\varepsilon}$$

$$\bar{f} = v - \pi_0 \qquad r = \bar{r}(1 + \nu)$$

These are the equations for motion in the orbit based on constant osculating elements and appropriately determined constants of integration.

If, in place of osculating elements and Hansen's $n\delta z$ and ν, v. Zeipel's elements and the corresponding perturbations are used, the equations are the same in form. In v. Zeipel's notation ε and f take the place of $\bar{\varepsilon}$ and $\bar{f}$. The omission of the dash over these variables is permissible, since the physically real values, with which they might be confused, do not occur in the theory except for the date of osculation, where the subscript zero is added. It is to be noted that, through v. Zeipel's choice of elements, the coordinates and the perturbations have values which are numerically different from the Hansen quantities of the same designation.

Let the time be the date of osculation and denote the true coordinates by ε_0, v_0, r_0. Then the preceding equations for undisturbed motion become Z 121, equations (206), (207), and Z 125, equation (230).

Let the disturbed eccentric anomaly and radius vector $(\varepsilon, \bar{r})$ be ε_1 and r_1, respectively. The relations for disturbed motion become Z 121, equation (209), and Z 122, equations (210).

The first derivatives of these expressions are given by equations (208) and (211), respectively, and the time rate of ε is given by the equation following (209).

The solution of the four equations (210), (211), with the aid of all the others, determines the four unknown constant elements, a, e, π, c, or, better, $a - a_0$, $e - e_0$, $\pi - \pi_0$, and c.

The fact that the adoption of the new elements in connection with the perturbations $n\delta z$ and ν, as developed in the preceding sections, is equivalent to the use of osculating elements, follows from the simultaneous solution of the equations for the disturbed coordinates and their velocities and the corresponding equations for undisturbed motion.

The method of calculating c from the equation

$$c = \varepsilon_1 - e \sin \varepsilon_1 - n\delta z$$

is given in the example, page 18.

After many laborious transformations the other three unknowns are expressed in terms of familiar functions in equations (233)–(236). In the verification of these equations slight differences in the numerical coefficients of certain unimportant terms were found. The magnitudes of these coefficients depend upon the number of the terms included in making the transformations. Since it makes little difference whether or not they are included and since v. Zeipel's values present a more symmetrical form of a later auxiliary function, we adopted his coefficients.

In the functions x, y, z the arguments and factors are functions of η, π, ε_1, θ_1, Δ, Σ, where

$$\theta_1 = \frac{1}{2}(\varepsilon_1 - e \sin \varepsilon_1) - g'$$

but at the beginning of the computation only η_0, π_0, ε_0, θ_0, Δ_0, Σ_0, the corresponding functions of osculating elements are known.[1]

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[1] There is a confusion of notation in v. Zeipel's developments. In Z 127, equation (238), θ_0 is defined to be the value of θ at the date of osculation when osculating elements are used for the planet, and θ_1 signifies the argument if the elements a, e, π, etc., are employed, or by Z 9, equation (43),

$$\theta_0 = \tfrac{1}{2}(\varepsilon_0 - e_0 \sin \varepsilon_0) - g'$$

$$\theta_1 = \tfrac{1}{2}(\varepsilon_1 - e \sin \varepsilon_1) - g'$$

and their difference is computed by Z 127, equation (238).
In the collection of formulae by Z 133,

$$\theta_0 = \tfrac{1}{2} c_0 - c'$$

This is an approximation for the above equation.
Again, in Z, 60,

$$\theta_0 = \tfrac{1}{2} c - c'$$

$$= \tfrac{1}{2}(\varepsilon_1 - e \sin \varepsilon_1) - n\delta z - c'$$

If the secular terms are counted from the date of osculation, the factor $(\theta - \theta_0)$ ought to be replaced by $(\theta - \theta_1)$.

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By equations Z (43), (235), (236) and the equations preceding (233), the factor η and the arguments Δ, ϵ_1, θ_1 are given in equations (238) in terms of osculating values and functions of perturbations, inclusive of first order.

To these should be added

$$\Sigma = \Sigma_0 - \frac{1}{4\eta_0} z + \frac{1}{2}\eta_0 z^2 + \ldots .$$

and

$$\Gamma_1 = \Gamma + \frac{3}{4}\left(1 - \frac{2}{3}\eta_0 \cos \epsilon_0\right)(y \sin \epsilon_0 - z \cos \epsilon_0) + \frac{1}{2}\eta_0 z^2 + \ldots .$$

where

$$\Gamma_1 = \frac{1}{2}\epsilon_1 + \theta_1 + \Delta_1$$

$$\Gamma = \frac{1}{2}\epsilon_0 + \theta_0 + \Delta_0$$

The equations (233), (235), (236), and (238) permit the construction of two tables which determine w, n or a, and e and π. From here on the developments differ in form from v. Zeipel's although they are the same in principle. If v. Zeipel's equations (237) and (239) are used, the term $(x_2'' - \eta y_2'')$ should read

$$(x_2'' + x_3'' + x_4'') - \eta(y_2'' + y_3'' + y_4'')$$

in agreement with Z 91, line 14.

Suppose that $w - w_0$ has been computed by equation (233) and the argument Γ has been introduced. The arguments and factors are unknown.

$$w - w_0 = f(\eta, \Gamma_1, \theta_1, \Delta, \Sigma)$$

By Taylor's theorem

$$w - w_0 = f(\eta_0, \Gamma, \theta_0, \Delta_0, \Sigma_0) + \frac{\delta f}{\delta \eta_0}\Delta\eta_0 + \frac{\delta f}{\delta \Gamma}\Delta\Gamma_0 + \frac{\delta f}{\delta \theta_0}\Delta\theta_0 + \frac{\delta f}{\delta \Delta_0}\Delta\Delta_0 + \frac{\delta f}{\delta \Sigma_0}\Delta\Sigma_0 + \ldots .$$

Inclusive of second order in m', the differentiation is for first order terms.

Substituting the values of $\Delta\eta$, $\Delta\Gamma$, $\Delta\theta_0$, $\Delta\Delta_0$, $\Delta\Sigma_0$ from equations (238) and the additional equations above,

$$w - w_0 = f(\eta_0, \Gamma, \theta_0, \Delta_0, \Sigma_0) + \left(\frac{\delta f}{\delta(2\theta_0)} - \frac{\delta f}{\delta \Delta_0} - \frac{\delta f}{\delta \Sigma_0}\right)\frac{1}{4\eta_0} z + \left(\frac{\delta f}{\delta \Delta_0} + \frac{\delta f}{\delta \Gamma} + \frac{\delta f}{\delta \Sigma_0}\right)\frac{1}{2}\eta_0 z^2$$

$$- \frac{\delta f}{\delta \eta_0}\frac{1}{4} y + \left\{\frac{1}{2}(1 - \eta_0 \cos \epsilon_0)\frac{\delta f}{\delta \theta_0} + \frac{3}{4}\left(1 - \frac{2}{3}\eta_0 \cos \epsilon_0\right)\frac{\delta f}{\delta \Gamma}\right\}(y \sin \epsilon_0 - z \cos \epsilon_0) + \ldots .$$

The order of calculation is: computation of equation (233), in which the arguments and the factors are given the subscript zero, differentiation of first order terms, computation of the second order terms in the above equation, and the additon of these second order terms to the first calculation.

With some foresight the computation can be simplified. The arguments should be arranged in groups like the following:

$$\begin{array}{r} -n\Gamma + 2\theta + 2\Delta \\ -(n-1)\Gamma + 2\theta + 2\Delta \\ -(n-2)\Gamma + 2\theta + 2\Delta \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ (n-1)\Gamma + 2\theta + 2\Delta \\ n\Gamma + 2\theta + 2\Delta \end{array}$$

Then, for whole groups of arguments,

$$\frac{\delta f}{\delta(2\theta_0)} - \frac{\delta f}{\delta \Delta_0} - \frac{\delta f}{\delta \Sigma_0} = 0$$

Also for some particular argument in a group, the condition

$$\frac{\delta f}{\delta \Delta_0} + \frac{\delta f}{\delta \Gamma} + \frac{\delta f}{\delta \Sigma_0} = 0$$

may be satisfied.

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Finally, by inspection of the arguments, considerable computation can be avoided if

$$\frac{1}{\eta_0}\left(\frac{\partial f}{\partial(2\theta_0)}-\frac{\partial f}{\partial \Delta_0}-\frac{\partial f}{\partial \Sigma_0}\right)z=\frac{\partial f}{\partial \eta_0}\,y$$

The function $w-w_0$ is tabulated in Table LVI. Since it is unavoidably a function of w itself, the determination of w for a given case must be made by successive trials, the first approximation being

$$w=w_0$$

TABLE LVI.

Logarithmic. $w-w_0$ Unit = 1 radian.

	Cos	w^{-3}	w^{-2}	w^{-1}	w^0	w	w^2
			4.360	[5.1966$_n$]	[5.7767]		
	Γ			4.766	6.6599	7.3732$_n$	7.7492
	2Γ			4.446	7.1194	7.7572$_n$	8.0553
	3Γ			4.412	6.8442	7.5458$_n$	7.9060
	4Γ			4.484	6.5883	7.3450$_n$	7.7602
	5Γ				6.3437	7.1490$_n$	7.6136
	7Γ				5.875	6.7632$_n$	7.3134
γ_0	$-5\Gamma+2\theta_0+2\Delta_0$				6.5090	6.6325$_n$	7.4746$_n$
	$-4\Gamma+2\theta_0+2\Delta_0$			4.161$_n$	6.169	7.0658	7.8698$_n$
	$-3\Gamma+2\theta_0+2\Delta_0$			3.19	6.8821$_n$	7.6078	7.9975$_n$
	$-2\Gamma+2\theta_0+2\Delta_0$			3.52	7.0986$_n$	7.6970	7.9394$_n$
	$-\Gamma+2\theta_0+2\Delta_0$			5.1420	6.359	7.0722$_n$	7.4480
	$2\theta_0+2\Delta_0$		4.379	7.6355$_n$	8.2144	8.4125$_n$	
	$\Gamma+2\theta_0+2\Delta_0$			4.856$_n$	8.0894$_n$	8.9548	9.5668$_n$
	$2\Gamma+2\theta_0+2\Delta_0$			4.92$_n$	7.8150$_n$	8.6561	9.2006$_n$
	$3\Gamma+2\theta_0+2\Delta_0$			5.5174$_n$	7.6056$_n$	8.4650	9.0111$_n$
	$4\Gamma+2\theta_0+2\Delta_0$			5.4248$_n$	7.4128$_n$	[8.2958]	[8.8561$_n$]
	$5\Gamma+2\theta_0+2\Delta_0$				7.2254$_n$	8.1426	8.7346$_n$
	$7\Gamma+2\theta_0+2\Delta_0$				[6.8746$_n$]	[7.8484]	8.4936$_n$
η'	$-5\Gamma+2\theta_0+\Delta_0$				6.8776$_n$	7.5604	7.8425$_n$
	$-4\Gamma+2\theta_0+\Delta_0$			4.582	6.8815$_n$	7.4536	7.5238$_n$
	$-3\Gamma+2\theta_0+\Delta_0$			4.674	6.6271$_n$	6.7816	7.3174
	$-2\Gamma+2\theta_0+\Delta_0$			4.99	6.7985	7.4732$_n$	7.7966
	$-\Gamma+2\theta_0+\Delta_0$			5.4623$_n$			
	$2\theta_0+\Delta_0$		4.605$_n$	[7.1987]	7.8314$_n$	8.1061	
	$\Gamma+2\theta_0+\Delta_0$			5.0056	8.2964	9.1086$_n$	9.6833
	$2\Gamma+2\theta_0+\Delta_0$			4.38	8.0434	8.8316$_n$	9.3296
	$3\Gamma+2\theta_0+\Delta_0$			5.6251	7.8458	8.6564$_n$	9.1558
	$4\Gamma+2\theta_0+\Delta_0$			5.5812	7.6603	8.5030$_n$	9.0248
	$5\Gamma+2\theta_0+\Delta_0$				7.4778	8.3544$_n$	8.9050
	$7\Gamma+2\theta_0+\Delta_0$				7.1130	8.0545$_n$	8.6668
η_0^2		4.664	4.71	5.83			
	Γ				7.8102	8.6250$_n$	
	2Γ				7.7520$_n$	8.1242	
	3Γ				7.6172$_n$	6.6043$_n$	
	4Γ				7.7135$_n$	8.2308	
η_0^2	$-4\Gamma+4\theta_0+4\Delta_0$				7.1862	7.9072$_n$	
	$-3\Gamma+4\theta_0+4\Delta_0$				7.1804	7.8679$_n$	
	$-2\Gamma+4\theta_0+4\Delta_0$				6.817	7.456$_n$	
	$-\Gamma+4\theta_0+4\Delta_0$				8.4680$_n$	8.8822	
	$4\theta_0+4\Delta_0$	4.666	[5.807$_n$]	[8.0913]	8.8270$_n$	9.2073	
	$\Gamma+4\theta_0+4\Delta_0$				8.7850	9.8236$_n$	
	$2\Gamma+4\theta_0+4\Delta_0$				[8.5144]	9.4910$_n$	
	$3\Gamma+4\theta_0+4\Delta_0$				8.3274	9.3006$_n$	
	$4\Gamma+4\theta_0+4\Delta_0$				8.1627	9.1494$_n$	
	$5\Gamma+4\theta_0+4\Delta_0$				8.0050	9.0105$_n$	
$\eta_0\eta'$	$-4\Gamma+4\theta_0+3\Delta_0$				7.354$_n$	8.1083	
	$-3\Gamma+4\theta_0+3\Delta_0$				7.5708$_n$	8.2084	
	$-2\Gamma+4\theta_0+3\Delta_0$						
	$-\Gamma+4\theta_0+3\Delta_0$				8.8838	9.0548$_n$	
	$4\theta_0+3\Delta_0$	4.516$_n$	[6.2084]	8.5565$_n$	9.2180	9.5174$_n$	
	$\Gamma+4\theta_0+3\Delta_0$				9.2783$_n$	0.2833	
	$2\Gamma+4\theta_0+3\Delta_0$				9.0241$_n$	9.9635	
	$3\Gamma+4\theta_0+3\Delta_0$				8.8480$_n$	9.7850	
	$4\Gamma+4\theta_0+3\Delta_0$				8.6916$_n$	9.6434	
	$5\Gamma+4\theta_0+3\Delta_0$				8.5401$_n$	9.5128	
		m'^3	m'^2	m'^2, m'	m'^2, m'	m'	m'

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TABLE LVI—Continued.

Logarithmic. $w - w_0$ Unit=1 radian.

	Cos	w^{-2}	w^{-1}	w^{-1}	w^0	w	w'^2
$\eta_0\eta'$	$-4\Gamma+\Delta_0$				7.7640	7.8364_n	
	$-3\Gamma+\Delta_0$				7.4203	8.3915	
	$-2\Gamma+\Delta_0$				7.8104_n	8.6268	
	$-\Gamma+\Delta_0$				8.0479_n	8.8018	
	Δ_0	4.518_n	[5.886$_n$]	[5.70$_n$]			
	$\Gamma+\Delta_0$				7.1339	7.8500	
	$2\Gamma+\Delta_0$				7.8421	8.4293_n	
	$3\Gamma+\Delta_0$				7.9669	8.6796_n	
	$4\Gamma+\Delta_0$				7.9760	8.7576_n	
η'^2	$-4\Gamma+4\theta_0+2\Delta_0$				6.9002	7.6938_n	
	$-3\Gamma+4\theta_0+2\Delta_0$				7.1638	7.8502_n	
	$-2\Gamma+4\theta_0+2\Delta_0$						
	$-\Gamma+4\theta_0+2\Delta_0$				8.1860_n	8.4016	
	$4\theta_0+2\Delta_0$	3.76	6.0608$_n$	8.4157	8.9760_n	9.1661	
	$+\Gamma+4\theta_0+2\Delta_0$				9.1714	0.1382_n	
	$+2\Gamma+4\theta_0+2\Delta_0$				8.9358	9.8333_n	
	$+3\Gamma+4\theta_0+2\Delta_0$				8.7718	9.6681_n	
	$+4\Gamma+4\theta_0+2\Delta_0$				8.6236	9.5372_n	
η'^2		3.76	5.7516	4.7			
	Γ				7.8677	8.6727_n	
	2Γ				7.8610_n	8.2228	
	3Γ				8.1026_n	8.7296	
	4Γ				8.1538_n	8.8728	
j^2	Γ				7.9418_n	8.7337	
	2Γ				7.9312_n	8.7154	
	3Γ				7.7920_n	8.6154	
	4Γ				7.639_n	8.5001	
j^2	$-4\Gamma+4\theta_0+3\Delta_0-\Sigma_0$				7.446	8.1156_n	
	$-3\Gamma+4\theta_0+3\Delta_0-\Sigma_0$				7.1858	7.8677_n	
	$-2\Gamma+4\theta_0+3\Delta_0-\Sigma_0$						
	$-\Gamma+4\theta_0+3\Delta_0-\Sigma_0$				7.6176_n	7.9693	
	$4\theta_0+3\Delta_0-\Sigma_0$		4.804$_n$	7.168	7.9368_n	8.3724	
	$\Gamma+4\theta_0+3\Delta_0-\Sigma_0$				7.7887	8.8492_n	
	$2\Gamma+4\theta_0+3\Delta_0-\Sigma_0$				7.448	8.4531_n	
	$3\Gamma+4\theta_0+3\Delta_0-\Sigma_0$				7.1976	8.2026_n	
	$4\Gamma+4\theta_0+3\Delta_0-\Sigma_0$				6.978	7.9963_n	
η_0^2	$2\theta_0+2\Delta_0$	5.4181_n	6.292	7.4754_n	8.6636		
	$6\theta_0+6\Delta_0$	5.418_n	6.292	8.6328_n	9.4351		
$\eta_0^2\eta'$	$2\theta_0+\Delta_0$	5.885	6.719_n	8.5059	9.2804_n		
	$2\theta_0+3\Delta_0$	4.974	5.896_n	8.0326_n	8.1975		
	$6\theta_0+5\Delta_0$	5.935	6.780_n	9.2774	0.0330_n		
$\eta_0\,\eta'^2$	$2\theta_0$	5.744_n	6.535	[8.3811$_n$]	9.1030		
	$2\theta_0+2\Delta_0$	5.44_n	6.327	8.0917	8.6300		
	$6\theta_0+4\Delta_0$	5.919_n	6.744	9.4432_n	0.1464		
η'^2	$2\theta_0+\Delta_0$	5.301	6.149_n	8.2302	9.0152_n		
	$6\theta_0+3\Delta_0$	5.301	6.149_n	9.1294	9.7729_n		
$j^2\eta_0$	$2\theta_0+2\Delta_0$			8.5904	9.3492_n	9.8022	
	$2\theta_0+\Delta_0-\Sigma_0$	4.502_n	5.41	8.1011_n	8.8726		
	$6\theta_0+5\Delta_0-\Sigma_0$	4.502_n	5.41	8.0554_n	8.9263		
$j^2\eta'$	$2\theta_0+\Delta_0$			8.5592_n	9.3245		
	$2\theta_0+2\Delta_0-\Sigma_0$	4.057	5.021_n	6.887	8.1804_n		
	$6\theta_0+4\Delta_0-\Sigma_0$	4.057	5.021_n	8.2718	9.1021_n		
		m'^2	m'^2	m'^2, m'	m'^2, m'	m'	m'

$w - w_0 = \Sigma C w^s \eta^{p'} \eta^q j^{2t}$ cos Agr., where C represents the respective coefficient.

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Turning now to the determination of e and π, let equations (235), (236) be written in the form (244), where

$$S = -\frac{1}{2}z + \left(\frac{1}{4e_0} - \frac{e_0}{3}\right) yz + \frac{1}{6}xz + \ldots\ldots$$

$$C = -\frac{1}{2}y - \frac{z^2}{4e_0} + \frac{1}{6}xy - \frac{1}{3}e_0y^2 + \ldots\ldots$$

Multiplying the first of these by $\sin\phi$, the second by $\cos\phi$ and adding,

$$S\sin\phi + C\cos\phi = -\frac{1}{2}(y\cos\phi + z\sin\phi) + \frac{1}{6}x(y\cos\phi + z\sin\phi)$$

$$-\frac{1}{3}e_0y(y\cos\phi + z\sin\phi) + \frac{1}{4e_0}z(y\sin\phi - z\cos\phi) + \ldots$$

Here, again, the arguments and factors are functions of the elements a, e, π, c, and the expansion in a Taylor's series is necessary.

Let

$$S\sin\phi + C\cos\phi = f(\eta, \Gamma_1, \theta_1, \Delta, \Sigma)$$

Then the form of Taylor's series is the same as the expression for $w - w_0$, (p. 148), with the following modification. Within first order quantities,

$$f(\eta, \Gamma_1, \theta_1, \Delta, \Sigma) = -\frac{1}{2}(y\cos\phi + z\sin\phi)$$

$$\frac{\overline{\delta f}}{\delta\psi} = \frac{\overline{\delta f}}{\delta\phi_{\epsilon=\epsilon}} = \frac{1}{2}(y\sin\epsilon - z\cos\epsilon)$$

Hence,

$$S\sin\phi + C\cos\phi = f(\eta_0, \Gamma, \theta_0, \Delta_0, \Sigma_0) + \left(\frac{\delta f}{\delta(2\theta_0)} - \frac{\delta f}{\delta\Delta_0} - \frac{\delta f}{\delta\Sigma_0}\right)\frac{1}{4\eta_0}z$$

$$+\left(\frac{\delta f}{\delta\Delta_0} + \frac{\delta f}{\delta\overline{\Gamma}} + \frac{\delta f}{\delta\Sigma_0}\right)\frac{1}{2}\eta_0 z - \frac{\delta f}{\delta\eta}\frac{1}{4}y$$

$$+\left\{(1-\eta_0\cos\epsilon_0)\frac{\delta f}{\delta\theta_0} + \frac{3}{2}\left(1 - \frac{2}{3}\eta_0\cos\epsilon_0\right)\frac{\delta f}{\delta\overline{\Gamma}}\right\}\frac{\overline{\delta f}}{\delta\psi} + \ldots\ldots$$

The order of computation is: calculation of

$$-\frac{1}{2}(y\cos\phi + z\sin\phi)$$

by inspection of the table for W, in which the arguments are to be given the subscript zero, differentiation of the first order terms, calculation of the necessary products of functions of y, z, and the partial derivatives, and the addition of these products to the first calculation. The required function is given in Table LVII.

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TABLE LVII.

Logarithmic $S \sin \phi + C \cos \phi$ Unit=1″.

	Cos	w^{-3}	w^{-2}	w^{-1}	w^{0}	w	w^{2}
	$\phi-5\Gamma+2\theta_0+2\Delta_0$			8.81	1.082_n	1.5710	1.612_n
	$\phi-4\Gamma+2\theta_0+2\Delta_0$			9.009	1.2314_n	1.5492	0.989_n
	$\phi-3\Gamma+2\theta_0+2\Delta_0$			9.318	0.931	1.604_n	1.916
	$\phi-2\Gamma+2\theta_0+2\Delta_0$			9.207	[1.6478]	2.1070_n	2.2333
	$\phi-\Gamma+2\theta_0+2\Delta_0$			9.711	1.950	2.3426_n	2.3713
	$\phi+2\theta_0+2\Delta_0$		9.196	2.1712_n	2.5678	2.565_n	
	$\phi+\Gamma+2\theta_0+2\Delta_0$			9.230_n	$[2.3541_n]$	3.1493	3.7107_n
	$\phi+2\Gamma+2\theta_0+2\Delta_0$			9.220_n	$[1.9114_n]$	2.6867	3.1657_n
	$\phi+3\Gamma+2\theta_0+2\Delta_0$			9.724_n	$[1.5372_n]$	2.3831	2.8623_n
	$\phi+4\Gamma+2\theta_0+2\Delta_0$			9.494_n	$[1.2544_n]$	2.1315	2.6333_n
	$\phi+5\Gamma+2\theta_0+2\Delta_0$			9.100_n	1.018_n	1.9034	2.4348_n
η_0	$\phi-5\Gamma+4\theta_0+4\Delta_0$			9.771_n	1.042_n	1.868	2.357_n
	$\phi-4\Gamma+4\theta_0+4\Delta_0$			0.064_n	1.723_n	2.3515	2.6814_n
	$\phi-3\Gamma+4\theta_0+4\Delta_0$			0.3185_n	2.1626_n	2.6961	2.9214_n
	$\phi-2\Gamma+4\theta_0+4\Delta_0$			0.497_n	$[2.7787_n]$	[3.0649]	3.0993_n
	$\phi-\Gamma+4\theta_0+4\Delta_0$			1.0286_n	3.2379_n	3.1223	3.9385_n
	$\phi+4\theta_0+4\Delta_0$	9.199	9.04_n	[2.6172]	[3.2511$_n$]	[3.4930]	
	$\phi+\Gamma+4\theta_0+4\Delta_0$			0.7226	3.1702	4.1580_n	4.9365
	$\phi+2\Gamma+4\theta_0+4\Delta_0$			0.669	2.7877	3.7083_n	4.3605
	$\phi+3\Gamma+4\theta_0+4\Delta_0$			0.9435	2.5117	3.4261_n	4.0450
	$\phi+4\Gamma+4\theta_0+4\Delta_0$			0.5122	2.2732	3.2042_n	
η_0	$\phi-5\Gamma$			9.814_n	1.925	2.634_n	2.984
	$\phi-4\Gamma$			0.0434_n	2.0527	2.6896_n	2.9432
	$\phi-3\Gamma$			0.3541_n	2.145	2.675_n	2.744
	$\phi-2\Gamma$		9.140	0.362_n	2.1351	2.3850_n	2.4864_n
	$\phi-\Gamma$			0.4164_n	2.3504_n	3.0929	3.5397_n
	ϕ		9.274_n	0.1436_n			
	$\phi+\Gamma$			0.3102_n	2.497	3.1875_n	3.5978
	$\phi+2\Gamma$		9.137_n	9.918	1.9006_n	1.0453	2.8834
	$\phi+3\Gamma$			9.465	0.812_n	2.5218_n	3.3564
	$\phi+4\Gamma$			9.20_n	1.406_n	1.729	
η'	$\phi-5\Gamma+4\theta_0+3\Delta_0$			9.476	1.327	1.889_n	2.2299
	$\phi-4\Gamma+4\theta_0+3\Delta_0$			9.781	1.447	2.1506_n	2.5419
	$\phi-3\Gamma+4\theta_0+3\Delta_0$			9.811	2.1070	2.6309_n	2.8608
	$\phi-2\Gamma+4\theta_0+3\Delta_0$			0.3489	2.5095	2.9557_n	3.0952
	$\phi-\Gamma+4\theta_0+3\Delta_0$			0.9511	3.3599	2.7758	3.9726
	$\phi+4\theta_0+3\Delta_0$	8.76_n	[0.158]	$[2.7932_n]$	3.3085	3.4526_n	
	$\phi+\Gamma+4\theta_0+3\Delta_0$			9.961_n	3.3609_n	4.3114	5.0691_n
	$\phi+2\Gamma+4\theta_0+3\Delta_0$			0.491_n	2.9943_n	3.8728	4.4922_n
	$\phi+3\Gamma+4\theta_0+3\Delta_0$			1.0464_n	2.7293_n	3.6067	4.1945_n
	$\phi+4\Gamma+4\theta_0+3\Delta_0$			0.678_n	2.4992_n	3.3946	
η'	$\phi-5\Gamma+\Delta_0$			9.848	2.0766_n	2.712	2.9697_n
	$\phi-4\Gamma+\Delta_0$			0.0792	2.1609_n	2.6968	2.7976_n
	$\phi-3\Gamma+\Delta_0$			0.3941	2.157_n	2.491	1.51
	$\phi-2\Gamma+\Delta_0$		9.013_n	0.248	2.0455	2.7898_n	3.2380
	$\phi-\Gamma+\Delta_0$			9.901	2.584	3.2539_n	3.6434
	$\phi+\Delta_0$		9.885_n	0.8518			
	$\phi+\Gamma+\Delta_0$			0.1664	1.836	2.448	3.3029_n
	$\phi+2\Gamma+\Delta_0$		9.009	9.76_n	2.1633	2.6170_n	2.2433
	$\phi+3\Gamma+\Delta_0$			9.38_n	2.1064	2.7194_n	2.9212
	$\phi+4\Gamma+\Delta_0$				1.9892	2.6870_n	
η_0^2	$\phi-5\Gamma+6\theta_0+6\Delta_0$				2.3144	2.9730_n	
	$\phi-4\Gamma+6\theta_0+6\Delta_0$				2.9538	3.3785_n	
	$\phi-3\Gamma+6\theta_0+6\Delta_0$				3.3102	3.5843_n	
	$\phi-2\Gamma+6\theta_0+6\Delta_0$				[3.4970]	$[3.8423_n]$	
	$\phi-\Gamma+6\theta_0+6\Delta_0$				3.9455	3.7269_n	
	$\phi+6\theta_0+6\Delta_0$	9.95_n	1.1109_n	3.1673_n	[3.9296]	$[4.3377_n]$	
	$\phi+\Gamma+6\theta_0+6\Delta_0$				3.9144_n	5.0372	
	$\phi+2\Gamma+6\theta_0+6\Delta_0$				3.5594_n	4.5942	
	$\phi+3\Gamma+6\theta_0+6\Delta_0$				3.3121_n	4.3236	

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TABLE LVII—Continued.

$S \sin \phi + C \cos \phi$

Logarithmic Unit=1″.

	Cos	w^{-2}	w^{-1}	w^{0}	w	w^{1}
η_0^2	$\phi-5\Gamma+2\theta_0+2\Delta_0$			2.1657	2.7221_n	
	$\phi-4\Gamma+2\theta_0+2\Delta_0$			2.1255	2.8004_n	
	$\phi-3\Gamma+2\theta_0+2\Delta_0$			2.234	3.1304_n	
	$\phi-2\Gamma+2\theta_0+2\Delta_0$			2.576	3.3804_n	
	$\phi-\Gamma+2\theta_0+2\Delta_0$			3.1995	3.8325_n	
	$\phi+2\theta_0+2\Delta_0$	0.344_n	1.017	2.689_n	[3.4822]	3.9938_n
	$\phi+\Gamma+2\theta_0+2\Delta_0$			2.2480	3.2839_n	
	$\phi+2\Gamma+2\theta_0+2\Delta_0$		9.45	3.1612	3.8424_n	
η_0^2	$\phi-5\Gamma-2\theta_0-2\Delta_0$			2.700_n	3.5481	
	$\phi-4\Gamma-2\theta_0-2\Delta_0$			2.817_n	3.6251	
	$\phi-3\Gamma-2\theta_0-2\Delta_0$			2.9247_n	3.6905	
	$\phi-2\Gamma-2\theta_0-2\Delta_0$		9.59_n	3.0241_n	3.7470	
	$\phi-\Gamma-2\theta_0-2\Delta_0$			3.1364_n	3.8346	
	$\phi-2\theta_0-2\Delta_0$	0.117	0.95_n	2.297_n	[2.7856]	3.6614
	$\phi+\Gamma-2\theta_0-2\Delta_0$			2.8942_n	3.5604	
	$\phi+2\Gamma-2\theta_0-2\Delta_0$			2.297_n	3.1129	
$\eta_0\,\eta'$	$\phi-5\Gamma+6\theta_0+5\Delta_0$			2.4885_n	3.1691	
	$\phi-4\Gamma+6\theta_0+5\Delta_0$			2.976_n	3.5560	
	$\phi-3\Gamma+6\theta_0+5\Delta_0$			3.6541_n	3.8829	
	$\phi-2\Gamma+6\theta_0+5\Delta_0$			[3.9514]	[4.1632]	
	$\phi-\Gamma+6\theta_0+5\Delta_0$			4.3903_n	4.0037_n	
	$\phi+6\theta_0+5\Delta_0$	0.295	1.366	3.6364	[4.3301]	[4.6662]
	$\phi+\Gamma+6\theta_0+5\Delta_0$			4.4005	5.4966_n	
	$\phi+2\Gamma+6\theta_0+5\Delta_0$			4.0582	5.0612_n	
	$\phi+3\Gamma+6\theta_0+5\Delta_0$			3.8204	4.8027_n	
$\eta_0\,\eta'$	$\phi-5\Gamma+2\theta_0+\Delta_0$			2.426_n	3.0684	
	$\phi-4\Gamma+2\theta_0+\Delta_0$			2.399_n	3.0310	
	$\phi-3\Gamma+2\theta_0+\Delta_0$			2.410_n	3.1305	
	$\phi-2\Gamma+2\theta_0+\Delta_0$			2.701_n	3.4602	
	$\phi-\Gamma+2\theta_0+\Delta_0$			3.2842_n	3.8558	
	$\phi+2\theta_0+\Delta_0$	0.444	1.188_n	3.0569	[3.7266]	4.1122
	$\phi+\Gamma+2\theta_0+\Delta_0$			2.8541	3.5823_n	
	$\phi+2\Gamma+2\theta_0+\Delta_0$			3.2191_n	3.7635	
$\eta_0\,\eta'$	$\phi-5\Gamma-2\theta_0-\Delta_0$			3.1551	3.9530_n	
	$\phi-4\Gamma-2\theta_0-\Delta_0$			3.2454	3.9948_n	
	$\phi-3\Gamma-2\theta_0-\Delta_0$			3.3100	4.0023_n	
	$\phi-2\Gamma-2\theta_0-\Delta_0$		9.93	3.3277	3.9401_n	
	$\phi-\Gamma-2\theta_0-\Delta_0$			3.1976	3.4598_n	
	$\phi-2\theta_0-\Delta_0$	0.490_n	1.324	3.0145_n	3.7326	4.2787
	$\phi+\Gamma-2\theta_0-\Delta_0$			3.3632	3.9402_n	
	$\phi+\Gamma-2\theta_0-\Delta_0$			2.7792	3.5224_n	
$\eta_0\,\eta'$	$\phi-5\Gamma+2\theta_0+3\Delta_0$			2.2738_n	2.847	
	$\phi-4\Gamma+2\theta_0+3\Delta_0$			2.116_n	3.0290	
	$\phi-3\Gamma+2\theta_0+3\Delta_0$			2.5858_n	3.3787	
	$\phi-2\Gamma+2\theta_0+3\Delta_0$			2.809_n	3.5429	
	$\phi-\Gamma+2\theta_0+3\Delta_0$			2.650_n	3.7297	
	$\phi+2\theta_0+3\Delta_0$	9.98	0.60_n	2.873_n	[2.685]	3.7980
	$\phi+\Gamma+2\theta_0+3\Delta_0$			3.5126_n	4.2856	
	$\phi+2\Gamma+2\theta_0+3\Delta_0$		9.46_n	3.3438_n	4.1208	
η'^2	$\phi-5\Gamma+6\theta_0+4\Delta_0$			1.9950	2.7422_n	
	$\phi-4\Gamma+6\theta_0+4\Delta_0$			2.6112	3.1949_n	
	$\phi-3\Gamma+6\theta_0+4\Delta_0$			3.0556	3.5583_n	
	$\phi-2\Gamma+6\theta_0+4\Delta_0$			3.7934	3.7947_n	
	$\phi-\Gamma+6\theta_0+4\Delta_0$			4.2260	4.4064	
	$\phi+6\theta_0+4\Delta_0$	9.98_n	0.76_n	3.5017_n	4.1098	4.3552_n
	$\phi+\Gamma+6\theta_0+4\Delta_0$			4.2852_n	5.3521	
	$\phi+2\Gamma+6\theta_0+4\Delta_0$			3.9567_n	4.9249	
η'^2	$\phi-5\Gamma+2\theta_0+2\Delta_0$			2.5018	3.0963_n	
	$\phi-4\Gamma+2\theta_0+2\Delta_0$			2.453	3.0935_n	
	$\phi-3\Gamma+2\theta_0+2\Delta_0$			2.4799	3.2779_n	
	$\phi-2\Gamma+2\theta_0+2\Delta_0$			2.9375	3.6294_n	
	$\phi-\Gamma+2\theta_0+2\Delta_0$			3.2833	3.8982_n	
	$\phi+2\theta_0+2\Delta_0$	0.025_n	0.60	2.634	3.2781	4.0439_n
	$\phi+\Gamma+2\theta_0+2\Delta_0$				3.5607	4.2381_n
	$\phi+2\Gamma+2\theta_0+2\Delta_0$				3.4629	4.1704_n

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TABLE LVII—Continued.

Logarithmic $S \sin \phi + C \cos \phi$ Unit=1″.

	Cos	w^{-3}	w^{-2}	w^{-1}	w^0	w	w^2
η'^2	$\phi-5\Gamma-2\theta_0$				3.0090_n	3.7477	
	$\phi-4\Gamma-2\theta_0$				3.0676_n	3.7445	
	$\phi-3\Gamma-2\theta_0$				3.0764_n	3.6664	
	$\phi-2\Gamma-2\theta_0$				2.958_n	3.3121	
	$\phi-\Gamma-2\theta_0$				3.1140	4.0201_n	
	$\phi\quad-2\theta_0$	0.305	1.127_n	2.912	3.5491_n	3.9085	
	$\phi+\Gamma-2\theta_0$				3.0396_n	3.6320	
	$\phi+2\Gamma-2\theta_0$				2.4706_n	3.2330	
j^2	$\phi-5\Gamma+6\theta_0+5\Delta_0-\Sigma_0$				2.006	2.7505_n	
	$\phi-4\Gamma+6\theta_0+5\Delta_0-\Sigma_0$				2.335	2.981_n	
	$\phi-3\Gamma+6\theta_0+5\Delta_0-\Sigma_0$				2.544	3.1436_n	
	$\phi-2\Gamma+6\theta_0+5\Delta_0-\Sigma_0$				2.718	3.2445_n	
	$\phi-\Gamma+6\theta_0+5\Delta_0-\Sigma_0$				2.970	2.9116_n	
	$\phi\quad+6\theta_0+5\Delta_0-\Sigma_0$	8.6_n	9.7	2.114_n	2.923	3.4067_n	
	$\phi+\Gamma+6\theta_0+5\Delta_0-\Sigma_0$				2.7948_n	3.9420	
	$\phi+2\Gamma+6\theta_0+5\Delta_0-\Sigma_0$				2.3824_n	3.4488	
j^2	$\phi-5\Gamma+2\theta_0+2\Delta_0$				9.6	2.387	
	$\phi-4\Gamma+2\theta_0+2\Delta_0$				1.916_n	2.911	
	$\phi-3\Gamma+2\theta_0+2\Delta_0$				2.5178_n	3.3047	
	$\phi-2\Gamma+2\theta_0+2\Delta_0$				2.938_n	3.6294	
	$\phi-\Gamma+2\theta_0+2\Delta_0$				3.3406_n	3.9330	
	$\phi\quad+2\theta_0+2\Delta_0$		0.5910	3.1266	3.8021_n	4.1894	
	$\phi+\Gamma+2\theta_0+2\Delta_0$				3.4070	4.3178_n	
	$\phi+2\Gamma+2\theta_0+2\Delta_0$				3.0472	3.9308_n	
j^2	$\phi-5\Gamma-2\theta_0-\Delta_0+\Sigma_0$				0.732_n	1.085	
	$\phi-4\Gamma-2\theta_0-\Delta_0+\Sigma_0$				0.35	1.895_n	
	$\phi-3\Gamma-2\theta_0-\Delta_0+\Sigma_0$				1.463	2.5146_n	
	$\phi-2\Gamma-2\theta_0-\Delta_0+\Sigma_0$				2.064	3.0255_n	
	$\phi-\Gamma-2\theta_0-\Delta_0+\Sigma_0$				2.6816	3.6280_n	
	$\phi\quad-2\theta_0-\Delta_0+\Sigma_0$	9.04	0.11_n	2.636	3.3284_n	3.7399	
	$\phi+\Gamma-2\theta_0-\Delta_0+\Sigma_0$				3.0572_n	3.6430	
	$\phi+2\Gamma-2\theta_0-\Delta_0+\Sigma_0$				2.9121_n	3.5491	
η_0^2	$\phi\quad+4\theta_0+4\Delta_0$	0.775	1.65_n	3.1052_n	3.0342_n		
	$\phi\quad-4\theta_0-4\Delta_0$	0.29_n	1.10	3.1888	3.6104_n		
	$\phi\quad+8\theta_0+8\Delta_0$	0.65	1.54_n	3.7520	4.5812_n		
$\eta_0^2\eta'$	$\phi\quad+4\theta_0+5\Delta_0$			3.7577	4.3244_n		
	$\phi\quad+4\theta_0+3\Delta_0$	1.260_n	2.081	3.1240	4.1388		
	$\phi\quad-4\theta_0-3\Delta_0$	1.005	1.77_n	3.5356_n	3.3560		
	$\phi\quad+8\theta_0+7\Delta_0$	1.228_n	2.093	4.3980_n	5.1827		
$\eta_0\,\eta'^2$	$\phi\quad+4\theta_0+4\Delta_0$			4.1155$_n$	4.5547		
	$\phi\quad+4\theta_0+2\Delta_0$	1.106	1.88_n	2.831	4.1803_n		
	$\phi\quad-4\theta_0-2\Delta_0$	1.146_n	1.88	3.0422	4.0180		
	$\phi\quad+8\theta_0+6\Delta_0$	1.321	2.152_n	4.5658	5.3010_n		
η'^3	$\phi\quad+4\theta_0+3\Delta_0$			3.8375	4.0446_n		
	$\phi\quad-4\theta_0-\Delta_0$			3.2197	3.9650_n		
	$\phi\quad+8\theta_0+5\Delta_0$			4.2553_n	4.9349		
$j^2\,\eta_0$	$\phi\quad+4\theta_0+3\Delta_0-\Sigma_0$			3.0024	3.8634_n		
	$\phi\quad-4\theta_0-3\Delta_0+\Sigma_0$	9.98_n	0.8	2.956$_n$	3.8331		
	$\phi\quad+8\theta_0+7\Delta_0-\Sigma_0$			3.0757	3.9759_n		
	$\phi\quad+4\theta_0+4\Delta_0$	0.46_n	1.32	3.8514_n	4.6436		
$j^2\quad\eta'$	$\phi\quad+4\theta_0+4\Delta_0-\Sigma_0$			2.442	1.846_n		
	$\phi\quad-4\theta_0-2\Delta_0+\Sigma_0$			3.2486	4.0585_n		
	$\phi\quad+8\theta_0+6\Delta_0-\Sigma_0$			3.2818_n	4.1441		
	$\phi\quad+4\theta_0+3\Delta_0$	0.27	1.15_n	3.9421	4.6972_n		

$S \sin \phi + C \cos \phi = \Sigma C_1 w^a \eta^p \eta'^q j^{2t} \cos \text{Arg}$, where C_1 represents the coefficient.

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COMPARISON OF TABLES.

Table LVI.—Unless there are errors of calculation, all the discrepancies are due to the accumulation of other discrepancies already discussed. Without going into the details of the construction, it is sufficient to remark that our table is built from practically all of the available auxiliary material. Our table includes many more terms than v. Zeipel's table, but it is wanting in the two arguments 6Γ and 8Γ in the first block of terms. These arguments contain 3ε and 4ε, respectively, and our series were not inclusive of these higher multiples. It would be more consistent to include them, since the argument 7Γ is included.

Table LVII.—Unless there are errors of calculation, all the discrepancies are due to the accumulation of discrepancies already discussed. Our table is built from practically all the available auxiliary material. Large disagreements are to be explained by v. Zeipel's use of the formula following Z 131, equation (244). In this equation the following functions are omitted:

$$-\tfrac{1}{2}(y_2' \cos\psi + z_2' \sin\psi) - \tfrac{1}{2}([y_2]\cos\psi + [z_2]\sin\psi).$$

ERRATA[1] IN H. v. ZEIPEL, ANGENÄHERTE JUPITERSTÖRUNGEN FÜR DIE HECUBA-GRUPPE.

With the exception of § 6, Störungen des Radius-vector, all the developments have been checked.

Page.	Line.[2]	For—	Read—
1	8a	$\frac{d\Omega}{dx}$	$\frac{\partial\Omega}{\partial x}$
1	9a	$\frac{d\Omega}{dy}$	$\frac{\partial\Omega}{\partial y}$
3ff[3]		ν	$\frac{d\nu}{ds}$
5	5b	(15)	(16)
9	2a	$\frac{\partial U}{\partial\psi}$	$\frac{\partial U}{\partial\psi}$
9	1b	$\omega+(1-\omega)\frac{W+\nu^2}{1+W}$	$w+(1-w)\frac{W+\nu^2}{1+W}$
12	2a	$\left(\frac{a}{\Delta}\right)^s$	$\left(\frac{a}{\Delta_0}\right)^s$
12	9a	β_{n+i}^{2i+1}	$\beta_{n+i}^{(2i+1)}$
12	10a	β_p^q	$\beta_p^{(q)}$
12	6b	$(2n+4i+1)\gamma_i^{1,n}(4i+4)\gamma_{i+1}^{1,n}$	$(2n+4i+1)\gamma_i^{1,n}+(4i+4)\gamma_{i+1}^{1,n}$
13	5a	$(2n+4i+3)\gamma_i^{2,n}(4i+4)\gamma_{i+1}^{2,n}$	$(2n+4i+1)\gamma_i^{2,n}+(4i+4)\gamma_{i+1}^{2,n}$
14	2b	$n'g'$	ng'
15	8bff	Σ	$\sum_{n=\infty}^{n=s}$
16	10a	sin	cos
16	6b	$\frac{1}{i}a_2\frac{\partial\Omega}{\partial z}$	$\frac{1}{i}a^2\frac{\partial\Omega}{\partial z}$
19	6a	$\frac{dF}{da_0}$	$\frac{dF}{d\alpha_0}$
20	5b	$\gamma_{i-1}^{3,n}$	$\bar{\gamma}_{i-1}^{2,n}$
21	4a	$\bar{\delta}_i^{m,n}$	$\bar{\delta}_i^{2,n}$
21	4b	Metoden	Methoden
24	9b	$\Sigma P^i{}_{p,q}$	$-\Sigma P^i{}_{p,q}$
27	21a	$P_{0,0}[n-1.-n+1]_{-s}$	$P_{0,0}[n-1.-n+1]_{-s}$
34	21a	$P_{0,0}(n-1.-n+1)_{-s}$	$P_{0,0}(n-1.-n+1)_{-s}$
42	5b	$P_{0,2}(n.-n)$	$F_{0,2}(n.-n)$
44	18b	$H_{1,1}(n+1.-n+1)$	$H_{1,1}(n+1.-n-1)$
45	20a	$_{1,0}(n.-2-n-1)_{-s}$	$_{1,0}(n-2.-n-1)_{-s}$
46	8a	H	G
46	10a	$_{1,0}(n-1\cdot-n+2)_{-\delta}$	$_{0,1}(n-1\cdot-n+2)_{-\delta}$
46	11a	$_{1,0}(n-1\cdot-n)_{+\delta}$	$_{0,1}(n-1\cdot-n)_{-\delta}$

[1] Inclusive of those tabulated by v. Zeipel.
[2] The number of the line counting from the top of the page is indicated by a, counting from the bottom of the page by b.
[3] On page 3 and all following pages ν' is defined by $\nu'=\frac{d\nu}{ds}$. The error consists in the omission of a statement announcing a change of notation. See definition of ν' given on page 2.

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Errata in H. v. Zeipel, Angenäherte Jupiterstörungen für die Hecuba-Gruppe—Continued.

Page.	Line.[1]	For—	Read—
49	7b	$\frac{\partial\Omega}{r\partial\varepsilon}$	$\frac{\partial\Omega}{\partial r}$
50	6b	$\frac{r^2}{a^2}$	$\frac{r^2}{a^2}$
50	6b	$3+\eta^2$	$3+14\eta^2$
51	1b	$S_{0\cdot1}(n.-n+1)$	$S_{0\cdot1}(n.-n-1)$
53	11b	Σ	Σ
54	5a	Σ_t	Σ_t
56	4b	$H_{1\cdot1}(n-1\cdot n-1)$	$H_{1\cdot1}(n-1\cdot-n-1)$
61	11b	$2\theta+2\theta$	$2\theta+2\Delta$
62	17a	$\psi+6\theta+40$	$\psi+6\theta+4\Delta$
62	5b	$+436$	$+439$
63	9a	$[(1-e\cos\varepsilon)W'_2]$	$[(1-e\cos\varepsilon)\overline{W}_2']$
65	3a	(106)	(106a)
65	5a	(106)	(106a)
68	3a	$w^{-4}\,w^{-3}\,w^{-2}\,w^{-1}\,w$	$w^{-4}\,w^{-3}\,w^{-2}\,w^{-1}\,w\,w^2$
69	6b	$\sin A$	$\eta^p\eta'^q j^{2t}\sin A$
69	5b	$\sin(A-\psi+\varepsilon)$	$\eta^p\eta'^q j^{2t}\sin(A-\psi+\varepsilon)$
69	4b	$\sin(A+\psi-\varepsilon)$	$\eta^p\eta'^q j^{2t}\sin(A+\psi-\varepsilon)$
70	1b	$\overline{W}_4'''$	$\overline{W}_4''$
70	1b	$\cos A$	$\eta^p\eta'^q j^{2t}\cos A$
71	7a	$\cos A$	$\eta^p\eta'^q j^{2t}\cos A$
75	15a	$A_{0\cdot3}$	$A_{0\cdot2}$
75	18a	$A_{0\cdot1}$	$A_{0\cdot0}$
75	2b	$A_{1\cdot0}$	$A_{0\cdot1}$
75	1b	$A_{1\cdot0}$	$A_{0\cdot1}$
79	10b	θ	θ_4
81	8b	$1-e\cos\varepsilon)$	$(1-e\cos\varepsilon)$
83	12a	$+3744$	$+3344$
86	4a	(128_3) und (130)	(128_2), (128_3) und (130)
86	6a	$\frac{\partial\overline{W}_1}{\partial\delta}$	$\frac{\partial\overline{W}_1}{\partial\vartheta}$
91	9a	$e\cos\varepsilon$	$e\cos\varepsilon$
91	11a	W'_2	$\overline{W}_2'$
92	3a	$\eta_1\cos\varepsilon$	$y_1\cos\varepsilon$
92	10b	$-\frac{1}{2}(1-e\cos\varepsilon)\,(\overline{W}-\frac{1}{3}\Sigma)\,(\overline{W}+\frac{1}{3}\Sigma)$	$-\frac{1}{2}[(1-e\cos\varepsilon)\,(\overline{W}-\frac{1}{3}\Sigma)\,(\overline{W}+\frac{1}{3}\Sigma)]$
92	4b	$\frac{\partial W}{\partial\delta}$	$\frac{\partial\overline{W}}{\partial\vartheta}$
93	10a	$\sin A$	$\eta^p\eta'^q j^{2t}\sin A$
93	10a	Σ'	Σ
94	19b	$\frac{\partial W}{\partial\delta}$	$\frac{\partial\overline{W}}{\partial\vartheta}$
97	15a	(156)	(154)
99	4b	$-\eta w\sin\varepsilon)$	$-\eta w\sin\varepsilon\}$
100	5a	A'	A
100	6a	$-\frac{1}{2}u_2$	$-\frac{1}{2}\bar{u}_2$
115	4b	(191)	(192)
116	7a	(195_1)	(195_{-1})
116	10b	$\frac{\partial}{\partial\varepsilon}(S_1-[S_1])$	$\frac{\partial}{\partial\varepsilon}(S_1-[S_1])$
119	(1)	$U_{q\cdot q}$	$U_{p\cdot q}$
122	3a	$\nu_0-\pi$	$v_0-\pi$
123	4b	$(1-\xi'^2-\xi'^2)$	$(1-\xi^2-\xi'^2)$
125	3a	$1-e_0\cos\varepsilon_0^2$	$1-e_0\cos\varepsilon_0$
128	7a	Δ^0	Δ_0
129	5a	μ	p
131	7b	$(\psi+A-\varepsilon)$	$(\psi-A-\varepsilon)$
131	6b	$(\psi+A+\varepsilon)$	$(\psi+A-\varepsilon)$
132	8a	2.9227_m	1.9227_m
132	26a	5.3376	5.0376
134	9a	$(2\vartheta+f_4)$	$(4\vartheta+f_4)$
135	10a	$\frac{w}{2}$	$\frac{w}{4}$
135	11a	$\frac{w}{2}$	$\frac{w}{4}$
140	26a	$[n\delta z]$	$[n\delta z]_1$
141	6a	0.'8998	0.'8998

[1] The number of the line counting from the top of the page is indicated by a, counting from the bottom of the page by b.

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ERRATA IN KARL BOHLIN, SUR LE DÉVELOPPEMENT DES PERTURBATIONS PLANÉTAIRES, § 1–7, AND TABLES I–XX.

Page.	Line.[1]	For—	Read—
3	5a	$\mu^3=(1+m)\alpha^2$	$\mu^2=(1+m)\,a^3$
11	1b	$\frac{\partial W}{\partial \zeta}$	$\frac{\partial W}{\partial \zeta}$
14	11a	$1+\nu^2$	$1-\nu^2$
20	3a	$+\frac{1}{2}e^2\cos 2\varepsilon$	$-\frac{1}{2}e^2\cos 2\varepsilon$
29	8b	y'^n	y'^{-n}
29	2b	$e^{\sqrt{-1}f'}$	$e^{-\sqrt{-1}f'}$
30	11a	$\frac{a}{r'}$	$\frac{a'}{r'}$
30	11a	$-\frac{e'}{\kappa'}$	$+\frac{e'}{\kappa'}$
30	12a	$2n+m-1$	$2n+m+1$
30	13a	$-\left(\frac{e'}{\kappa'}\right)^2$	$+\left(\frac{e'}{\kappa'}\right)^2$
30	13a	$2n+m-1$	$2n+m+1$
30	13a	$2n+m-2$	$2n+m+2$
30	14a	$2n+m-1$	$2n+m+1$
30	5b	$e^{\sqrt{-1}n(\tau-\tau')}$	$e^{\sqrt{-1}n(\tau-\tau')}$
33	9a	$K_{1.0}^{2i+s}(n-1.\ n)$	$K_{1.0}^{2i+s}(n-1.-n)$
35	4a	(73)	(74)
36	1a	$2\Gamma_s^{s\cdot n}e^{\sqrt{-1}n(\tau-\tau)}$	$2\Gamma_i^{s\cdot n}e^{\sqrt{-1}n(\tau-\tau')}$
36	10a	$\overline{K}_{1\cdot1}(n-1.+n+1)$	$\overline{K}_{1\cdot1}(n-1.-n+1)$
38	13a	a	α
38	10b	$e^{-\sqrt{-1}\tau-\tau')}$	$e^{-\sqrt{-1}(\tau-\tau')}$
38	3b	$2(\eta')y'^{-1}$	$2(\eta')y^{-1}$
38	2b	$2\eta)y'^{-1}$	$2(\eta)y'^{-1}$
40	3a	$K_{1\cdot0}(n.-1-n)$	$K_{1\cdot0}(n-1.-n)$
41	13a	α	a
45	2a	$K_{0\cdot0}(0.-n)$	$K_{0\cdot0}(n.-n)$
45	9b	à	
46	7a	$-\frac{r(r-1)^2x^{r-r'}}{1.1.2}$	$+\frac{r(r-1)^2x^{r-1}}{1.1.2}$
46	7b	$-\frac{(n-s)(n-s+2)}{2}\eta'^2x'^{-2}$	$+\frac{(n-s)(n-s+2)}{2}\eta'^2x'^{-2}$
46	5b	$(n-3)$	$(n-s)$
46	4b	η^4	η'^4
48	14a	$P^1_{210}(n-2.-n)$	$P^1_{2\cdot0}(n-2.-n)$
48	7b	$P^1_{1\cdot0}\|n+1.-n-2\|$	$P^1_{1\cdot2}\|n+1.-n-2\|$
48	7b	$P^1_{1\cdot1}(n+1.-n-2)$	$P^1_{1\cdot2}(n+1.-n-2)$
48	6b	$P^1_{1\cdot1}\|n-1.-n-2\|$	$P^1_{1\cdot2}\|n-1.-n-2\|$
50	5a	$R^1_{1\cdot0}(n+1.-n-1)-\varepsilon$	$R^1_{1\cdot0}(n+1.-n-1)-\varepsilon'$
50	9b	$R^1_{1\cdot0}(n-1.-n+1)+\varepsilon'$	$R^1_{1\cdot0}(n-1.-n+1)+\varepsilon'$
50	3b	$R^1_{0\cdot0}(n.-n+2)+\varepsilon'$	$R^1_{0\cdot1}(n.-n+2)+\varepsilon'$
51	1b	$P^1(n+r.-n+s\ \|$	$P^1\|n+r.-n+s\|$
59	5a	$Q^{3\cdot1}_{1\cdot0}[n+1.-n]$	$Q^{3\cdot1}_{1\cdot0}[n-1.-n]$
59	8a	$Q^{3,1}_{0\cdot1}[n.-n+1]$	$Q^{3\cdot1}_{0\cdot1}[n.-n+1]$
60	6a	g'	g'
60	8a	(See footnote [2])	
60	9b	$\frac{-(m\mu)^4}{24}$	$\frac{+(m\mu)^4}{24}$
60	5b	$y^{(-n+s)}$	$y^{(-n+s)\mu}$
61	7b	$P_{0\cdot1}[n.+n+1]$	$P_{0\cdot1}[n.-n+1]$
61	5b	$P_{0\cdot1}[n.+n-1]$	$P_{0\cdot1}[n.-n-1]$
62	1a	$\frac{n^2\mu^3}{6}$	$\frac{n^3\mu^3}{6}$
62	7a	$P_{2\cdot1}(n+2.-n-1)$	$P_{2\cdot1}(n+2.-n+1)$
62	7a	$\frac{(n-2)^2\mu^2}{2}$	$\frac{(n-1)^2\mu^2}{2}$
62	8a	$P_{1\cdot1}[n-1.-n+1]$	$P_{1\cdot1}[n+1.-n+1]$
63	5a	$P_{0\cdot0}(n-1.-n+1)+\vartheta$	$P_{0\cdot0}(n-1.-n+1)-\vartheta$
63	11a	$P_{1\cdot0}(n+2.-n-1)+\sigma$	$P_{1\cdot0}(n+2.-n-1)+\vartheta$
63	13a	$P_{0\cdot0}[n-1.-n-1]-\vartheta$	$P_{0\cdot0}[n-1.-n+1]-\vartheta$
63	14a	$P_{0\cdot0}[n-1.-n-1]-\vartheta$	$P_{0\cdot0}[n-1.-n+1]-\vartheta$
63	5b	$R_{0\cdot0}[n.-n+1]-\varepsilon'$	$R_{0\cdot0}[n.-n-1]-\varepsilon'$
63	2b	$R_{0\cdot0}[n.-u-1]-\varepsilon'$	$R_{0\cdot0}[n.-n-1]-\varepsilon'$
63	1b	$R_{0\cdot0}[n._n-1]-\varepsilon'$	$R_{0\cdot0}[n.-n-1]-\varepsilon'$

[1] The number of the line counting from the top of the page is indicated by a, counting from the bottom of the page by b.
[2] The argument θ is defined first by eq. (31), p. 20, secondly by eq. (105), p. 60. The first of these definitions is used in § 8.

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Errata in Karl Bohlin, Sur le Développement des Perturbations Planétaires, § 1-7, and Tables I-XX—Continued.

Page.	Line.[1]	For—	Read—
64	6a	$R_{2\cdot0}[n+1.-n]+\pi'$	$R_{2\cdot0}[n.-n+1]+\pi'$
64	12a	$R_{0\cdot0}[u.-u-1]-\pi'$	$R_{0\cdot0}[n.-n-1]-\pi'$
64	14a	$(n-n)$	$(n-2)$
64	15a	$R_{1\cdot1}[n+1.-n]-\pi'$	$R_{1\cdot1}[n+1.-n]+\pi'$
66	7a	$F(n+r.-n+s$	$F(n+r.-n+s)$
66	8a	$G(n+r.-n+s$	$G(n+r.-n+s)$
70	1a	$+s$ $-s\ P_{1\cdot2}(n+1.-n-2)$ -2	$+s$ $-2\ P_{1\cdot2}(n+1.-n-2)$ -2
71	4a	$F_{1\cdot0}(n.\ n+1)+\sigma$ $+s$	$F_{1\cdot0}(n.-n+1)+\sigma$ $+s$
71	9b	$-2\ P_{1.0}(n.-n+1)-\vartheta$ -2	$-2\ P_{1\cdot0}(n.-n+1)-\vartheta$ -2
73	2a	$F_{2\cdot0}(n.-n+1)-\pi'$	$F_{2\cdot0}(n.-n-1)-\pi'$
73	18a, ff.	See foot note.[2]	j^2
73	4b	$R_{0\cdot0}(n.-n+1)+\pi$	$R_{0\cdot0}(n.-n+1)+\pi'$
73	3b	$R_{0\cdot0}(n.-n+1)+\pi$	$R_{0\cdot0}(n.-n+1)+\pi'$
74	8a	$R_{0\cdot0}n.-n+1)+\pi'$	$R_{0\cdot0}(n.-n+1)+\pi'$
75	1a	$G_{0\cdot0}(n+1.-n)$	$G_{0\cdot0}(n+1.-n)+\sigma+\pi'$
75	11b	$R_{0\cdot0}(n.-n+1)-\pi'$	$R_{0\cdot0}(n.-n-1)-\pi'$
78	1b	$\gamma_0^{1\cdot n}$	$\gamma_1^{1\cdot n}$
79	1b	$\gamma_i^{m+2\cdot n}$	$\gamma_{i-1}^{m+2\cdot n}$
79	*)	$n=1$	$n=0$
80	9b	$\gamma_{i+1}^{m\cdot n}$	$=\gamma_{i+1}^{m\cdot n}$
81	12a	*)	
81	13a	(120)	(120)*)
135	7a	$K_{0\cdot2}^{1\cdot i}$	$K_{0\cdot2}^{1\cdot i}$
139	3a	$\Gamma_i^{1\cdot n}$	$2\Gamma_i^{1\cdot n}$
140	2a	$\Gamma_i^{1\cdot n}$	$2\Gamma_i^{1\cdot n}$
154	1a	(86)	(93)
161	1a	$-ar$	ar
169	8b	»	$\frac{3}{4a^2}2\Gamma_i^{3\cdot n}$
170	3a	$\frac{1}{2a}2\Gamma_i^{2\cdot n}$	$2\gamma_i^{1\cdot n}$
170	4b	»	$\frac{1}{2a}2\gamma_i^{2\cdot n}$
171 ff.		See foot note.[3]	
185	2a	3. 27886	3. 27887
185	13b	4	3
188	6b	$2.017\ 3_n$	2.01703_n
189	14a	3. 27886	3. 27887
197	16a	0.146128_n	1.146128_n
197	18b	1. 505151	1. 505150
198	15b	1.662759_n	1.662758_n
198	2b	0. 477121	0.477121_n

[1] The number of the line counting from the top of the page is indicated by a, counting from the bottom of the page by b.
[2] The space between lines 18 and 19 should read j^3.
[3] Tables XII, XIII, XIV give the same coefficients in numbers as Tables XVI, XVII, XVIII give in logarithms, respectively. The same factor should therefore occur in the former.

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